Knowledge Networks

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KNOWLEDGE NETWORKS

Knowledge networks as noted by Beckmann (1995) is a concept invented and utilized by Swedish economists in an atmosphere of growing international competition, which has led Sweden to focus on high technology industries which are knowledge intensive.

In today's *Network Economy*, the existence of highly skilled workers is essential for

- innovation;
- research and development;
- and for increasing the competitive position of regions and nations.

A Knowledge Network Equilibrium Model

We assume that there are two distinct types of workers, "knowledge" workers and "goods" workers.

A Firm's Production Function

Each firm's production function is composed of the product of the knowledge production function, denoted here by $g_i(D_i, G)$ for firm i and the conventional goods or commodity production function, denoted by $f_i(K_i, L_i)$ for firm i,

$$q_i = g_i(D_i, G) f_i(K_i, L_i), \tag{1}$$

where q_i denotes the quantity produced by firm i, D_i represents the capacity of the information systems, $G = (G_1, \ldots, G_m) \in R^m_+$ is the column vector of knowledge workers at the nodes, K_i denotes the amount of capital held by firm i, and L_i denotes the amount of goods workers at firm i.

The production function is quite general and can also incorporate different measures of *knowledge accessibility* which depend on the telecommunication and transportation networks.

For example, one may define a telecommunication accessibility measure, which we denote by TC_i for firm i, and which is defined as follows:

$$TC_i = \sum_{j=1}^{m} \sigma_1 f_{ij1} G_j^{\gamma_1}, \quad \forall i, \tag{2}$$

where σ_1 and γ_1 are parameters, $f_{ij1} = e^{-\beta d_{ij1}}$, where β denotes the distance friction associated with knowledge exchange across the telecommunication network between firms i and j and d_{ij1} is the distance between firms i and j.

Also, one may define a transportation accessibility measure, which we denote by TR_i for firm i, and which is defined as:

$$TR_i = \sum_{j=1}^{m} (\sigma_2 f_{ij2} W_j^{\gamma_2} + \sigma_3 f_{ij2} G_j^{\gamma_3}), \quad \forall i, \tag{3}$$

where σ_2, σ_3, γ_2, and γ_3 are parameters, W_j is the scale of firm j's public research & development units, assumed to be fixed at each node, and f_{ij2} is the distance friction for knowledge exchange on the transportation network between firms i and j.

Transportation distance plays an important role in impeding the movement of individuals for purposes of information and knowledge exchange. Distance, in terms of knowledge exchange on telecommunication networks, on the other hand, plays a less critical role.

We assume that the firms are perfectly competitive in that they take the price of the good produced as fixed, which we denote by $\bar{p}_i$ for firm i.

Let ω_i denote the rent of capital for firm i, θ_i – the wage rate for the goods workers at firm i, η – the rent of information systems, and ζ – the wage rate of the knowledge workers, which we assume to be uniform in this economy.

We assume also that the firms compete for the knowledge workers in a noncooperative fashion.

The utility function facing a firm i can be expressed as:

$$u_i(D_i, G, K_i, L_i) =$$

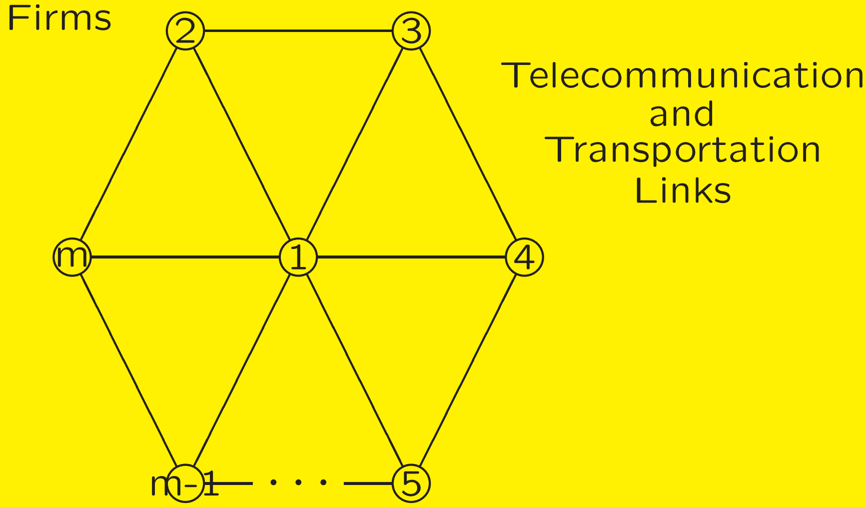
$$\bar{p}_i g_i(D_i, G) f(K_i, L_i) - \omega_i K_i - \theta_i L_i - \eta D_i - \zeta G_i. \quad (4)$$

Hence, the objective function facing such a profit-maximizing firm is:

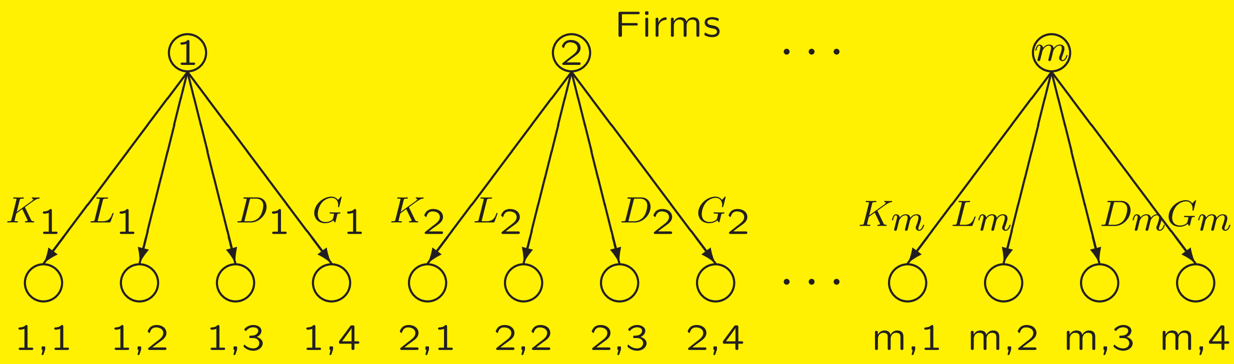
$$\text{Maximize} \quad u_i(D_i, G, K_i, L_i) \quad (5)$$

subject to:

$$D_i \geq 0, \quad G_i \geq 0, \quad K_i \geq 0, \quad L_i \geq 0. \quad (6)$$



Hypothetical knowledge network topology



Network structure of the firms' decisions

Definition 1 (Knowledge Network Equilibrium)

A knowledge network equilibrium is a vector of information system capacities, amounts of knowledge workers, capital, and goods workers $(D^*, G^*, K^*, L^*) \in R_+^{4m}$*:*

$$u_i(D_i^*, G_i^*, \hat{G}_i^*, K_i^*, L_i^*) \geq u_i(D_i, G_i, \hat{G}_i^*, K_i, L_i), \quad (7)$$

$$\forall D_i \in R_+, \quad \forall G_i \in R_+, \quad \forall K_i \in R_+, \quad \forall L_i \in R_+, \quad (8)$$

where $\hat{G}_i \equiv (G_1, \ldots, G_{i-1}, G_{i+1}, \ldots, G_m)$.

Theorem 1 (Variational Inequality Formulation)

Assume that each u_i is continously differentiable on R^4_+ and concave with respect to its arguments. Then (D^*, G^*, K^*, L^*) is a knowledge network equilibrium if and only if it satisfies the variational inequality problem

$$\sum_{i=1}^{m}(\eta - \bar{p}_i f_i(K_i^*, L_i^*)\frac{\partial g_i(D_i^*, G^*)}{\partial D_i}) \times (D_i - D_i^*)$$

$$+\sum_{i=1}^{m}(\zeta - \bar{p}_i f_i(K_i^*, L_i^*)\frac{\partial g_i(D_i^*, G^*)}{\partial G_i}) \times (G_i - G_i^*)$$

$$+\sum_{i=1}^{m}(\omega_i - \bar{p}_i g_i(D_i^*, G^*)\frac{\partial f_i(K_i^*, L_i^*)}{\partial K_i}) \times (K_i - K_i^*)$$

$$+\sum_{i=1}^{m}(\theta_i - \bar{p}_i g_i(D_i^*, G^*)\frac{\partial f_i(K_i^*, L_i^*)}{\partial L_i}) \times (L_i - L_i^*) \geq 0,$$

$$\forall (D, G, K, L) \in R^{4m}_+. \quad (9)$$

We may put variational inequality (9) into standard form. Let $x \equiv (D, G, K, L)$, and define $F(x)$ as the column vector with the first m components consisting of: $(\eta - \bar{p}_i f_i(K_i, L_i) \frac{\partial g_i(D_i,G)}{\partial D_i})$ for $i = 1, \ldots, m$, the second m components consisting of the terms: $(\zeta - \bar{p}_i f_i(K_i, L_i) \frac{\partial g_i(D_i,G)}{\partial G_i})$ for $i = 1, \ldots, m$, the next m components consisting of the terms: $(\omega_i - \bar{p}_i g_i(D_i, G) \frac{\partial f_i(K_i,L_i)}{\partial K_i})$ and, finally, the last m terms consisting of the terms: $(\theta_i - \bar{p}_i g_i(D_i, G) \frac{\partial f_i(K_i,L_i)}{\partial L_i})$ for $i = 1, \ldots, m$.

We further define the feasible set $\mathcal{K} \equiv R_+^{4m}$. Then variational inequality (9) may be expressed in standard form as:

Determine $x^* \in \mathcal{K}$, such that

$$\langle F(x^*)^T, x - x^* \rangle \geq 0, \quad \forall x \in \mathcal{K}. \tag{10}$$

We now provide an existence result. In particular, we have the following:

Theorem 2 (Existence)

Assume that the negative of the gradient of the utility function of each firm is coercive. Then there exists a solution to variational inequality (9).

We now state a uniqueness result.

Theorem 3 (Uniqueness)

Assume that the negative of the gradient of the utility functions is strictly monotone. Then there exists a unique equilibrium pattern (D^*, G^*, K^*, L^*).

Proof: Assume that there are two distinct solutions to variational inequality (9), denoted by (D^1, G^1, K^1, L^1) and (D^2, G^2, K^2, L^2), that is, we have that

$$\sum_{i=1}^{m}(\eta - \bar{p}_i f_i(K_i^1, L_i^1)\frac{\partial g_i(D_i^1, G^1)}{\partial D_i}) \times (D_i - D_i^1)$$

$$+\sum_{i=1}^{m}(\zeta - \bar{p}_i f_i(K_i^1, L_i^1)\frac{\partial g_i(D_i^1, G^1)}{\partial G_i}) \times (G_i - G_i^1)$$

$$+\sum_{i=1}^{m}(\omega_i - \bar{p}_i g_i(D_i^1, G^1)\frac{\partial f_i(K_i^1, L_i^1)}{\partial K_i}) \times (K_i - K_i^1)$$

$$+\sum_{i=1}^{m}(\theta_i - \bar{p}_i g_i(D_i^1, G^1)\frac{\partial f_i(K_i^1, L_i^1)}{\partial L_i}) \times (L_i - L_i^1) \geq 0, \quad (11)$$

$$\forall (D, G, K, L) \in R_+^{4m}$$

and

$$\sum_{i=1}^{m}(\eta - \bar{p}_i f_i(K_i^2, L_i^2)\frac{\partial g_i(D_i^2, G^2)}{\partial D_i}) \times (D_i - D_i^2)$$

$$+\sum_{i=1}^{m}(\zeta - \bar{p}_i f_i(K_i^2, L_i^2)\frac{\partial g_i(D_i^2, G^2)}{\partial G_i}) \times (G_i - G_i^2)$$

$$+\sum_{i=1}^{m}(\omega_i - \bar{p}_i g_i(D_i^2, G^2)\frac{\partial f_i(K_i^2, L_i^2)}{\partial K_i}) \times (K_i - K_i^2)$$

$$+\sum_{i=1}^{m}(\theta_i - \bar{p}_i g_i(D_i^2, G^2)\frac{\partial f_i(K_i^2, L_i^2)}{\partial L_i}) \times (L_i - L_i^2) \geq 0, \quad (12)$$

$$\forall (D, G, K, L) \in R_+^{4m}.$$

Let $(D, G, K, L) = (D^2, G^2, K^2, L^2)$ and substitute into (11). Similarly, let $(D, G, K, L) = (D^1, G^1, K^1, L^1)$ and substitute into (12). Adding the two resulting inequalities yields:

$$\sum_{i=1}^{m}(\bar{p}_i f_i(K_i^1, L_i^1)\frac{\partial g_i(D_i^1, G^1)}{\partial D_i} - \bar{p}_i f_i(K_i^2, l_i^2)\frac{\partial g_i(D_i^2, G^2)}{\partial D_2})$$

$$\times(D_i^1 - D_i^2)$$

$$+\sum_{i=1}^{m}(\bar{p}_i f_i(K_i^1, L_i^1)\frac{\partial g_i(D_i^1, G^1)}{\partial G_i} - \bar{p}_i f_i(K_i^2, L_i^2)\frac{\partial g_i(D_i^2, G^2)}{\partial G_i})$$

$$\times(G_i^1 - G_i^2)$$

$$+\sum_{i=1}^{m}(\bar{p}_i g_i(D_i^1, G^1)\frac{\partial f_i(K_i^1, L_i^1)}{\partial K_i} - \bar{p}_i G_i(D_i^2, G^2)\frac{\partial f_i(K_i^2, L_i^2)}{\partial K_i})$$

$$\times(K_i^1 - K_i^2)$$

$$+\sum_{i=1}^{m}(\bar{p}_i g_i(D_i^1, G^1)\frac{\partial f_i(K_i^1, L_i^1)}{\partial L_i} - \bar{p}_i g_i(D_i^2, G^2)\frac{\partial f_i(K_i^2, L_i^2)}{\partial L_i})$$

$$\times(L_i^1 - L_i^2) \qquad \geq 0, \tag{13}$$

which is in contradiction to the assumption of strict monotonicity of the negative of the gradient of the utility functions.

Hence, we must have that

$$(D^1, G^1, K^1, L^1) = (D^2, G^2, K^2, L^2)$$

We now provide some sensitivity analysis results.

Theorem 4

Assume that the negative of the gradient of the utility functions is monotone.

Consider a change $\Delta\omega_i$ to the rent of capital for firm i, keeping all other data as before. Let ΔK_i denote the subsequent change in the equilibrium amount of capital held by firm i. Then

$$\Delta\omega_i \times \Delta K_i \leq 0.$$

Similarly, consider a change $\Delta\theta_i$ to the wage rate for the goods workers at firm i, keeping all other data as before. Let ΔL_i denote the subsequent change in the equilibrium amount of goods workers at firm i. Then

$$\Delta\theta_i \times \Delta L_i \leq 0.$$

From these inequalities we have that if the rent of capital for firm i increases (decreases) then the amount of capital for firm i can not increase (decrease). Analogously, we have that if the wage rate for goods workers at firm i increases (decreases) then the amount of goods workers can not increase (decrease).

Theorem 5

Assume that the negative of the gradient of the utility functions is monotone.

Consider a change $\Delta\eta$ to the rent of information systems, keeping all other data as before. Let ΔD_i denote the subsequent change in the equilibrium capacity of information. Then

$$\Delta\eta \times \sum_{i=1}^{m} \Delta D_i \leq 0.$$

Similarly, consider a change $\Delta\zeta$ to the wage rate of knowledge workers, keeping all other data as before. Let ΔG_i denote the subsequent change in equilibrium amounts of knowledge workers. Then

$$\Delta\zeta \times \sum_{i=1}^{m} \Delta G_i \leq 0.$$

Hence: If the rent of information systems increases (decreases) then the total capacity of information in the network economy can not increase (decrease). If the wage rate of knowledge workers increases (decreases) then the total number of knowledge workers in the network economy can not increase (decrease).

For additional background on this subject, see Kobayashi (1995). This topic of application is relatively new and we can expect more research in this area in the future. Additional references are included below to further research on this topic.

References

Andersson, A. E., and Karlqvist, A., "Population and capital in geographical space. The problem of general equilibrium allocation," in **Computing Equilibria: How and Why**, pp. 183-195, J. Los and W. Los, editors, North-Holland, Amsterdam, The Netherlands, 1976.

Andersson, A. E., and Persson, O., "Networking scientists," *The Annals of Regional Science* **27** (1993) 11-21.

Batten, D. F., Kobayashi, K., and Andersson, A. E., "Knowledge, nodes, and networks: An analytical perspective," in **Knowledge and Industrial Organization**, pp. 31-46, A. E. Andersson, D. F. Batten, and D. Karlsson, editors, Springer-Verlag, Berlin, Germany, 1989.

Beckmann, M. J., "Knowledge networks. The case of scientific interaction at a distance," *The Annals of Regional Science* **27** (1993) 5-9.

Beckmann, M. J., "On knowledge networks in science: collaboration among equals," *The Annals of Regional Science* **28** (1994) 233-242.

Beckmann, M. J., "Economic models of knowledge networks," in **Networks in Action**, pp. 159-174, D. Batten, J. Casti, and R. Thord, editors, Springer-Verlag, Berlin, Germany, 1995.

Karlqvist, A., and Lundqvist, L., "A contract model for spatial allocation," *Regional Studies* **4** (1972) 401-419.

Kobayashi, K., "Knowledge network and market structure: An analytical perspective," in **Networks in Action**, pp. 127-158, D. Batten, J. Casti, and R. Thord, editors, Springer-Verlag, Berlin, Germany, 1995.

Machlup, F., **The Production and Distribution of Knowledge in the United States**, Princeton University Press, Princeton, New Jersey, 1962.

Machlup, F., **The Economics of Information and Human Capital: The Third Volume of Knowledge: Its Creation, Distribution and Economic Significance**, Princeton University Press, Princeton, New Jersey, 1984.

Nagurney, A., and Zhang, D., **Projected Dynamical Systems and Variational Inequalities with Applications**, Kluwer Academic Publishers, Boston, Massachusetts, 1996.