Optimal Endogenous Carbon Taxes
for
Electric Power Supply Chains with Power Plants

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Abstract: In this paper, we develop a modeling and computational framework that allows for the determination of optimal carbon taxes applied to electric power plants in the context of electric power supply chain (generation/distribution/consumption) networks. The adoption of carbon/pollution taxes both internationally and regionally has been fueled by global climate change and fuel security risks with a significant portion of such policy interventions directed at the electric power industry. The general framework that we develop allows for three distinct types of carbon taxation environmental policies, beginning with a completely decentralized scheme in which taxes can be applied to each individual power generator/power plant in order to guarantee that each assigned emission bound is not exceeded, to two versions of a centralized scheme, one which assumes a fixed bound over the entire

electric power supply chain in terms of total carbon emissions and the other which allows the bound to be a function of the tax. The behavior of the various decision-makers in the electric power supply chain network is described, along with the three taxation schemes, and the governing equilibrium conditions, which are formulated as finite-dimensional variational inequality problems. Twelve numerical examples are presented in which the optimal carbon taxes, as well as the equilibrium electric power flows and demands, are computed. The numerical results demonstrate, as the theory predicts, that the carbon taxes achieve the desired goal, in that the imposed bounds on the carbon emissions are not exceeded. Moreover, they illustrate the spectrum of scenarios that can be explored in terms of changes in he bounds on the carbon emissions; changes in emission factors; changes in the demand functions, etc.

1. Introduction

Electric power is a fundamental resource that has fueled modern economies and societies from its role in the lighting and heating of homes and offices to the running of computers, which underpin our communications, manufacturing processes and financial services, and even transportation. Electric power is so essential to our every day lives that when failures arise the impact is wide and vividly apparent as the biggest blackout in US history in 2003 graphically illustrated. Indeed, in modern societies there are few goods or services that do not depend directly on electricity. Globally, in 2002, 16.1 trillion kilowatt hours were supplied, with United States being the largest producer and consumer of electric energy (see [1]). In the past half-century, the total annual electricity use in the US alone has grown every year but two. The US electrical industry has more than half a trillion dollars of net assets, $220 billion in annual sales, and consumes almost 40% of domestic primary energy (coal, natural gas, uranium, and oil), or about 40 quadrillion British Thermal Units (BTUs) (see [2] – [4]). This industry is growing, with the expectation that the total global consumption of electricity will reach 23.1 trillion kilowatt hours in 2025. For background on the electric power industry, see [5] – [7].

However, despite the major positive economic impacts of electric power, the heavy reliance on fossil fuel sources before conversion to electricity has had, at the same time, an immense environmental impact. For example, of the total US emissions of carbon dioxide and nitrous oxide, more than a third arises from generating electricity. In China, in turn, the electric power sector currently accounts for more than one-third of its annual coal consumption with such power plants generating over 75% of the air pollution in China (cf. [8]). Fossil fuels, which include coal, are expected to be used in 36% of the electricity production into 2020, and presently account for 39% of the electricity generated worldwide. With growing accumulating evidence of global warming, any policy aimed at mitigating the immense risks of unstable climate must directly consider the electricity industry ([9], [10]).

As noted in Wu et al. [11], pollution taxes, and, in particular, carbon taxes are a powerful policy mechanism that can address market failures in energy. Currently, market prices for energy do not capture its many external costs in the form of regional and global pollution and also hide market distortions. The usefulness of carbon taxes has been noted by among

others, Baranzini, Goldemberg, and Speck [12], whereas Painuly [13] has argued for the encouragement of power generation from renewable sources, including solar power and wind power through the use of green credits. Such credits are now being utilized in the European Union as well as in several states in the US (cf. [14], [15]). Hence, a mathematical modeling framework that can capture the interactions among decision-makers in an electric power supply chain network from power generators, along with the power plant production options; the suppliers as well as ultimate consumers, coupled with the incorporation of environmental policies, such as carbon taxes, is of great practical as well as policy-making importance.

Recently, Wu et al. [11] proposed an electric power supply chain network equilibrium model with carbon taxes that are applied a priori to distinct power generator/power plant combinations and demonstrated that the model could be reformulated and solved as a transportation network equilibrium problem with elastic demands. However, in that model, the government authority would have to conduct simulation exercises to determine the carbon tax assignment in order to achieve some goal. In this paper, in contrast, we demonstrate how carbon taxes can be determined optimally and endogenously within a generalized electric power supply chain network equilibrium model. In particular, we allow the government to impose bound(s) on the total amount of carbon emissions and the optimal carbon taxes guarantee that the bound(s) are not exceeded.

Since electric power plants may be under different governmental jurisdictions either in the US or abroad we propose both a decentralized carbon taxation scheme as well as two centralized schemes. In the former environmental policy framework, a bound on carbon emissions can be applied on each power generator/power plant with a resulting distinct tax; in the latter framework there is a single carbon emission bound imposed on the entire electric power supply chain network with a resulting single tax. We note that Nagurney and Matsypura [16] were the first to introduce a general electric power supply chain network equilibrium model, which was, subsequently, shown by Nagurney and Liu [17] to be transformable into a transportation network equilibrium model over an appropriately constructed abstract network. This identification validated a hypothesis made over half a century ago in the book by Beckmann, McGuire, and Winsten [18].

The paper is organized as follows. In Section 2, we present the electric power supply chain

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network equilibrium model with three distinct carbon taxation schemes. We also demonstrate the relationship between carbon taxes and weights associated with environmental criteria and the concommitant multicriteria decision-making akin to the work of Nagurney and Toyasaki [19]. In Section 3, we present twelve numerical examples in which the optimal taxes and the equilibrium electric power flows between tiers of decision-makers and the demands for electric power are computed. We also report the prices at the demand markets at the equilibrium. In Section 4, we summarize the results in this paper and present our conclusions.

2. The Electric Power Supply Chain Network Model with Power Plants and Optimal Endogenous Carbon Taxes

In this Section we develop the electric power supply chain network equilibrium model that includes power plants as well as carbon taxes under three distinct taxation schemes. We begin with a decentralized taxation scheme outlined in Section 2.1 and then turn to the centralized schemes in Section 2.2. The presentation of the model follows the description of the electric supply chain network model with pollution taxes (which, however, were fixed and assigned a priori) in [11].

In particular, we consider G power generators (sometimes also referred to as "gencos"), each of which, typically, owns and operates M power plants. A specific power plant may use a different primary energy fuel such as, for example, coal, natural gas, uranium, oil, sun, wind, etc., with different associated costs. In addition, each power plant will also have associated costs that fully reflect policy objectives, which here will be in the form of carbon taxes. There are S power suppliers, T transmission service providers, and K consumer markets, as depicted in Figure 1. The majority of the needed notation is given in Table 1. An equilibrium solution is denoted by "*". All vectors are assumed to be column vectors, except where noted otherwise.

The top-tiered nodes in the electric power supply chain network in Figure 1, enumerated by $1, \ldots, g \ldots, G$, represent the G electric power generators, who are the decision-makers who own and operate the electric power generating facilities or power plants denoted by the second tier of nodes in the network. The gencos produce electric power using the different power plants and sell to the power suppliers in the third tier. Node gm in the second tier corresponds to genco g's power plant m, with the second tier of nodes enumerated as: $11, \ldots, GM$. We assume that each electric power generator seeks to determine his optimal production portfolio across his power plants and his sales allocations of the electric power to the suppliers in order to maximize his own profit.

Table 1: Notation for the Electric Power Supply Chain Network Model

Notation	Definition
q_{gm}	quantity of electricity produced by generator g using power plant m, where $g = 1, \ldots, G; m = 1, \ldots, M$
q_m	G-dimensional vector of electric power generated by the gencos using power plant m with components: $q_{1m}, \ldots, q_{Gm}$
q	GM-dimensional vector of all the electric power outputs generated by the gencos at the power plants
Q^1	GMS-dimensional vector of electric power flows between the power plants of the power generators and the power suppliers with component gms denoted by q_{gms}
Q^2	STK-dimensional vector of power flows between suppliers and demand markets with component stk denoted by q_{sk}^t and denoting the flow between supplier s and demand market k via transmission provider t
d	K-dimensional vector of market demands with component k denoted by d_k
$f_{gm}(q_m)$	power generating cost function of power generator g using power plant m with marginal power generating cost with respect to q_{gm} denoted by $\frac{\partial f_{gm}}{\partial q_{gm}}$
$c_{gms}(q_{gms})$	transaction cost incurred by power generator g using power plant m in transacting with power supplier s with marginal transaction cost denoted by $\frac{\partial c_{gms}(q_{gms})}{\partial q_{gms}}$
e_{gm}	amount of carbon emitted by genco g using power plant m per unit of electric power produced
h	S-dimensional vector of the power suppliers' supplies of the electric power with component s denoted by h_s, with $h_s \equiv \sum_{g=1}^{G} \sum_{m=1}^{M} q_{gms}$
$c_s(h) \equiv c_s(Q^1)$	operating cost of power supplier s with marginal operating cost with respect to h_s denoted by $\frac{\partial c_s}{\partial h_s}$ and the marginal operating cost with respect to q_{gms} denoted by $\frac{\partial c_s(Q^1)}{\partial q_{gms}}$
$c_{sk}^t(q_{sk}^t)$	transaction cost incurred by power supplier s in transacting with demand market k via transmission provider t with marginal transaction cost with respect to q_{sk}^t denoted by $\frac{\partial c_{sk}^t(q_{sk}^t)}{\partial q_{sk}^t}$
$\hat{c}_{gms}(q_{gms})$	transaction cost incurred by power supplier s in transacting with power generator g for power generated by plant m with marginal transaction cost denoted by $\frac{\partial \hat{c}_{gms}(q_{gms})}{\partial q_{gms}}$
$\hat{c}_{sk}^t(Q^2)$	unit transaction cost incurred by consumers at demand market k in transacting with power supplier s via transmission provider t
$\rho_{3k}(d)$	demand market price function at demand market k

Power Generators

Power Plants

Suppliers

Transmission Service
Providers

Demand Markets
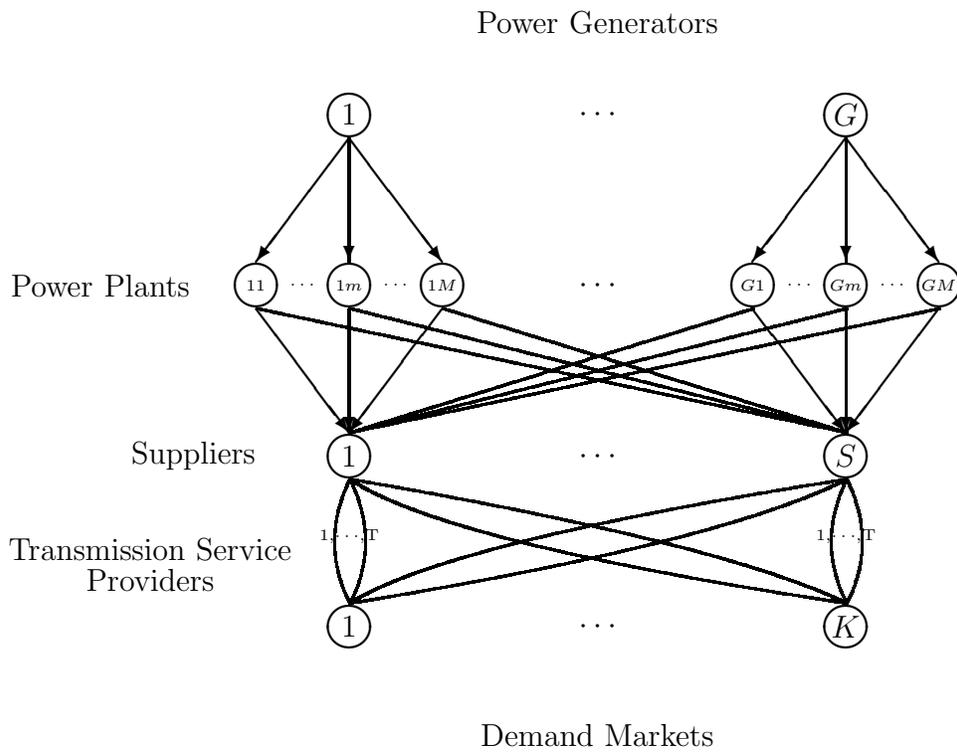
Figure 1: The Electric Power Supply Chain Network with Power Plants

Power suppliers, which are represented by the third tiered nodes in Figure 1, function as intermediaries. The nodes corresponding to the power suppliers are enumerated as: $1, \ldots, s, \ldots, S$ with node s corresponding to supplier s. They purchase electric power from the power generators and are aware as to the types of power plants used by the generators. They also sell the electric power to the consumers at the different demand markets. We assume that the power suppliers compete with one another in a noncooperative manner. However, the suppliers do not physically possess electric power at any stage of the supplying process; they only hold and trade the right for the electric power.

The bottom-tiered nodes in Figure 1 represent the demand markets, which can be distinguished from one another by their geographic locations or the type of associated consumers such as whether they correspond, for example, to businesses or to households. There are K bottom-tiered nodes with node k corresponding to demand market k.

A transmission service is necessary for the physical delivery of electric power from the power generators to the points of consumption. The transmission service providers are the entities who own and operate the electric power transmission and distribution systems, and distribute electric power from power generators to the consumption markets. Since the transmission service providers do not make decisions such as to where or from whom the electric power will be delivered, they are not represented by nodes in the network model. The structure of the network in Figure 1 guarantees that the conservation of flow equations associated with the electric power production and distribution are satisfied. The flows on the links joining the genco nodes in Figure 1 to the power plant nodes are respectively: $q_{11}, \ldots, q_{gm}, \ldots, q_{GM}$; the flows on the links from the power plant nodes to the supplier nodes are given, respectively, by the components of the vector Q^1, whereas the flows on the links joining the supplier nodes with the demand markets are given by the respective components of the vector: Q^2.

2.1 A Decentralized Carbon Taxation Scheme

We now describe the behavior of the electric power generators, the suppliers, and the consumers at the demand markets. We first consider a decentralized carbon taxation scheme. We then state the equilibrium conditions of the electric power supply chain network and

provide the variational inequality formulation. Subsequently, we discuss how centralized taxation schemes can be substituted for the decentralized scheme.

The Behavior of the Power Generators and their Optimality Conditions

Let e_{gm}; $g = 1, \ldots, G$; $m = 1, \ldots, M$ denote the carbon emissions generated per unit of electric power produced by genco g using his power plant m. Hence, the total amount of carbon emissions associated with genco g and power plant m is $e_{gm}q_{gm}$.

Let ρ^*_{1gms} denote the unit price charged by power generator g for the transaction with power supplier s for power produced at plant m with $g = 1, \ldots, G$; $m = 1, \ldots, M$, and $s = 1, \ldots, S$. ρ^*_{1gms} is an endogenous variable and can be determined once the complete electric power supply chain network equilibrium model is solved.

Let τ_{gm}; $g = 1, \ldots, G$; $m = 1, \ldots, M$, denote the unit tax that the governmental or responsible environmental authority will charge genco g operating power plant m for his carbon emissions and group all these taxes into the vector τ. The optimal values of these taxes, denoted by the components of the vector τ^*, are endogenous and are determined once the complete model is solved. As we will later see, the taxes guarantee that bounds on the carbon emissions, also imposed by the responsible authority, are not exceeded. The bounds are denoted by $\bar{B}_{gm}$; $g = 1, \ldots, G$; $m = 1, \ldots, M$ and reflect the maximum carbon emissions allowed by the responsible authority for the particular genco/power plant combination. The complete electric power supply chain network equilibrium model captures the complex interactions among the various decision-makers and its solution yields the equilibrium electric power flows and demands as well as the optimal carbon taxes.

Since we have assumed that each individual power generator is a profit-maximizer, the optimization problem of power generator g can be expressed as follows:

$$\text{Maximize} \quad \sum_{m=1}^{M}\sum_{s=1}^{S}\rho^*_{1gms}q_{gms} - \sum_{m=1}^{M}f_{gm}(q_m) - \sum_{m=1}^{M}\sum_{s=1}^{S}c_{gms}(q_{gms}) - \sum_{m=1}^{M}\tau^*_{gm}e_{gm}q_{gm} \quad (1)$$

subject to:

$$\sum_{s=1}^{S}q_{gms} = q_{gm}, \quad m = 1, \ldots, M, \quad (2)$$

$$q_{gms} \geq 0, \quad m = 1, \ldots, M; s = 1, \ldots, S. \quad (3)$$

The first term in the objective function (1) represents the revenue and the next two terms represent the power generation cost and transaction costs, respectively. The last term in (1) denotes the total payout in carbon taxes by the genco based on the total carbon pollution emitted by his power plants. Conservation of flow equation (2) states that the amount of power generated at a particular power plant (and corresponding to a particular genco) is equal to the electric power transacted by the genco from that power plant with all the suppliers, and this holds for each of the power plants.

Assume, as was done in [11], that the generating cost and the transaction cost functions for each power generator are continuously differentiable and convex, and that the power generators compete in a noncooperative manner in the sense of Nash [20,21]. The optimality conditions for all power generators simultaneously, under the above assumptions (cf. [22]), coincide with the solution of the following variational inequality: determine $(q^*, Q^{1*}) \in \mathcal{K}^1$ satisfying

$$\sum_{g=1}^{G} \sum_{m=1}^{M} \left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \tau_{gm}^* e_{gm} \right] \times [q_{gm} - q_{gm}^*]$$

$$+ \sum_{g=1}^{G} \sum_{m=1}^{M} \sum_{s=1}^{S} \left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} - \rho_{1gms}^* \right] \times [q_{gms} - q_{gms}^*] \geq 0, \quad \forall (q, Q^1) \in \mathcal{K}^1, \tag{4}$$

where $\mathcal{K}^1 \equiv \{(q, Q^1) | (q, Q^1) \in R_+^{GM+GMS} \text{ and } (2) \text{ holds}\}$.

The Behavior of Power Suppliers and their Optimality Conditions

The power suppliers, in turn, are involved in transactions both with the power generators and with the consumers at demand markets through the transmission service providers.

Since electric power cannot be stored, the total amount of electricity sold by a power supplier is equal to the total electric power that he purchased from the generators and produced via the different power plants available to the generators, that is:

$$\sum_{k=1}^{K} \sum_{t=1}^{T} q_{sk}^t = \sum_{g=1}^{G} \sum_{m=1}^{M} q_{gms}, \quad s = 1, \ldots, S. \tag{5}$$

Let ρ_{2sk}^{t*} denote the price charged by power supplier s to demand market k via transmission service provider t. This price is determined endogenously in the model once the entire

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network equilibrium problem is solved. As noted above, it is assumed that each power supplier seeks to maximize his own profit. Hence the optimization problem faced by supplier s may be expressed as follows:

$$\text{Maximize} \quad \sum_{k=1}^{K} \sum_{t=1}^{T} \rho_{2sk}^{t*} q_{sk}^t - c_s(Q^1) - \sum_{g=1}^{G} \sum_{m=1}^{M} \rho_{1gms}^* q_{gms} - \sum_{g=1}^{G} \sum_{m=1}^{M} \hat{c}_{gms}(q_{gms}) - \sum_{k=1}^{K} \sum_{t=1}^{T} c_{sk}^t(q_{sk}^t)$$

(6)

subject to:

$$\sum_{k=1}^{K} \sum_{t=1}^{T} q_{sk}^t = \sum_{g=1}^{G} \sum_{m=1}^{M} q_{gms} \tag{7}$$

$$q_{gms} \geq 0, \quad g = 1, \ldots, G, \quad m = 1, \ldots, M, \tag{8}$$

$$q_{sk}^t \geq 0, \quad k = 1, \ldots, K; t = 1, \ldots, T. \tag{9}$$

The first term in (6) denotes the revenue of supplier s; the second term denotes the operating cost of the supplier; the third term denotes the payments for the electric power to the various gencos, and the final two terms represent the various transaction costs.

We assume that the transaction costs and the operating costs in (6) are all continuously differentiable and convex, and that the power suppliers compete in a noncooperative manner. Hence, the optimality conditions for all suppliers, simultaneously, under the above assumptions (see [16]), can be expressed as the following variational inequality: determine $(Q^{2*}, Q^{1*}) \in \mathcal{K}^2$ such that

$$\sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t=1}^{T} \left[\frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} - \rho_{2sk}^{t*} \right] \times [q_{sk}^t - q_{sk}^{t*}]$$

$$+ \sum_{g=1}^{G} \sum_{m=1}^{M} \sum_{s=1}^{S} \left[\frac{\partial c_s(Q^{1*})}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}} + \rho_{1gms}^* \right] \times [q_{gms} - q_{gms}^*] \geq 0, \quad \forall (Q^2, Q^1) \in \mathcal{K}^2, \tag{10}$$

where $\mathcal{K}^2 \equiv \{(Q^2, Q^1) | (Q^2, Q^1) \in R_+^{STK+GMS} \text{ and } (7) \text{ holds}\}$.

For notational convenience, and as was done in [11], we let

$$h_s \equiv \sum_{g=1}^{G} \sum_{m=1}^{M} q_{gms}, \quad s = 1, \ldots, S. \tag{11}$$

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As defined in Table 1, the operating cost of power supplier s, c_s, is a function of the total electricity inflows to the power supplier, that is:

$$c_s(h) \equiv c_s(Q^1), \quad s = 1, \ldots, S. \tag{12}$$

Hence, his marginal cost with respect to h_s is equal to the marginal cost with respect to q_{gms}:

$$\frac{\partial c_s(h)}{\partial h_s} \equiv \frac{\partial c_s(Q^1)}{\partial q_{gms}}, \quad s = 1, \ldots, S, \quad m = 1, \ldots, M, \quad g = 1, \ldots, G. \tag{13}$$

After the substitution of (11) and (13) into (10), and algebraic simplification, we obtain a variational inequality equivalent to (10), as follows: determine $(h^*, Q^{2*}, Q^{1*}) \in \mathcal{K}^3$ such that

$$\sum_{s=1}^{S} \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*] + \sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t=1}^{T} \left[\frac{\partial \hat{c}_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} - \rho_{2sk}^{t*} \right] \times [q_{sk}^t - q_{sk}^{t*}]$$

$$+ \sum_{g=1}^{G} \sum_{m=1}^{M} \sum_{s=1}^{S} \left[\frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}} + \rho_{1gms}^* \right] \times [q_{gms} - q_{gms}^*] \geq 0, \qquad \forall(h, Q^1, Q^2,) \in \mathcal{K}^3, \tag{14}$$

where $\mathcal{K}^3 \equiv \{(h, Q^2, Q^1) | (h, Q^2, Q^1) \in R_+^{S(1+TK+GM)} \text{and (7) and (11) hold}\}$.

Equilibrium Conditions for the Demand Markets

At each demand market k the following conservation of flow equation must be satisfied:

$$d_k = \sum_{s=1}^{S} \sum_{t=1}^{T} q_{sk}^t, \quad k = 1, \ldots, K. \tag{15}$$

The market equilibrium conditions at demand market k take the form: for each power supplier s; $s = 1, ..., S$ and transaction mode t; $t = 1, ..., T$:

$$\rho_{2sk}^{t*} + \hat{c}_{sk}^t(Q^{2*}) \begin{cases} = \rho_{3k}(d^*), & \text{if} \quad q_{sk}^{t*} > 0, \\ \geq \rho_{3k}(d^*), & \text{if} \quad q_{sk}^{t*} = 0. \end{cases} \tag{16}$$

According to (16) (see also [11]), consumers at a demand market will purchase power from a supplier via a transmission provider, provided that the purchase price plus the unit transaction cost is equal to the price that the consumers are willing to pay at that demand market. If the purchase price plus the unit transaction cost exceeds the price the consumers

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are willing to pay, then there will be no transaction between that supplier and demand market via that transmission provider. The equivalent variational inequality takes the form: determine $(Q^{2*}, d^*) \in \mathcal{K}^4$, such that

$$\sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t=1}^{T} \left[\rho_{2sk}^{t*} + \hat{c}_{sk}^t(Q^{2*}) \right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^{K} \rho_{3k}(d^*) \times [d_k - d_k^*] \geq 0, \quad \forall(Q^2, d) \in \mathcal{K}^4, \ (17)$$

where $\mathcal{K}^4 \equiv \{(Q^2, d) | (Q^2, d) \in R_+^{K(ST+1)} \text{and } (15) \text{ holds}\}$.

Decentralized Carbon Tax Equilibrium Conditions

Unlike the model of Wu et al. [11], which considered pollution taxes applied to each genco/power plant combination that were fixed and assigned a priori, we here assume that the taxes are endogenous and optimal in the following sense. We assume, as is commonly done in real-life environmental policy-making (see also, e.g., the book by Dhanda, Nagurney, and Ramanujam [23]), and as was noted earlier, that bounds are applied in terms of the maximum carbon emissions that are allowed for each genco/power plant combination. If the particular genco/power plant emits fewer carbons than the imposed limit, then it is not taxed; a tax is assigned if the emissions equal the bound. Mathematically, this statement corresponds to the following carbon tax equilibrium conditions under a decentralized tax scheme: for all power generators g; $g = 1, \ldots, G$, and for all power plants m; $m = 1, \ldots, M$, a carbon tax policy is said to be in equilibrium if:

$$\bar{B}_{gm} - e_{gm}q_{gm}^* \begin{cases} = 0, & \text{if} \quad \tau_{gm}^* > 0, \\ \geq 0, & \text{if} \quad \tau_{gm}^* = 0. \end{cases} \tag{18}$$

Such a decentralized carbon taxation scheme would be reasonable, for example, if each power plant would reside in a different jurisdiction. Of course, if a given genco owns a subset of power plants (which may correspond to different fuel source options), then those links would just be eliminated from the network in Figure 1 and the notation altered accordingly.

The Equilibrium Conditions for the Electric Power Supply Chain Network

In equilibrium, the optimality conditions for all the power generators, the optimality conditions for all the power suppliers, and the equilibrium conditions for all the demand markets,

as well as the carbon tax equilibrium conditions must be simultaneously satisfied so that no decision-maker has any incentive to alter his transactions. We now formally state the equilibrium conditions for the entire electric power supply chain network with endogenous, decentralized carbon taxes.

Definition 1: Electric Power Supply Chain Network Equilibrium with Endogenous, Decentralized Carbon Taxes

The equilibrium state of the electric power supply chain network with power plants and endogenous, decentralized carbon taxes is one where the electric power flows between the tiers of the network coincide and the electric power flows, the demands, and the carbon taxes satisfy the sum of conditions (4), (14), (17), and the decentralized carbon tax equilibrium conditions (18).

We now state and prove:

Theorem 1: Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Decentralized Carbon Taxes

The equilibrium conditions governing the electric power supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*, \tau^*) \in \mathcal{K}^5$ satisfying:

$$\sum_{g=1}^{G} \sum_{m=1}^{M} \left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \tau_{gm}^* e_{gm} \right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^{S} \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*]$$

$$+ \sum_{g=1}^{G} \sum_{m=1}^{M} \sum_{s=1}^{S} \left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}} \right] \times [q_{gms} - q_{gms}^*]$$

$$+ \sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t=1}^{T} \left[\frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*}) \right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^{K} \rho_{3k}(d^*) \times [d_k - d_k^*]$$

$$+ \sum_{g=1}^{G} \sum_{m=1}^{M} \left[\bar{B}_{gm} - e_{gm} q_{gm}^* \right] \times \left[\tau_{gm} - \tau_{gm}^* \right] \geq 0,$$

$$\forall (q, h, Q^1, Q^2, d, \tau) \in \mathcal{K}^5, \tag{19}$$

where

$$\mathcal{K}^5 \equiv \{(q, h, Q^1, Q^2, d, \tau) | (q, h, Q^1, Q^2, d, \tau) \in R_+^{2GM+S+GMS+TSK+K}$$

$$and \ (2), \ (5), \ (11), \ and \ (15) \ hold\}.$$

Proof: We first prove that an equilibrium according to Definition 1 coincides with the solution of variational inequality (19). Note that (see also, e.g., [22]), if $\tau^* \in R_+^{GM}$ satisfies equilibrium conditions (18) then it also satisfies the following inequality:

$$\sum_{g=1}^{G} \sum_{m=1}^{M} \left[\bar{B}_{gm} - e_{gm} q_{gm}^* \right] \times \left[\tau_{gm} - \tau_{gm}^* \right] \geq 0, \quad \forall \tau \in R_+^{MG}. \tag{20}$$

Summation of (4), (14), (17), and (20) (which is equivalent to the satisfaction of the carbon tax equilibrium conditions (18)), after algebraic simplifications, yields variational inequality (19).

We now prove the converse, that is, a solution to variational inequality (19) satisfies the sum of conditions (4), (14), (17), and the carbon tax equilibrium conditions (18), and is, therefore, an electric power supply chain network equilibrium pattern according to Definition 1.

First, we add the term $\rho_{1gms}^* - \rho_{1gms}^*$ to the first term in the third summand expression in (19). Then, we add the term $\rho_{2sk}^{t*} - \rho_{2sk}^{t*}$ to the first term in the fourth summand expression in (19). Since these terms are all equal to zero, they do not change (19). Hence, we obtain the following inequality:

$$\sum_{g=1}^{G} \sum_{m=1}^{M} \left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \tau_{gm}^* e_{gm} \right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^{S} \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*]$$

$$+ \sum_{g=1}^{G} \sum_{m=1}^{M} \sum_{s=1}^{S} \left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}} + \rho_{1gms}^* - \rho_{1gms}^* \right] \times [q_{gms} - q_{gms}^*]$$

$$+ \sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t=1}^{T} \left[\frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(q_{sk}^{t*}) + \rho_{2sk}^{t*} - \rho_{2sk}^{t*} \right] \times [q_{sk}^t - q_{sk}^{t*}]$$

$$- \sum_{k=1}^{K} \rho_{3k}(d^*) \times [d_k - d_k^*]$$

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$$+ \sum_{g=1}^{G} \sum_{m=1}^{M} \left[\bar{B}_{gm} - e_{gm} q_{gm}^* \right] \times \left[\tau_{gm} - \tau_{gm}^* \right] \geq 0, \quad \forall (q, h, Q^1, Q^2, d, \tau) \in \mathcal{K}^5, \qquad (21)$$

which can be rewritten as:

$$\sum_{g=1}^{G} \sum_{m=1}^{M} \left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \tau_{gm}^* e_{gm} \right] \times [q_{gm} - q_{gm}^*] + \sum_{g=1}^{G} \sum_{m=1}^{M} \sum_{s=1}^{S} \left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} - \rho_{1gms}^* \right] \times [q_{gms} - q_{gms}^*]$$

$$+ \sum_{s=1}^{S} \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*] + \sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t=1}^{T} \left[\frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} - \rho_{2sk}^{t*} \right] \times [q_{sk}^t - q_{sk}^{t*}]$$

$$+ \sum_{s=1}^{S} \sum_{m=1}^{M} \sum_{g=1}^{G} \left[\frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}} + \rho_{1gms}^* \right] \times [q_{gms} - q_{gms}^*]$$

$$+ \sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t=1}^{T} \left[\rho_{2sk}^{t*} + \hat{c}_{sk}^t(q_{sk}^{t*}) \right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^{K} \rho_{3k}(d^*) \times [d_k - d_k^*]$$

$$+ \sum_{g=1}^{G} \sum_{m=1}^{M} \left[\bar{B}_{gm} - e_{gm} q_{gm}^* \right] \times \left[\tau_{gm} - \tau_{gm}^* \right] \geq 0, \quad \forall (q, h, Q^1, Q^2, d, \tau) \in \mathcal{K}^5. \qquad (22)$$

Clearly, (22) is the sum of the optimality conditions (4) and (14) and the inequality formulations of equilibrium conditions (16) and (18) given, respectively, by (17) and (20), and is, hence, according to Definition 1 an electric power supply chain network equilibrium with endogenous, decentralized carbon taxes. □

Note that the solution of variational inequality (19) yields the equilibrium electric power flows (see also the network in Figure 1), the equilibrium demands, as well as the optimal carbon taxes. If desired, once variational inequality (19) is solved, one can also determine the prices associated with the power generators; ρ_{1gms}^* for $g = 1, \ldots, G$; $m = 1, \ldots, M$, and $s = 1, \ldots, S$, as well as the prices associated with the suppliers/transmission providers/demand markets and denoted by ρ_{2sk}^{t*}; $s = 1, \ldots, S$; $k = 1, \ldots, K$, and $t = 1, \ldots, T$. The procedure would be similar to that described in [11].

Remark

Note that in the case of objective function (1) and, hence, also variational inequality (19), the taxes could also be interpreted as "weights" asssociated with the minimization of the

carbon emissions for each genco and power plant. Weights associated with environmental criteria in the form of total emission generation in the context of multicriteria decision-making and supply chains have been proposed and discussed in Nagurney and Toyasaki [19]. Of course, such weights would reflect that the gencos assign such weights to pollution emission minimization themselves and are self-directed, whereas environmental policies, in the form of carbon taxes, are imposed by an authority. It is interesting to see, however, that positive, self-induced decision-making in terms of environmental decision-making could have the same result as the imposition of taxes. However, the weights would have to be determined endogenously to have the precise correspondence between the electric supply chain network with multicriteria decision-makers and the model with carbon taxes proposed above. The same analogies can be made for the two carbon taxation schemes below.

2.2 Centralized Carbon Taxation Schemes

In the case of a centralized carbon taxation scheme, we first consider the case in which there is a fixed bound $\bar{B}$ for all the carbon emissions generated in the electric power supply chain and the carbon tax is now denoted by $\mathcal{T}$. We then turn to the case in which the bound can vary as a function of the tax.

Centralized Carbon Tax Equilibrium Conditions – Fixed Bound

In this case, we have, analogous to equilibrium conditions (18), the following centralized carbon tax equilibrium conditions, which state that if the amount of carbon produced in equilibrium is less than the carbon emission bound then the tax will be zero; if the tax is positive, then the emissions are at the bound:

$$\bar{B} - \sum_{g=1}^{G} \sum_{m=1}^{M} e_{gm} q_{gm}^* \begin{cases} = 0, & \text{if} \quad \mathcal{T}^* > 0, \\ \geq 0, & \text{if} \quad \mathcal{T}^* = 0. \end{cases} \tag{23}$$

Clearly equilibrium conditions (23) can be formulated as the inequality:

$$\left[\bar{B} - \sum_{g=1}^{G} \sum_{m=1}^{M} e_{gm} q_{gm}^* \right] \times [\mathcal{T} - \mathcal{T}^*] \geq 0, \quad \forall \mathcal{T} \geq 0. \tag{24}$$

We, hence, have the following definition.

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Definition 2: Electric Power Supply Chain Network Equilibrium with Endogenous, Centralized Carbon Taxes - Fixed Upper Bound Case

The equilibrium state of the electric power supply chain network with power plants and endogenous, centralized carbon taxes with a fixed upper bound on carbon emissions is one where the electric power flows between the tiers of the network coincide and the electric power flows, the demands, and the carbon taxes satisfy the sum of conditions (4), (14), and (17) with $\mathcal{T}^*$ substituted for each τ_{gm}^*; $g = 1, \ldots, G$; $m = 1, \ldots, M$, and the centralized carbon tax equilibrium conditions (23) also hold.

The variational inequality for this centralized carbon taxation scheme (cf. (19)) is immediate and it takes the form:

Theorem 2: Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Centralized Carbon Taxes and a Fixed Upper Bound

The equilibrium conditions governing the electric power supply chain network according to Definition 2 coincide with the solution of the variational inequality given by: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*, \mathcal{T}^*) \in \mathcal{K}^6$ satisfying:

$$\sum_{g=1}^{G} \sum_{m=1}^{M} \left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \mathcal{T}^* e_{gm} \right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^{S} \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*]$$

$$+ \sum_{g=1}^{G} \sum_{m=1}^{M} \sum_{s=1}^{S} \left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}} \right] \times [q_{gms} - q_{gms}^*]$$

$$+ \sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t=1}^{T} \left[\frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*}) \right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^{K} \rho_{3k}(d^*) \times [d_k - d_k^*]$$

$$+ \left[\bar{B} - \sum_{g=1}^{G} \sum_{m=1}^{M} e_{gm} q_{gm}^* \right] \times [\mathcal{T} - \mathcal{T}^*] \geq 0, \quad \forall (q, h, Q^1, Q^2, d, \mathcal{T}) \in \mathcal{K}^4, \quad (25)$$

where

$$\mathcal{K}^5 \equiv \{(q, h, Q^1, Q^2, d, \mathcal{T}) | (q, h, Q^1, Q^2, d, \mathcal{T}) \in R_+^{GM+S+GMS+TSK+K+1}$$

$$\text{and (2), (5), (11), and (15) hold}\}.$$

Centralized Carbon Taxation Scheme - Elastic Bound

We now consider a carbon taxation scheme in which the carbon emission bound is elastic in that it is no longer fixed but is a function of the carbon tax. Such a scheme would reflect, for example, the situation where the government might be willing to assign a different bound on carbon emissions, depending upon the size of the tax. Hence, we now assume rather than a bound $\bar{B}$ as in (23), a bound denoted by $B(\mathcal{T})$ where this function is assumed to be continuous.

Centralized Carbon Tax Equilibrium Conditions – Elastic Bound

In this case, the carbon tax equilibrium conditions (cf. (23)) would be as follows.

$$B(\mathcal{T}^*) - \sum_{g=1}^{G}\sum_{m=1}^{M} e_{gm} q_{gm}^* \begin{cases} = 0, & \text{if} \quad \mathcal{T}^* > 0, \\ \geq 0, & \text{if} \quad \mathcal{T}^* = 0. \end{cases} \tag{26}$$

Clearly equilibrium conditions (23) can be formulated as the inequality:

$$\left[B(\mathcal{T}^*) - \sum_{g=1}^{G}\sum_{m=1}^{M} e_{gm} q_{gm}^* \right] \times [\mathcal{T} - \mathcal{T}^*] \geq 0, \quad \forall \mathcal{T} \geq 0. \tag{27}$$

As for the behavior of the electric power supply chain network decision-makers, it remains as described in Section 2.1, except that now we substitute $\mathcal{T}^*$ for each τ_{gm}^*; $g = 1, \ldots, G$; $m = 1, \ldots, M$, as we did for the centralized scheme in the case of a fixed upper bound for carbon emissions. Such a substitution (again) needs to be made in (1) and in (4) with the resulting definition below:

Definition 3: Electric Power Supply Chain Network Equilibrium with Endogenous, Centralized Carbon Taxes - Elastic Bound Case

The equilibrium state of the electric power supply chain network with power plants and endogenous, centralized carbon taxes with an elastic carbon emission bound is one where the electric power flows between the tiers of the network coincide and the electric power flows,

the demands, and the carbon taxes satisfy the sum of conditions (4), (14), (17), with $\mathcal{T}^*$ substituted for each τ^*_{gm}; $g = 1, \ldots, G$; $m = 1, \ldots, M$, and the centralized carbon tax equilibrium conditions (26) also holds.

The variational inequality for the centralized carbon taxation scheme with an elastic bound is also immediate:

Theorem 3: Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Centralized Carbon Taxes and an Elastic Carbon Emission Bound

The equilibrium conditions governing the electric power supply chain network according to Definition 3 coincide with the solution of the variational inequality given by: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*, \mathcal{T}^*) \in \mathcal{K}^6$ satisfying:

$$\sum_{g=1}^{G} \sum_{m=1}^{M} \left[\frac{\partial f_{gm}(q^*_m)}{\partial q_{gm}} + \mathcal{T}^* e_{gm} \right] \times [q_{gm} - q^*_{gm}] + \sum_{s=1}^{S} \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h^*_s]$$

$$+ \sum_{g=1}^{G} \sum_{m=1}^{M} \sum_{s=1}^{S} \left[\frac{\partial c_{gms}(q^*_{gms})}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q^*_{gms})}{\partial q_{gms}} \right] \times [q_{gms} - q^*_{gms}]$$

$$+ \sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t=1}^{T} \left[\frac{\partial c^t_{sk}(q^{t*}_{sk})}{\partial q^t_{sk}} + \hat{c}^t_{sk}(Q^{2*}) \right] \times [q^t_{sk} - q^{t*}_{sk}] - \sum_{k=1}^{K} \rho_{3k}(d^*) \times [d_k - d^*_k]$$

$$+ \left[B(\mathcal{T}^*) - \sum_{g=1}^{G} \sum_{m=1}^{M} e_{gm} q^*_{gm} \right] \times [\mathcal{T} - \mathcal{T}^*] \geq 0,$$

$$\forall (q, h, Q^1, Q^2, d, \mathcal{T}) \in \mathcal{K}^6. \tag{28}$$

Each of the variational inequality problems (19), (25), and (28) can be put into standard variational inequality form (cf. Nagurney [22]) given by: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K} \tag{29}$$

by defining the vector X and the function F that enters the variational inequality accordingly. Qualitative properties of existence of a solution can then be obtained for variational

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inequalities (19) and (25) by noting, first, that, due to the bound(s) on the carbon emissions, the electric power outputs are bounded. If one further assumes that the imposed tax can be reasonably assumed to be bounded (although the bound may be vary large), then we know that the feasible set will be compact and existence of a solution will then follow from the standard theory of variational inequalities (see [22]), since under our previously given assumptions, the corresponding function F is continuous. In the case of variational inequality (28), one can assume similar conditions or a coercivity condition on the corresponding function F. Monotonicity of F can be obtained for both variational inequalities (19) and (25) under analogous assumptions as those given in Nagurney and Matsypura [16]; the same holds in the case of F for variational inequality (28) with the additional assumption that the function $B(\mathcal{T})$ is monotone.

3. Numerical Examples

In this Section, we provide numerical examples to demonstrate how the theoretical results in this paper can be applied in practice.

Clearly, there are distinct variational inequality algorithms that may be applied to solve variational inequalities (19), (25), and/or (28), and, in particular, we note the modified projection method (see [24]) which was been successfully applied to solve variational inequality problems in which the function F (cf. (29)) is monotone and Lipschitz continuous; see, e.g., [16] and [25].

Wu et al. [11], in turn, proposed an Euler method for the electric power supply chain network equilibrium problem with power plants and preassigned carbon taxes. That Euler method was introduced by Dupuis and Nagurney [26] and is a special case of a general iterative scheme for the solution of variational inequalities as well as projected dynamical systems. Wu et al. [11] showed that such the electric power supply chain problem with preassigned taxes could be transformed into a transportation network equilibrium problem over an appropriately constructed abstract network or supernetwork.

In the models developed in this paper, however, in which the carbon taxes are no longer pre-assigned but, are endogenous and reflect particular goals of governmental/environmental authorities, we can no longer transform the variational inequalities (19), (25), and (28) directly into transportation network equilibrium problems (as was also done by [27] for supply chain network equilibrium problems in the case of products). However, we can still exploit the connection by noticing that the variational inequality problems in this paper are defined over feasible sets that are, in effect, decomposable into subproblems in the flows and subproblems in carbon taxes. Furthermore, the former subproblems retain the transportation network structure identified in [11]. Analogously, we can construct path flow versions of each of these variational inequalities over the corresponding abstract transportation network and there will be an additional term in each case to correspond to the particular carbon taxation scheme equilibrium condition. We can then apply the Euler method, whose general statement to solve a variational inequality given by: determine $x^* \in K$ such that

$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in K, \tag{30}$$

is given immediately following.

The Euler Method

The Euler method (see [26]) has been applied by Nagurney and Zhang [28] to solve the variational inequality governing elastic demand transportation network equilibrium problems in path flows and by Zhang and Nagurney [29] for the formulation in link flows. Convergence results can be found in the above references.

For the solution of (30), the Euler method takes the form: at iteration l compute x^{l+1} by solving the variational inequality problem:

$$x^{l+1} = P_K(x^l - a_l F(x^l)), \tag{31}$$

where P_K is the projection operator, and the sequence $\{a_l\}$ must satisfy the conditions: $\sum_{l=0}^{\infty} a_l = \infty$, $a_l > 0$, for all l, and $a_l \to 0$, as $l \to \infty$.

The Euler method was implemented in FORTRAN and the computer system used was a Sun system at the University of Massachusetts at Amherst. The convergence criterion utilized was that the absolute value of the electric flows and the carbon taxes between two successive iterations differed by no more than 10^{-6}. The sequence $\{\alpha_l\}$ in the Euler method cf. (31) was set to: $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$.

In all the numerical examples, the electric power supply chain network consisted of two power generators, with two power plants each, two power suppliers, one transmission provider, and two demand markets as depicted in Figure 2.

Power Generators
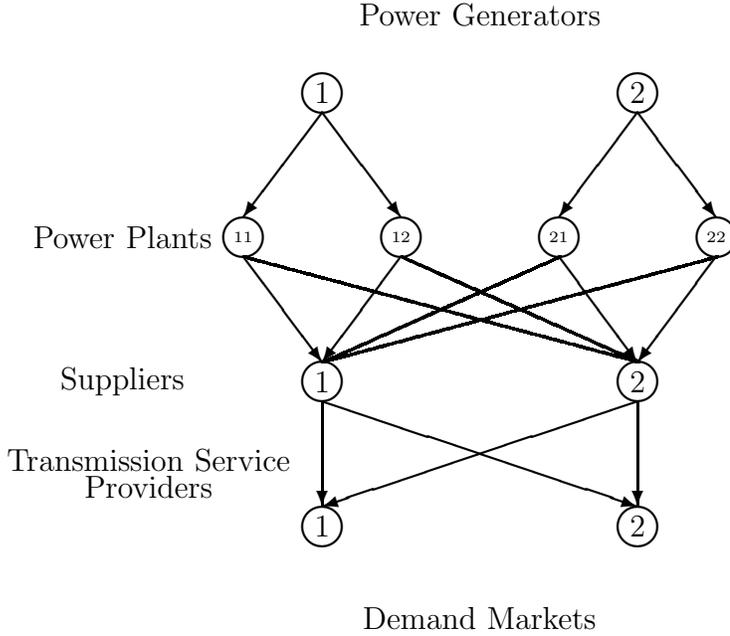
Figure 2: Electric Power Supply Chain Network for the Numerical Examples

Examples 1, 2, 3, and 4 correspond to instances of the electric power supply chain network equilibrium model with a decentralized carbon taxation scheme as outlined in Section 2.1 with variational inequality formulation given by (19). Examples 5, 6, 7, and 8, in turn, correspond to instances in the case of a centralized carbon taxation scheme with a fixed bound on the carbon emissions, as described in Section 2.2, with corresponding variational inequality given by (25). Examples 9, 10, 11, and 12 correspond to the centralized scheme but with a variable carbon emission bound and the variational inequality model is given by (28). The results of the computations are given in Tables 1, 2, and 3, respectively.

Example 1

The data for the first numerical example is given below. The functional forms of the power generating cost functions, the transaction cost functions, the operating cost functions, and the demand price functions are identical to those in Example 1 in [11].

The carbon emission terms: e_{gm}; $g = 1, 2$; $m = 1, 2$ were all equal to 1.

The power generating cost functions for the power generators were given by:

$$f_{11}(q_1) = 2.5q_{11}^2 + q_{11}q_{21} + 2q_{11}, \quad f_{12}(q_2) = 2.5q_{12}^2 + q_{11}q_{12} + 2q_{22}, \quad f_{21}(q_1) = .5q_{21}^2 + .5q_{11}q_{21} + 2q_{21},$$

$$f_{22}(q_2) = .5q_{22}^2 + q_{12}q_{22} + 2q_{22}.$$

The transaction cost functions faced by the power generators and associated with transacting with the power suppliers were given by:

$$c_{111}(q_{111}) = .5q_{111}^2 + 3.5q_{111}, \; c_{112}(q_{112}) = .5q_{112}^2 + 3.5q_{112}, \; c_{121}(q_{121}) = .5q_{121}^2 + 3.5q_{121},$$

$$c_{122}(q_{122}) = .5q_{122}^2 + 3.5q_{122},$$

$$c_{211}(q_{211}) = .5q_{211}^2 + 2q_{211}, \; c_{212}(q_{212}) = .5q_{212}^2 + 2q_{212}, \; c_{221}(q_{221}) = .5q_{221}^2 + 2q_{221},$$

$$c_{222}(q_{222}) = .5q_{222}^2 + 2q_{222}.$$

The operating costs of the power generators, in turn, were given by:

$$c_1(Q^1) = .5(\sum_{i=1}^{2} q_{i1})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2} q_{i2})^2.$$

The demand market price functions at the demand markets were:

$$\rho_{31}(d) = -1.33d_1 + 366.6, \quad \rho_{32} = -1.33d_2 + 366.6,$$

and the transaction costs between the power suppliers and the consumers at the demand markets were given by:

$$\hat{c}_{sk}^1(q_{sk}^1) = q_{sk}^1 + 5, \quad s = 1, 2; k = 1, 2.$$

All other transaction costs were assumed to be equal to zero.

In Example 1, the carbon emission bounds were: $\bar{B}_{11} = \bar{B}_{12} = \bar{B}_{21} = \bar{B}_{22} = 100$.

The computed equilibrium electric power flows and demands and the optimal taxes are given in Table 1. Note that the imposed bounds were sufficiently high so that all optimal taxes were identically equal to 0.00 since the carbon emissions generated by each genco and

power plant combination were less than the imposed bound. The total carbon emissions were: 22.56, 9.93, 22.90, and 92.38, respectively, for genco 1/power plant 1; genco 1/power plant 2, and so on.

The price of electric power at the first demand market was 268.33 and at the second demand market the price was also 268.33. The demand was 73.89 at each demand market.

The optimality/equilibrium conditions were satisfied with excellent accuracy.

Example 2

Example 2 was constructed from Example 1 as follows. It had the identical data but since genco 2/power plant 2 emitted over 92 units of carbon, we set $\bar{B}_{22} = 23$, which was approximately the next highest amount of carbon emitted by another genco/power plant combination. The computed solution is given in Table 2.

The total amounts of carbon emitted were now: 29.86, 31.17, 30.20, and 23.00 for genco 1/power plant 1; genco 1/power plant 2, and so on, respectively. Note that, as the theory predicts, the endogenous carbon taxes guaranteed that, in equilibrium, the amount of carbon emitted by each power plant would not exceed the imposed bound. Only the second genco was taxed and it was for the emissions at his second power plant. All other taxes were equal to zero since the amounts of carbon emitted did not exceed the corresponding imposed bound of 100 at each of those power plants.

The prices of electric power at the demand markets were now both equal to 290.63 and the demand for electric power was equal to 57.12 at each demand market. Hence, the consumers had an increase in the price relative to that in Example 1 since now there was an additional tax levied.

Example 3

Example 3 was constructed from Example 2. The data were identical to the data in Example 2, except that now we set all the bounds as follows: $\bar{B}_{11} = \bar{B}_{12} = \bar{B}_{21} = \bar{B}_{22} = 23$. Hence, we tightened the bounds on all the genco power plants, in comparison to the bounds imposed in Example 1 and in Example 2. Again, the computed solution is given in Table 1. Note that

in Example 3, all the optimal taxes are now positive since the amounts of carbon emitted by the various power plants are at the imposed bounds.

The price of electric power at each demand market was now equal to 305.42, whereas the demand was reduced to 46 at each demand market. Hence, tighter carbon emission bounds, as expected, resulted in higher demand market prices due to greater carbon taxes applied to all the gencos' power plants. Note that in our framework we are able to determine precisely the carbon taxes that should be assigned in order to achieve the desired environmental policy goal.

Example 4

Example 4 was constructed from Example 3 except that now the emission factor $e_{11} = 2$. Hence, its value was now doubled. The equilibrium solution is given in Table 1. Note that the electric power output of the first genco and the first power plant was essentially halved in comparison to the corresponding output in Example 3. Again, the endogenous carbon taxes had the desired effect in that the carbon emissions at each genco/power plant did not exceed the imposed bounds of 23. Interestingly, the carbon taxes increased for all the gencos and power plants although it was only the first genco and his first power plant that had its emissions factor doubled. The demand for electric power at the demand markets decreases to 40.30 at each demand market since the price of electric power has now increased and it is equal to 313.00 at both demand markets.

Example 5

Examples 5, 6, 7, and 8 assumed a single centralized carbon taxation scheme in which the bound on carbon emissions was fixed. The e_{gm}; $g = 1, 2$; $m = 1, 2$ were set equal to 1. The remaining data were as given in Example 1 except that in Example 5 we set $\bar{B} = 100$. Note that this bound represents the bound on the total amount of carbon emitted by all the power plants of all the gencos in the electric power supply chain network. The computed solution for Example 5 is given in Table 2. The demand was 50.00 at each of the two demand markets and the demand market price was 300.10.

Equilibrium Solution	Example 1	Example 2	Example 3	Example 4
Computed Equilibrium Power Flows				
q_{11}^*	22.56	29.86	23.00	11.51
q_{12}^*	9.93	31.17	23.00	23.02
q_{21}^*	22.90	30.20	23.00	23.02
q_{22}^*	92.38	23.00	23.00	23.05
q_{111}^*	11.28	14.93	11.50	5.76
q_{112}^*	11.28	14.93	11.50	5.76
q_{121}^*	4.97	15.59	11.50	11.51
q_{122}^*	4.97	15.59	11.50	11.51
q_{211}^*	11.45	15.10	11.50	11.51
q_{212}^*	11.45	15.10	11.50	11.51
q_{221}^*	46.19	11.50	11.50	11.52
q_{222}^*	46.19	11.50	11.50	11.52
h_1^*	73.89	57.12	46.00	40.30
h_2^*	73.89	57.12	46.00	40.30
q_{11}^{1*}	36.94	28.56	23.00	20.15
q_{12}^{1*}	36.94	28.56	23.00	20.15
q_{21}^{1*}	36.94	28.56	23.00	20.15
q_{22}^{1*}	36.94	28.56	23.00	20.15
Computed Equilibrium Demands				
d_1^*	73.89	57.12	46.00	40.30
d_2^*	73.89	57.12	46.00	40.30
Computed Optimal Taxes				
τ_{11}^*	0.00	0.00	76.43	77.86
τ_{12}^*	0.00	0.00	76.43	92.38
τ_{21}^*	0.00	0.00	77.93	105.41
τ_{22}^*	0.00	130.26	169.93	185.96

Table 2: Solutions to Examples 1, 2, 3, and 4

Example 6

Example 6 had data identical to that in Example 5 except that we now tightened the bound on the carbon emissions and had that $\bar{B} = 50$. The computed solution is given in Table 2. The demand for electric power at each demand market was 50.00 with the demand market price now rising to 333.35 at each demand market since the tax is now 236.38 as opposed to being only 115.50 in Example 5.

Example 7

Example 7, in turn, had the same data as Example 6 (and Example 5) except that we further tightened the carbon emission bound so that $\bar{B} = 20$. The computed equilibrium solution is given in Table 2. The demand is now 10.0 at each demand market and the demand market price rises to 353.28 with the new carbon tax being equal to 308.84.

Example 8

Example 8 was constructed from Example 7 and had the same data except that we changed the second demand price function to

$$\rho_{32}(d) = -1.33d_2 + 733.3,$$

which means that the consumers at the second demand market are willing to pay a higher price for electric power than in Examples 5 through 7. The computed solution is given in Table 2. The results are quite interesting in that now the first demand market has zero demand for electric power.

The final set of examples, consisting of Examples 9, 10, 11, and 12, assumed that the carbon taxation scheme was centralized but now the bound was a function of the tax.

Equilibrium Solution	Example 5	Example 6	Example 7	Example 8
Computed Equilibrium Power Flows				
q_{11}^*	15.20	7.48	2.85	2.87
q_{12}^*	6.63	3.17	1.10	1.10
q_{21}^*	15.53	7.82	3.19	3.20
q_{22}^*	62.65	31.53	12.86	12.91
q_{111}^*	7.60	3.74	1.43	1.43
q_{112}^*	7.60	3.74	1.43	1.43
q_{121}^*	3.31	1.59	0.55	0.55
q_{122}^*	3.31	1.59	0.55	0.55
q_{211}^*	7.76	3.91	1.59	1.60
q_{212}^*	7.76	3.91	1.59	1.60
q_{221}^*	31.32	15.77	6.43	6.46
q_{222}^*	31.32	15.77	6.43	6.46
h_1^*	50.00	25.00	10.00	10.00
h_2^*	50.00	25.00	10.00	10.00
q_{11}^{1*}	25.00	12.50	5.00	0.00
q_{12}^{1*}	25.00	12.50	5.00	10.00
q_{21}^{1*}	25.00	12.50	5.00	0.00
q_{22}^{1*}	25.00	12.50	5.00	10.00
Computed Equilibrium Demands				
d_1^*	50.00	25.00	10.00	0.00
d_2^*	50.00	25.00	10.00	20.00
Computed Optimal Tax				
$\mathcal{T}^*$	115.50	236.38	308.91	656.96

Table 3: Solutions to Examples 5, 6, 7, and 8

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Example 9

Example 9 had the same input data as Example 5 but the bound on the carbon emissions was now elastic and given by:

$$B(\mathcal{T}) = \mathcal{T} + 100.$$

The equilibrium solution is reported in Table 3. The total carbon emissions are equal to 133.79 with the value of $B(\mathcal{T}^*){=}133.79$. The demand is 66.90 at each demand market and the demand market price is 277.63 at each demand market. The optimal carbon tax $\mathcal{T}^* = 33.79$.

Example 10

Example 10 had the same data as Example 9 except that the carbon emission bound function was now given by:

$$B(\mathcal{T}) = \mathcal{T} + 50.$$

The computed equilibrium solution is reported in Table 3. The total carbon emissions are now equal to 119.16 and this is also the value of $B(\mathcal{T}^*)$. The demand for electric power is equal to 59.58 at each demand market and the demand market price is 287.36 at each demand market. The optimal carbon tax is now $\mathcal{T}^* = 69.16$. Hence, the tax is now higher than in Example 9 and the emissions are reduced. The demand market prices increase due to the higher tax.

Example 11

Example 11 had the same data as the two preceding examples but differed in the elastic carbon emission bound function which was now even "tighter" and given by:

$$B(\mathcal{T}) = \mathcal{T} + 20.$$

The computed electric power flows, demands, and the optimal carbon tax are reported in Table 3.

The demand for electric power is now 55.19 at each demand market. The optimal tax is now $\mathcal{T}^* = 90.39$ and the total carbon emissions generated equal to 110.39, which is

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equal to the bound $B(\mathcal{T}^*)$. Recall that in this example, as in the two preceding ones, the emission terms e_{gm}; $g = 1, 2$; $m = 1, 2$ are all equal to 1. The demand is now 55.19 at each demand market and the demand market price is now 293.19 at each demand market. Hence, the carbon tax increases as do the demand market prices whereas the demands for electric power decrease, in comparison to the values in Examples 9 and 10.

Example 12

Example 12 was constructed from Example 10 and had the identical data except that we now modified the second demand market demand function to be as in Example 8. The solution computed by the Euler method is given in Table 3. The demand for electric power at the first demand market now drops to zero (as it did in Example 8) whereas the demand for electric power at the second demand market is now equal to 178.61. The optimal carbon tax is now $\mathcal{T}^* = 128.61$ and the demand market price is equal to 406.34 at the first demand market and to 495.65 at the second demand market. The total carbon emitted is now equal to 178.61 which is also the value of the bound $B(\mathcal{T}^*)$.

Equilibrium Solution	Example 9	Example 10	Example 11	Example 12
Computed Equilibrium Power Flows				
q_{11}^*	20.41	18.15	16.80	27.32
q_{12}^*	8.96	7.95	7.35	12.06
q_{21}^*	20.74	18.48	17.13	27.65
q_{22}^*	83.68	74.58	69.11	111.57
q_{111}^*	10.20	9.08	8.40	13.66
q_{112}^*	10.20	9.08	8.40	13.66
q_{121}^*	4.48	3.98	3.67	6.03
q_{122}^*	4.48	3.98	3.67	6.03
q_{211}^*	10.37	9.24	8.57	13.83
q_{212}^*	10.37	9.24	8.57	13.83
q_{221}^*	41.84	37.29	34.56	55.79
q_{222}^*	1.84	37.29	34.56	55.79
h_1^*	66.90	59.58	55.19	89.31
h_2^*	66.90	59.58	55.19	89.31
q_{11}^{1*}	33.45	29.79	27.60	0.00
q_{12}^{1*}	33.45	29.79	27.60	89.31
q_{21}^{1*}	33.45	29.79	27.60	0.00
q_{22}^{1*}	33.45	29.79	27.60	89.31
Computed Equilibrium Demands				
d_1^*	66.90	59.58	55.19	0.00
d_2^*	66.90	59.58	55.19	178.61
Computed Optimal Tax				
$\mathcal{T}^*$	33.79	69.16	90.38	128.61

Table 4: Solutions to Examples 9, 10, 11, and 12

4. Summary and Conclusions

In this paper, we presented a modeling and computational framework that may help policy-makers to determine the optimal carbon taxes on the power plants in the electric power generation industry. This general electric power supply chain network modeling framework utilizes variational inequality theory and is capable of identifying the optimal carbon taxes as well as the equilibrium electric power transaction flows and the demands for electric power (along with the associated prices) under three distinct carbon taxation environmental policies. Specifically, the first model, a completely decentralized scheme, allows the policy-makers to determine the optimal tax for each individual electric power plant which guarantees that the emission bound or quota of each plant is not exceeded. The second and third models, on the other hand, both enforce a "global" emission bound on the entire industry by imposing a uniform tax rate on the generating plants. However, the second policy assumes that the global emission bound is fixed while the third policy allows the bound to be a function of the tax.

We then discovered that the three variational inequality models were decomposable into subproblems that can be efficiently solved using the Euler method. Twelve numerical examples were presented to illustrate the impacts of the three distinct carbon taxation policies on the electric power supply chain networks. These numerical examples also demonstrate how policy-makers can determine the optimal taxes in order to achieve environmental objectives.

This research is a contribution to the growing research in the development of rigorous mathematical frameworks for environmental-energy modeling.

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