

Lecture 6

Equilibration Algorithms for a General Transportation Network

Dr. Anna Nagurney

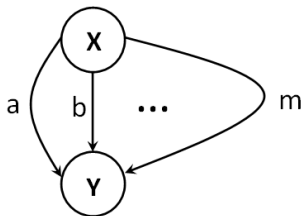
John F. Smith Memorial Professor
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

Equilibration Algorithms for a General Transportation Network

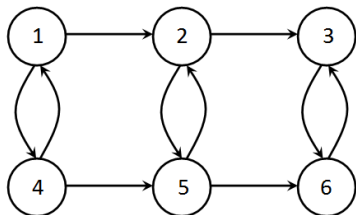
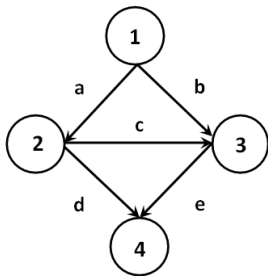
Assume that the user cost functions on the links are still linear and increasing, that is,

$$c_a(f_a) = g_a f_a + h_a, \quad g_a, h_a > 0$$

but now the transportation network need no longer look like this:



It can have any topology:



Equilibration Algorithms for a General Transportation Network

A General Equilibration Algorithm (U-O)

1. Initialization

Initialize with a feasible link and path flow pattern. Set $i = 1$.

2. Equilibration of O/D pair w_i :

Set

$r =$ path p with $F_p > 0$ and maximum C_p in w_i

$q =$ path p with minimum C_p in w_i

Compute reallocation of path flow

$$\delta' = \frac{C_r(f) - C_q(f)}{\sum_a g_a (\delta_{aq} - \delta_{ar})^2}$$

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The reallocation : $\delta = \min\{\delta', F_r\}$.

Update: new $F_q = F_q + \delta$

new $F_r = F_r - \delta$

All other F_p 's stay the same.

Continue Step 2 until O/D pair w_i satisfies U-O conditions within a prespecified tolerance.

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3. Go to next O/D pair:

Set $i = i + 1$; If $i \leq J$, where $J = \#$ of O/D pairs, goto Step 2; else goto Step 4.

4. Overall Convergence Verification

If all O/D pairs are equilibrated then stop; the U-O pattern has been computed; else set $i = 1$, and goto Step 2.



Massachusetts Turnpike

Photo by: Central Artery / Tunnel Project [Massachusetts Turnpike Authority]

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A General Equilibration Algorithm (S-O)

1. Initialization

Initialize with a feasible link and path flow pattern. Set $i = 1$.

2. Equilibration of O/D pair w_i :

Set

$r =$ path p with $F_p > 0$ and maximum \hat{C}'_p in w_i

$q =$ path p with minimum \hat{C}'_p in w_i

Compute reallocation of path flow

$$\delta' = \frac{\hat{C}'_r(f) - \hat{C}'_q(f)}{2 \sum_a g_a (\delta_{aq} - \delta_{ar})^2}$$

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The reallocation : $\delta = \min\{\delta', F_r\}$.

Update: new $F_q = F_q + \delta$

new $F_r = F_r - \delta$

All other F_p 's stay the same.

Continue Step 2 until O/D pair w_i satisfies S-O conditions within a prespecified tolerance.

Equilibration Algorithms for a General Transportation Network

3. Go to next O/D pair:

Set $i = i + 1$; If $i \leq J$, where $J = \#$ of O/D pairs, goto Step 2; else goto Step 4.

4. Overall Convergence Verification

If all O/D pairs are equilibrated then stop; the S-O pattern has been computed; else set $i = 1$, and goto Step 2.

- ▶ Nagurney A. (1990) Equilibrium Modeling, Analysis and Computation: the Contributions of Stella Dafermos. *Operations Research*, Vol. 39, No. 1, Jan-Feb 1991, pp. 9-12
- ▶ Nagurney A. (2005) Supernetworks; Invited Chapter for the *Handbook of Optimization in Telecommunications*, Pardalos P. and Resende M., Editors, Springer-Verlag
- ▶ **[ADVANCED]** Florian M. and Hearn D. (1995) Network Equilibrium Models and Algorithms; Chapter 6 of *Network Routing*; Ball M. et al., *Handbooks in OR & MS*, Vol. 8