Lecture 7
Basic Sensitivity Analysis

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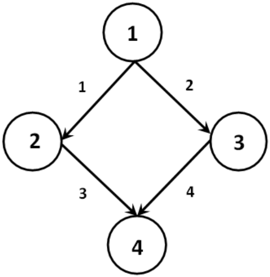
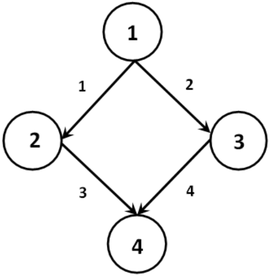
Professor Braess's visit to UMass, Spring 2006

http://supernet.som.umass.edu/cfoto/braess-visit/braessvisit.html

Basic Sensitivity Analysis Illustration via Braess Paradox



$$w_1 = (1, 4)$$
$$d_{w_1} = 6$$



User Cost Functions:

$c_1(f_1) = 10f_1$ $\quad$ $c_2(f_2) = f_2 + 50$

$c_3(f_3) = f_3 + 50$ $\quad$ $c_4(f_4) = 10f_4$

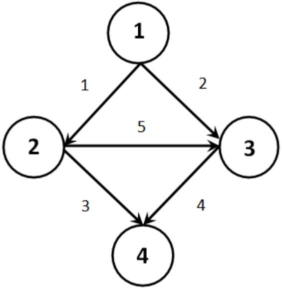
User-Optimized Solution

$F^*_{p_1} = F^*_{p_2} = 3$

$C_{p_1} = C_{p_2} = 83$

$p_1 = (1, 3), \; p_2 = (2, 4)$

Add now a new link, link 5:



with associated user link cost: $c_5(f_5) = f_5 + 10$.
Now we also have a new path: $p_3 = (1, 5, 4)$.

New User-Optimized Solution:

$F^*_{p_1} = F^*_{p_2} = F^*_{p_3} = 2$
with associated path travel times: $C_{p_1} = C_{p_2} = C_{p_3} = 92$!

Observe: Travel Cost has increased for all users of the network.

Note:

The addition of a new path on a network may: increase, decrease, or leave unchanged the equilibrium (U-O) travel path costs.

In the case of S-O solution, the addition of a new path can never increase the total system cost in the network.

Hence from the system point of view, the network is "improved" or at least not worsened.

Basic Sensitivity Analysis

Question:

Can you design a new path connecting O/D pair so that the travelers can never be worsened off, from a U-O perspective?

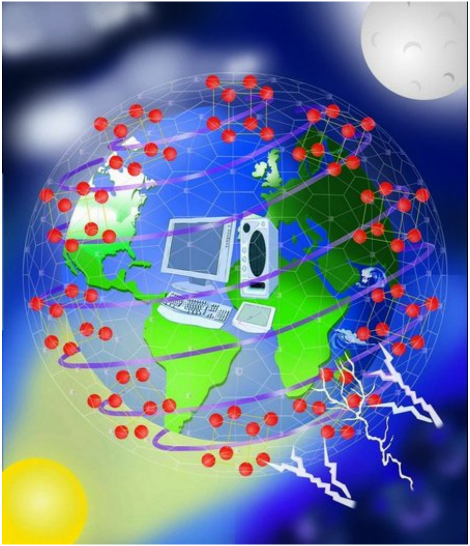
Basic Sensitivity Analysis

What can we say about the effect on users', travelers' costs with respect to:

- ◇ an increase in travel demand?
- ◇ a decrease in travel demand?
- ◇ an increase in the link cost function?
- ◇ a decrease in the link cost function?

Basic Sensitivity Analysis

The Braess paradox occurs not only in transportation networks but also in other networks where there is "selfish" type of behavior, such as the Internet.



Internet Traffic

www.dailygalaxy.com

References

⇒ Braess D., Nagurney A. and Wakolbinger T. (2005), On a Paradox of Traffic Planning, *Transportation Science*, Vol. 39, No. 4, November 2005, pp. 446-450

http://homepage.rub.de/Dietrich.Braess/Paradox-BNW.pdf

∗ Original article in German: Braess, D. (1968), ber ein Paradoxon aus der Verkehrsplanung, *Unternehmensforschung*, 12, pp. 258-268

http://homepage.ruhr-uni-bochum.de/Dietrich.Braess/paradox.pdf

∗ Additional information on Professor Braess's visit to UMass (Spring 2006) in the Supernetworks website:

http://supernet.som.umass.edu/cfoto/braess-visit/braessvisit.html

References (continued)

⇒ Nagurney A. (1999), *Network Economics: A Variational Inequality Approach, Revised Second Edition*, Springer

http://supernet.som.umass.edu/bookser/netbook.htm

⇒ Nagurney A. and Dong J. (2002), *Supernetworks: Decision-Making for the Information Age*, Edward Elgar Publishing

http://supernet.som.umass.edu/bookser/supbook.html

⇒ Roughgarden T. (2005), *Selfish Routing and the Price of Anarchy*, MIT Press

Additional Reading

For more advanced formulations and associated theory, see Professor Nagurney's Fulbright Network Economics lectures.

http://supernet.som.umass.edu/austria_lectures/fulmain.html