Network Economics:

An Introduction

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Throughout history, **networks** have served as the foundation for connecting humans to one another and their activities. Roads were laid, bridges built, and waterways crossed so that humans, be they on foot, on animal, or vehicle could traverse physical distance. The airways were conquered through flight.

Communications, in turn, were conducted using the available means of the period, from smoke signals, drum beats, and pigeons, to the telegraph, telephone, and computer networks of today.

We live in an era in which the freedom to choose is weighted by the immensity of the number of choices and possibilities:

Where should one live?

Where should one work? And when?

How should one travel? Or communicate? And with whom?

Where should one shop? And how?

Underlying the numerous choices available is the wealth of information that can be accessed through computer networks.

How should businesses avail themselves of the new opportunities made possible through the Information Age?

How can they effectively compete?

How has the landscape changed for consumers as well?

In this course on Network Economics, we tackle the questions surrounding decision-making in the Network Economy today.

Our approach is conceptual, graphical, theoretical, and, ultimately, analytical.

In particular, we lay the foundations for economic systems, as networks, with a focus on decision-making.

The approach adds a graphic dimension to the understanding of the fundamental underlying structure of complex economic systems and their ultimate evolution over time.

Networks thread through our lives, and provide the fabric for our societies and economies and the infrastructure for commerce, science and technology, social systems, and education.

Examples of networks which supply the basic foundation for economic and social activity are: transportation, communication, energy, and financial networks.

See the Table 1, for some basic, classical networks and the associated nodes, links, and flows. By *classical* we mean that the nodes correspond to physical locations in space and the links to physical connections between the nodes.

Examples of Classical Networks

Network System	Nodes	Links	Flows
Transportation			
Urban	Intersections, Homes, Places of Work	Roads	Autos
Air	Airports	Airline Routes	Planes
Rail	Railyards	Railroad Track	Trains
Manufacturing and Logistics	Distribution Points Processing Points	Routes Assembly Line	Parts, Products
Communication	Computers	Cables	Messages
	Satellites	Radio	Messages
	Phone Exchanges	Cables, Microwaves	Voice, Video
Energy	Pumping Stations	Pipelines	Water
	Plants	Pipelines	Gas, Oil

Transportation networks give us the means to cross physical distance in order to conduct our daily activities.

They provide us with access to food as well as to consumer products and come in a myriad of forms: road, air, rail, or waterway.

According to the U.S. Department of Transportation, the significance of transportation in dollar value alone as spent by US consumers, businesses, and governments was $950 billion in 1998.

Communication networks, in turn, allow us to communicate with friends and colleagues and to conduct the necessary transactions of life.

They, through such innovations as the Internet, have transformed the manner in which we live, work, and conduct business today.

Communication networks allow the transmission of voice, data/information, and/or video and can involve telephones and computers, as well as satellites and microwaves.

The trade publication *Purchasing* reports that corporate buyers alone spent $517.6 billion on telecommunications goods and services in 1999.

Energy networks, in addition, are essential to the very existence of the *Network Economy* and help to fuel not only transportation networks but in many settings also communication networks.

They provide electricity to run the computers and to light our businesses, oil and gas to heat our homes and to power vehicles, and water for our very survival.

In 1995, according to the U.S. Department of Commerce, the energy expenditures in the United States were $515.8 billion.

The topic of networks and the management thereof dates to ancient times with such classical examples including the publicly provided Roman road network and the “time of day” chariot policy, whereby chariots were banned from the ancient city of Rome at particular times of day.

The formal study of networks, consisting of nodes, links, and flows involves:

- how to model such applications (as well as numerous other ones) as mathematical entities,

- how to study the models qualitatively, and

- how to design algorithms to solve the resulting models effectively.

The study of networks is necessarily interdisciplinary in nature due to their breadth of appearance and is based on scientific techniques from applied mathematics, computer science, and engineering with applications as varied as economics, finance, and even biology.

Network models and tools are widely used by businesses and industries, as well as governments today (cf. Ahuja, Magnanti, and Orlin (1993), Nagurney and Siokos (1997), Nagurney (1999, 2000), Nagurney and Dong (2002), and the references therein).

Basic examples of network problems are:

the shortest path problem, in which one seeks to determine the most efficient path from an origin node to a destination node;

the maximum flow problem, in which one wishes to determine the maximum flow that one can send from an origin node to a destination node, given that there are capacities on the links that cannot be exceeded, and

the minimum cost flow problem, where there are both costs and capacities associated with the links and one must satisfy the demands at the destination nodes, given supplies at the origin nodes, at minimal total cost associated with shipping the flows, and subject to not exceeding the arc capacities.

Applications of the Shortest Path Problem:

- arise in transportation and telecommunications.

Other applications include:

- simple building evacuation models
- DNA sequence alignment
- dynamic lot-sizing with backorders
- assembly line balancing
- compact book storage in libraries.

Applications of the Maximum Flow Problem:

- machine scheduling
- network reliability testing
- building evacuation.

Applications of the Minimum Cost Flow Problem:

- warehousing and distribution
- vehicle fleet planning
- cash management
- automatic shromosome classification
- satellite scheduling.

Network problems also arise in other surprising and fascinating ways for problems, which at first glance and on the surface, may not appear to involve networks at all.

Hence, the study of networks is not limited to only physical networks but also to abstract networks in which nodes do not coincide to locations in space.

The advantages of a network formalism:

• many present-day problems are concerned with flows (material, human, capital, informational, etc.) over space and time and, hence, ideally suited as an application domain for network theory;

• provides a graphical or visual depiction of different problems;

• helps to identify similarities and differences in distinct problems through their underlying network structure;

• enables the application of efficient network algorithms; allows for the study of disparate problems through a unifying methodology.

One of the primary purposes of scholarly and scientific investigation is to structure the world around us and to discover patterns that cut across boundaries and, hence, help to unify diverse applications.

Network theory provides us with a powerful methodology to establish connections with different disciplines and to break down boundaries.

Early Networks in Economics

The concept of a network in economics was implicit as early as the classical work of Cournot (1838), who not only seems to have first explicitly stated that a competitive price is determined by the intersection of supply and demand curves, but had done so in the context of two spatially separated markets in which the cost of transporting the good between markets was considered.

Pigou (1920) also studied a network system in the setting of a transportation network consisting of two routes and noted that the "system-optimized" solution was distinct from the "user-optimized" solution.

Nevertheless, the first instance of an abstract network or supernetwork in the context of economic applications, was actually due to Quesnay (1758), who visualized the circular flow of funds in an economy as a network.

Since that very early contribution there have been numerous economic and financial models that have been constructed over abstract networks.

In particular, we note the work of Dafermos and Nagurney (1985) who identified the isomorphism between traffic network equilibrium problems and spatial price equilibrium problems, whose development had been originated by Samuelson (1952) (who, interestingly, focused on the bipartite network structure of the spatial price equilibrium problem) and Takayama and Judge (1971).

Zhao (1989) (see also Zhao and Dafermos (1991) and Zhao and Nagurney (1993)) identified the general economic equilibrium problem known as Walrasian price equilibrium as a network equilibrium problem over an abstract network with very simple structure. The structure consisted of a single origin/destination pair of nodes and single links joining the two nodes. This structure was then exploited for computational purposes.

Nagurney (1989), in turn, proposed a migration equilibrium problem over an abstract network with an identical structure. A variety of abstract networks in economics were studied in the book by Nagurney (1999), which also contains extensive references to the subject.

Characteristics of Many of Today's Networks

The characteristics of today's networks include:

- large-scale nature and complexity of network topology;

- congestion; alternative behavior of users of the network, which may lead to paradoxical phenomena; and

- the interactions among networks themselves such as in transportation versus telecommunications networks.

Moreover, policies surrounding networks today may have a major impact not only economically but also socially.

Large-Scale Nature and Complexity

Many of today's networks are characterized by both a large-scale nature and complexity of the underlying network topology. For example, in Chicago's Regional Transportation Network, there are 12,982 nodes, 39,018 links, and 2,297,945 origin/destination (O/D) pairs (see Bar-Gera (1999)), whereas in the Southern California Association of Governments' model there are 3,217 origins and/or destinations, 25,428 nodes, and 99,240 links, plus 6 distinct classes of users (cf. Wu, Florian, and He (2000)).

In terms of the size of existing telecommunications networks, AT&T's domestic network has 100,000 origin/-destination pairs (cf. Resende (2000)), whereas in their detail graph applications in which nodes are phone numbers and edges are calls, there are 300 million nodes and 4 billion edges (cf. Abello, Pardalos, and Resende (1999)).

Congestion

Congestion is playing an increasing role in not only transportation networks but also in telecommunication networks. For example, in the case of transportation networks in the United States alone, congestion results in $100 billion in lost productivity, whereas the figure in Europe is estimated to be $150 billion. The number of cars is expected to increase by 50 percent by 2010 and to double by 2030 (see Nagurney (2000)).

In terms of the Internet, with 275 million present users, the Federal Communications Commission reports that the volume of traffic is doubling every 100 days, which is remarkable given that telephone traffic has typically increased only by about 5 percent a year (cf. Labaton (2000)).

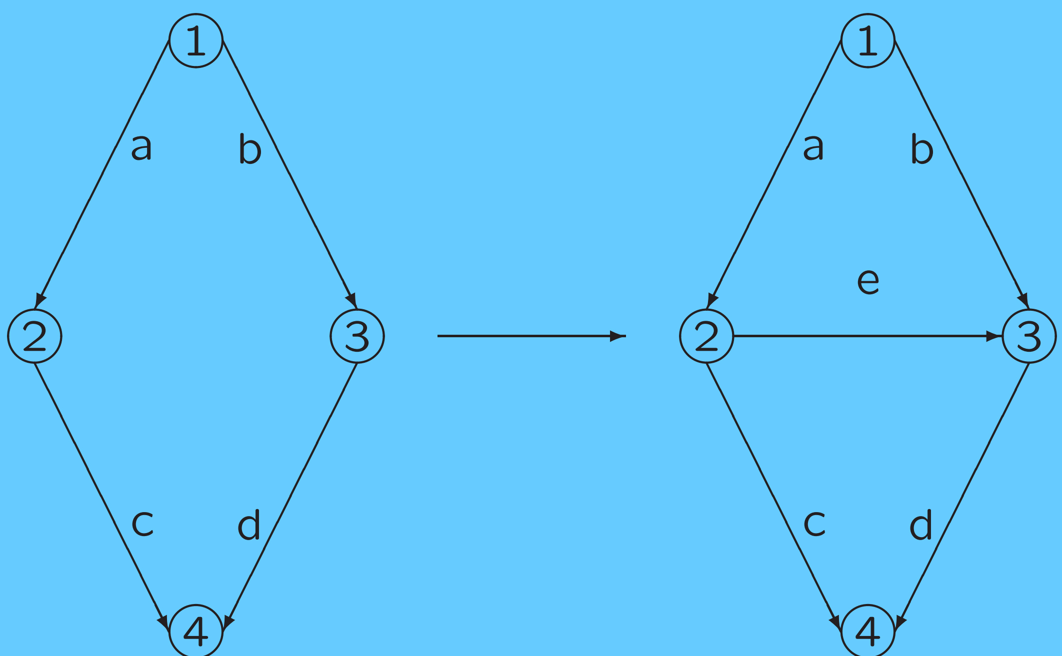
As individuals increasingly access the Internet through wireless communication such as through handheld computers and cellular phones, experts fear that the heavy use of airwaves will create additional bottlenecks and congestion that could impede the further development of the technology.

System-Optimization versus User-Optimization

In many of today's networks, not only is congestion a characteristic feature leading to nonlinearities, but the behavior of the users of the networks themselves may be that of noncooperation.

For example, in the case of urban transportation networks, travelers select their routes of travel from an origin to a destination so as to minimize their own travel cost or travel time, which although "optimal" from an individual's perspective (user-optimization) may not be optimal from a societal one (system-optimization) where one has control over the flows on the network and, in contrast, seeks to minimize the total cost in the network and, hence, the total loss of productivity. Consequently, in making any kind of policy decisions in such networks one must take into consideration the users of the particular network.

Indeed, this point is vividly illustrated through a famous example known as Braess's paradox, in which it is assumed that the underlying behavioral principle is that of user-optimization. In the Braess (1968) network, the addition of a new road with no change in the travel demand results in all travelers in the network incurring a higher travel cost and, hence, being worse off.



The Braess network example

The Braess Paradox

We now present the Braess's paradox example. For easy reference, see the two networks depicted in the Figure.

Assume a network as the first network depicted in the Figure in which there are 4 links: a, b, c, d; 4 nodes: 1,2,3,4; and a single O/D pair $w_1 = (1, 4)$. There are, hence, 2 paths available to travelers between this O/D pair: $p_1 = (a, c)$ and $p_2 = (b, d)$.

The link travel cost functions are:

$$c_a(f_a) = 10f_a \quad c_b(f_b) = f_b + 50$$

$$c_c(f_c) = f_c + 50 \quad c_d(f_d) = 10f_d.$$

Assume a fixed travel demand $d_{w_1} = 6$.

It is easy to verify that the equilibrium path flows are:

$$x^*_{p_1} = 3 \quad x^*_{p_2} = 3;$$

the equilibrium link flows are:

$$f^*_a = 3 \quad f^*_b = 3 \quad f^*_c = 3 \quad f^*_d = 3;$$

with associated equilibrium path travel costs:

$$C_{p_1} = 83 \quad C_{p_2} = 83.$$

Assume now that, as depicted in the Figure, a new link "e," joining node 2 to node 3, is added to the original network, with user cost $c_e(f_e) = f_e + 10$. The addition of this link creates a new path $p_3 = (a, e, d)$ that is available to the travelers. Assume that the travel demand d_{w_1} remains at 6 units of flow. Note that the original flow distribution pattern $x_{p_1} = 3$ and $x_{p_2} = 3$ is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, $C_{p_3} = 70$. Hence, users from paths p_1 and p_2 would switch to path p_3.

The equilibrium flow pattern on the new network is:

$$x^*_{p_1} = 2 \quad x^*_{p_2} = 2 \quad x^*_{p_3} = 2;$$

with equilibrium link flows:

$$f^*_a = 4 \quad f^*_b = 2 \quad f^*_c = 2 \quad f^*_e = 2 \quad f^*_d = 4;$$

and with associated equilibrium path travel costs:

$$C_{p_1} = 92 \quad C_{p_2} = 92.$$

Indeed, one can verify that any reallocation of the path flows would yield a higher travel cost on a path.

Note that the travel cost increased for every user of the network from 83 to 92!

The increase in travel cost on the paths is due, in part, to the fact that in this network two links are shared by distinct paths and these links incur an increase in flow and associated cost. Hence, Braess's paradox is related to the underlying topology of the networks. One may show, however, that the addition of a path connecting an O/D pair that shares no links with the original O/D pair will never result in Braess's paradox for that O/D pair.

Interestingly, as reported in the *New York Times* by Kolata (1990), this phenomenon has been observed in practice in the case of New York City when in 1990, 42nd Street was closed for Earth Day and the traffic flow actually improved. Just to show that it is not a purely New York or US phenomena concerning drivers and their behavior an analogous situation was observed in Stuttgart where a new road was added to the downtown but the traffic flow worsened and following complaints, the new road was torn down (see Bass (1992)).

This phenomenon is also relevant to telecommunications networks (see Korilis, Lazar, and Orda (1999)) and, in particular, to the Internet which is another example of a "noncooperative network" and, therefore, network tools have wide application in this setting as well, especially in terms of congestion management and network design (see also Cohen and Kelly (1990)).

Network Interactions

Clearly, one of the principal facets of the Network Economy is the interaction among the networks themselves. For example, the increasing use of e-commerce especially in business to business transactions is changing not only the utilization and structure of the underlying logistical networks but is also revolutionizing how business itself is transacted and the structure of firms and industries.

Cellular phones are being using as vehicles move dynamically over transportation networks resulting in dynamic evolutions of the topologies themselves.

This course also, under "advanced topics" explores the network interactions among such networks as transportation networks and telecommunication networks, as well as financial networks.

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