A Bi-Criteria Measure to Assess
Supply Chain Network Performance for Critical Needs
Under Capacity and Demand Disruptions

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Abstract: In this paper, we develop a supply chain/logistics network model for critical needs in the case of disruptions. The objective is to minimize the total network costs, which are generalized costs that may include the monetary, risk, time, and social costs. The model assumes that disruptions may have an impact on both the network link capacities as well as on the product demands. Two different cases of disruption scenarios are considered. In the first case, we assume that the impacts of the disruptions are mild and that the demands can be met. In the second case, the demands cannot all be satisfied. For these two cases, we propose two individual performance measures. We then construct a bi-criteria measure to assess the supply chain network performance for critical needs. An algorithm is described which is applied to solve a spectrum of numerical examples in order to illustrate the new concepts.

Key words: supply chains, disruptions, critical needs, network vulnerability, performance measurement, disaster planning, emergency preparedness
1. Introduction

Critical needs may be defined as those products and supplies that are essential to human health and life. Examples include food, water, medicines, and vaccines. The demand for critical needs is always present and, hence, the disruption to the production, storage, transportation/distribution, and ultimate delivery of such products can result not only in discomfort and human suffering but also in loss of life.

Critical needs supply chains also play a pivotal role during and post disasters during which severe disruptions can be expected to have occurred. Indeed, the past few decades have visibly demonstrated that disasters, whether natural or man-made, may severely damage infrastructure networks, such as transportation and logistical networks, may cause great loss to human life, and also may result in tremendous damage to a nation’s economy. Furthermore, according to Braine (2006), from January to October 2005 alone, an estimated 97,490 people were killed in disasters globally; 88,117 of them lost their lives because of natural disasters. Some of the deadliest examples of disasters that have been witnessed in the past few years were the September 11 attacks in 2001, the tsunami in South Asia in 2004, Hurricane Katrina in 2005, Cyclone Nargis in 2008, the earthquake and aftershocks in Sichuan, China in 2008 (see, e.g., Nagurney and Qiang (2009a)), and the earthquake in Haiti that struck in January 2010. The Haiti earthquake was the worst earthquake in the region in more than 200 years and according to The New York Times (2011), a study by the Inter-American Development Bank estimated that the total cost of the disaster was between $8 billion to $14 billion, based on a death toll from 200,000 to 250,000, which was revised in 2011 by Haiti’s government to 316,000.

Critical needs supply chains, hence, are essential in both healthcare as well as in humanitarian logistics operations. Given their importance also in terms of emergency preparedness and planning, special attention to them is needed, since their functions are so important to the well-being and, in effect, the very survival of our societies.

Specifically, in the case of disruptions to critical needs supply chains, there are two primary parameters that may be affected. These are: 1. the capacities of the various supply chain network activities (production, storage, transportation, etc.) and 2. the demands for the products. Indeed, as shown by numerous recent incidences, disruptions may tremendously reduce supply chain capacities as well as impact the demands for critical needs products.

For example, in 2004, Chiron Corporation, a vaccine manufacturer with a plant in Bristol, England, experienced contamination in its production processes which resulted in flu vaccine supplies to the US being reduced by 50% (cf. Fink (2004)). As another example, the winter
storm in China in 2008 destroyed crop supplies (over 40,000km²), which, in turn, resulted in sharp food price inflation. Furthermore, more than 80 million people in China suffered from power outages as a consequence of that winter storm due to missing coal deliveries, which were disrupted because of the storm’s blockage of the transportation infrastructure (BBC News (2008)). Although some researchers have studied random capacities when suppliers cannot satisfy orders (see, e.g., Zimmer (2002) and Babich (2009)), there are few studies that address the impact of disruptions on supply chain capacities (cf. Snyder et al. (2006)).

Moreover, since demands may also be highly uncertain in critical needs supply chains (cf. Sheu (2007)) uncertainty in demands is also a big challenge. In the case of Hurricane Katrina, for example, overestimation of the demand for certain products resulted in a surplus of supplies, with, ultimately, $81 million of MREs (Meals Ready to Eat) being destroyed by FEMA (cf. Stamm and Villarreal (2009)). On the other hand, governmental and medical organizations did not anticipate the chronic medical needs of those suffering in the aftermath of Hurricane Katrina. Of the approximately 1,000,000 individuals evacuated after Katrina, about 100,000 suffered from diabetes, which requires daily medical supplies and attention. Such needs and demand, especially in shelters to which the victims had been evacuated, caught the logistics chain completely off-guard (see Cefalu et al. (2006)). This was a dramatic example in which the demands for critical need products were severely underestimated. Another example occurred as a consequence of the tsunami in 2004 with pharmaceutical storage areas damaged, resulting in significant short-term shortages of medications (cf. Yamada et al. (2006)). Indeed, as noted in Developments (2006), “thousands of lives could have been saved in the tsunami and other recent disasters if simple, cost effective measures like evacuation training and storage of food and medical supplies had been put in place to protect vulnerable communities.”

In 2009, the World Health Organization declared an H1N1 (swine) flu pandemic (see World Health Organization (2009)). Hence, we continue to see additional examples of critical needs supply chains that did not achieve goals of demand satisfaction – most significantly, in the healthcare arena, in the case of flu vaccine production as evidenced from flu vaccine shortages (both seasonal and H1N1 (swine) ones) in parts of the globe with citizens clamoring for vaccines. Flu medicines were also in short supply in 2009 in parts of the world (cf. Belluck (2009)). According to McNeil (2009), the five corporations that are licensed to make seasonal flu vaccine shots for the US (see also Dooren (2009)): GlaxoSmithKline, Novartis, Sanofi-Aventis, CSL, and Medimmune, originally planned on producing only slightly more than 118 million units of the seasonal flu vaccine that they produced the year before. However, GlaxoSmithKline, because of production problems, cut its run by half, whereas Novartis’s
yield was reduced by 10 percent. Subsequently, all five producers had to switch their vaccine production from the seasonal flu to the H1N1 (swine) flu vaccine. Shortages of seasonal flu vaccine were chronic in the US in 2009 in nursing homes with federal officials beginning to intervene since the elderly are the most vulnerable to seasonal flu. From the above examples, we can see that when facing disruptions, the demands and capacities for critical needs supply chain can both change, oftentimes with demands increasing and capacities decreasing. This makes supply chain management under disruptions even more difficult.

In the case of the earthquake that struck Haiti on January 12, 2010, as reported in Robbins (2010), many of the immediate challenges involved transportation and logistics, with planes not being able to land, only a single warehouse available for the distribution of relief supplies, the major port closed, and many roads decimated. Moreover, there was no coordinated plan by which the relief supplies that did ultimately arrive at the airport could be distributed.

Since the goals of supply chains for critical needs are quite different from those of commercial supply chains, they should be evaluated by distinct sets of metrics (cf. Beamon (1998, 1999), Lee and Whang (1999), Lambert and Pohlen (2001), and Lai, Ngai, and Cheng (2002)). As pointed out by Beamon and Balci̇k (2008), the goals for humanitarian relief chains, for example, include cost reduction, capital reduction, and service improvement (see also Altay and Green (2006)). Tomasini and van Wassenhove (2004), similarly, argued that “A successful humanitarian operation mitigates the urgent needs of a population with a sustainable reduction of their vulnerability in the shortest amount of time and with the least amount of resources.”

Qiang, Nagurney, and Dong (2009) studied supply chain disruptions and uncertain demands and developed a model to capture the impacts of disruptions as the random parameters in the cost functions. The authors further proposed a comprehensive performance measure to assess commercial supply chains. Wilson (2007), in turn, overviewed the impact of transportation disruptions on supply chain performance. For a recent edited volume on supply chain disruptions, see Wu and Blackhurst (2009). In this paper, however, we are interested in investigating supply chains for critical needs and the corresponding appropriate performance measures.

In this paper, hence, we first define a performance measure in the case that the demands can be satisfied. We then consider the case when not all the demands can be satisfied and define another performance measure. In order to assist cognizant organizations, such as governments, relevant corporations, and NGOs, to better manage critical needs supply
chains, a bi-criteria performance measure is, subsequently, proposed in this paper. This measure synthesizes the preceding two in that it considers the following factors:

- Supply chain capacities may be affected by disruptions;
- Demands may be affected by disruptions; and
- Disruption scenarios are categorized into two types.

Due to their special nature and characteristics, supply chains for critical needs (cf. Nagurney (2008) and Nagurney, Yu, and Qiang (2011)) deliver critical products, especially in times of crises. Therefore, a system-optimization approach is mandated since the demands for critical supplies should be met (as nearly as possible) at minimal total cost (see also Nagurney, Woolley, and Qiang (2008) and Tomasini and van Wassenhove (2009)). The use of a profit maximization criterion may not appeal to stakeholders, whereas a cost minimization one demonstrates social responsibility and sensitivity. The modeling approach in this paper relies on advances both in transportation network modeling and supply chain analysis since it has been shown that there exist close relationships between these two network systems (cf. Nagurney (2006, 2009)). Our work differs from the model in Nagurney, Yu, and Qiang (2011), in that here we provide measures for existing supply chain networks for critical needs, that are subject to disruptions, rather than designing such networks from scratch or redesigning existing ones, in terms of adding capacity.

In the transportation literature, researchers have proposed methodologies to analyze transportation network vulnerability and robustness. For example, Taylor, Sekhar, and D’Este (2006) applied three accessibility indices, namely, the change in generalized travel cost, the Hansen integral accessibility index, and the ARIA index, to study the vulnerability of the Australian road network. A node is considered to be vulnerable if the loss (or substantial degradation) of a small number of links significantly diminishes the accessibility of the node while a link is deemed to be critical if the loss (or substantial degradation) of the link significantly diminishes the accessibility of the network or of particular nodes. In a series of papers, Nagurney and Qiang (2007a, 2009b) studied transportation network robustness when the link capacities degrade in both user-optimized and system-optimized settings.

Nagurney and Qiang (2007b,c, 2008a), in turn, proposed a network efficiency / performance measure to incorporate such crucial network characteristics as decision-making induced flows and costs, in order to assess the importance of network components in a plethora of network systems, including transportation networks. This network measure has
significant advantages in that it explicitly considers congestion, which is a major problem of network systems and, in particular, of transportation networks today. Moreover, the efficiency measure, which is based on user-optimization, can handle both fixed and elastic demand network problems (cf. Qiang and Nagurney (2008)) plus time-dependent, dynamic networks (see Nagurney and Qiang (2008b)), of specific relevance to the Internet (see Nagurney, Parkes, and Daniele (2007)). In addition, the measure allows for the ranking of network components, that is, the nodes and links, or combinations thereof, in terms of their importance from an efficiency / performance standpoint. This has significant implications for planning, maintenance, and emergency and disaster preparedness purposes as well as for national security, in scenarios in which the outright destruction of nodes and links, or combinations thereof may occur. Jenelius, Petersen, and Mattsson (2006) also studied transportation network vulnerability and proposed several link importance indicators. The authors applied these indicators to the road transportation network in northern Sweden. These indicators are distinct, depending upon whether or not there exist disconnected origin and destination pairs in a network. However, it is worth pointing out that in this paper, we investigate the disruption impact on capacities and on demands in critical needs supply chains, with a focus on system-optimization, and we do not discuss the importance of individual network components. For additional literature, please see the book by Nagurney and Qiang (2009a).

This paper is organized as follows: in Section 2, we introduce the supply chain network model for critical needs and discuss two different disruption cases. The bi-criteria supply chain performance measure is then defined in Section 3. In Section 4, we illustrate the concepts via a spectrum of numerical examples. The paper concludes with Section 5.
2. The Supply Chain Network Model for Critical Needs Under Disruptions

In this Section, we develop the supply chain network model for critical needs. We assume that the organization (such as the government, cognizant corporation, humanitarian organization, etc.) responsible for ensuring that the demand for the essential product be met is considering the possible supply chain activities, associated with the product (be it food, water, medicine, vaccine, etc.), which are represented by a network. For clarity and definiteness, we consider the network topology depicted in Figure 1 but emphasize that the modeling framework developed here is not limited to such a network. Indeed, as will become apparent, what is required, to begin with, is the appropriate network topology with a top level (origin) node 1 corresponding to the organization and the bottom level (destination) nodes corresponding to the demand points that the organization must supply.

The paths joining the origin node to the destination nodes represent sequences of supply chain network activities that ensure that the product is produced and, ultimately, delivered to those in need at the demand points. Hence, different supply chain network topologies to that depicted in Figure 1 correspond to distinct supply chain network problems. For example, if a product can be delivered directly to the demand points from a manufacturing plant, then there would be a direct link joining the corresponding nodes.

In particular, as depicted in Figure 1, we assume that the organization is considering \( n_M \) manufacturing facilities/plants; \( n_D \) distribution centers, but must serve the \( n_R \) demand points with respective demands given by: \( d_{R_1}, d_{R_2}, \ldots, d_{R_{n_R}} \). The links from the top-tiered node 1 are connected to the possible manufacturing nodes of the organization, which are denoted, respectively, by: \( M_1, \ldots, M_{n_M} \), and these links represent the manufacturing links. The links from the manufacturing nodes, in turn, are connected to the possible distribution center nodes of the organization, and are denoted by \( D_{1,1}, \ldots, D_{n_D,1} \). These links correspond to the possible transportation links between the manufacturing plants and the distribution centers where the product will be stored. The links joining nodes \( D_{1,1}, \ldots, D_{n_D,1} \) with nodes \( D_{1,2}, \ldots, D_{n_D,2} \) correspond to the possible storage links. Finally, there are possible transportation links joining the nodes \( D_{1,2}, \ldots, D_{n_D,2} \) with the demand nodes: \( R_1, \ldots, R_{n_R} \).

We denote the supply chain network consisting of the graph \( G = [N, L] \), where \( N \) denotes the set of nodes and \( L \) the set of links. Let \( L \) denote the links associated with supply chain activities. We further denote \( n_L \) as the number of links in the link set. Note that \( G \) represents the topology of the full supply chain network.

As mentioned in the Introduction, the formalism that we utilize is that of system-optimization, where the organization wishes to determine which manufacturing plants it
should operate and at what level; the same for the distribution centers. We assume that the organization seeks to minimize the total costs associated with its production, storage, and transportation activities.

We assume that there are $\omega$ different disruption scenarios that can affect the capacities of the links on the supply chain network as well as demands. We further assume that among these $\omega$ disruption scenarios, there are $\omega^1$ cases where the demands at the demand points can be satisfied while there are $\omega^2$ scenarios where the demands cannot be met. Hence, we define two disruption scenario sets, namely, $\Xi^1 \equiv \{\xi_1^1, \xi_2^1, \ldots, \xi_{\omega^1}^1\}$ and $\Xi^2 \equiv \{\xi_1^2, \xi_2^2, \ldots, \xi_{\omega^2}^2\}$. We further denote $\xi^0$ as the scenario where there is no disruption. Moreover, we associate each disruption scenario with a probability, which are defined, respectively, as $p_{\xi_1^1}, p_{\xi_2^1}, \ldots, p_{\xi_{\omega^1}^1}$ and $p_{\xi_1^2}, p_{\xi_2^2}, \ldots, p_{\xi_{\omega^2}^2}$. In addition, $p_{\xi^0}$ is defined as the probability where no disruption happens. These disruption scenarios are assumed to be independent.

Note that we use discrete probabilities to study disruption scenarios in this paper. This is due to the fact that in the case of the majority of relevant disruptions, especially those caused by natural disasters, there is a lack of historical data that would enable the generation of more detailed probabilities, as in the case of continuous probabilities. Therefore, discrete probabilities, which sometimes may rely on experts’ subjective judgment, provide us with a good starting point. Obviously, if we are equipped with enough historical data, then we will be able to study supply chain performance under disruptions more comprehensively.

Associated with each link (cf. Figure 1) of the network is a total cost that reflects the total cost of operating the particular supply chain activity, that is, the manufacturing of the product, the transportation of the product, the storage of the product, etc. We denote, without any loss in generality, the links by $a, b$, etc., and the total cost on a link $a$ by $\hat{c}_a$. For the sake of generality, we note that the total costs are generalized costs and may include, for example, risk, time, etc. We also emphasize that the model is, in effect, not restricted to the topology in Figure 1.

A path $p$ in the network (see, e.g., Figure 1) joining node 1, which is the origin node, to a demand node, which is a destination node, represents the activities and their sequence associated with producing the product and having it, ultimately, delivered to those in need. Let $w_k$ denote the pair of origin/destination (O/D) nodes $(1, R_k)$ and let $P_{w_k}$ denote the set of paths, which represent alternative associated possible supply chain network processes, joining $(1, R_k)$. $P$ then denotes the set of all paths joining node 1 to the demand nodes. Let $n_P$ denote the number of paths from the organization to the demand points.
Figure 1: Topology of Supply Chain Network for Critical Needs
Let \( x_p \) represent the nonnegative flow of the product on path \( p \) joining (origin) node 1 with a (destination) demand node that the organization is to supply with the critical product.

2.1 Case I: Demands Can be Satisfied Under Disruptions

In this case, we know that the impacts of disruptions are milder in that the demands can be satisfied. As discussed above, we are referring, in this case, to the disruption scenario set \( \Xi^1 \).

For convenience of expression, let

\[
v_{\xi^1_i}^{k} \coloneqq \sum_{p \in P_{\xi^1_i}} x_{\xi^1_i}^p \delta_{ap}, \quad k = 1, \ldots, n_R, \forall \xi^1_i \in \Xi^1; \ i = 1, \ldots, \omega^1,
\]

where \( v_{\xi^1_i}^{k} \) is the demand at demand point \( k \) under disruption scenario \( \xi^1_i \); \( k = 1, \ldots, n_R \) and \( i = 1, \ldots, \omega^1 \).

In addition, let \( f_{\xi^1_i}^a \) denote the flow of the product on link \( a \) under disruption scenario \( \xi^1_i \). Hence, we must have the following conservation of flow equations satisfied:

\[
f_{\xi^1_i}^a = \sum_{p \in P} x_{\xi^1_i}^p \delta_{ap}, \quad \forall a \in L, \forall \xi^1_i \in \Xi^1; \ i = 1, \ldots, \omega^1,
\]

that is, the total amount of a product on a link is equal to the sum of the flows of the product on all paths that utilize that link.

Of course, we also have that the path flows must be nonnegative, that is,

\[
x_{\xi^1_i}^p \geq 0, \quad \forall p \in P, \forall \xi^1_i \in \Xi^1; \ i = 1, \ldots, \omega^1,
\]

since the product will be produced in nonnegative quantities.

We group the path flows, the link flows, and the demands into the respective vectors \( x_{\xi^1_i}, f_{\xi^1_i}, \) and \( v_{\xi^1_i} \).

The total cost on a link, be it a manufacturing/production link, a transportation link, or a storage link is assumed to be a function of the flow of the product on the link; see, for example, Nagurney (2006, 2009) and the references therein. We have, thus, that

\[
\hat{c}_a = \hat{c}_a(f_{\xi^1_i}^a), \quad \forall a \in L, \forall \xi^1_i \in \Xi^1; \ i = 1, \ldots, \omega^1.
\]

We further assume that the total cost on each link is convex and continuously differentiable. We denote the nonnegative capacity on a link \( a \) under disruption scenario \( \xi^1_i \) by \( u_{\xi^1_i}^a \), \( \forall a \in L, \forall \xi^1_i \in \Xi^1 \), with \( i = 1, \ldots, \omega^1 \).
The supply chain network optimization problem for critical needs faced by the organization can be expressed as follows. The organization seeks to determine the optimal levels of product processed on each supply chain network link subject to the minimization of the total cost. Hence, under the disruption scenario $\xi^1$, the organization must solve the following problem:

$$\text{Minimize } TC_{\xi^1} = \sum_{a \in L} \hat{c}_a(f^1_{\xi^1})$$

subject to: constraints (1), (2), (3), and

$$f^1_{\xi^1} \leq u^1_{\xi^1}, \quad \forall a \in L. \quad (6)$$

Constraint (6) guarantees that the product flow on a link does not exceed that link’s capacity. We let $TC^0$ denote the minimum total network cost when there is no disruption, which is obtained by solving the above optimization problem under no disruptions, that is, under the original capacities and demands.

Clearly, the solution of the above optimization problem will yield the product flows that minimize the total supply chain costs faced by the organization. Under the above imposed assumptions, the optimization problem is a convex optimization problem.

We associate the Lagrange multiplier $\lambda^1_{\xi^1}$ with constraint (6) for link $a \in L$ and we denote the associated optimal Lagrange multiplier by $\lambda^1_{\xi^1 \ast}$. $\lambda^1_{\xi^1}$ may also be interpreted as the shadow price or value of an additional unit of capacity on link $a$ under disruption scenario $\xi^1$. We group these Lagrange multipliers under disruption scenario $\xi^1$ into the vector $\lambda^1_{\xi^1}$.

Let $K^1_{\xi}$ denote the feasible set such that

$$K^1_{\xi} \equiv \{(x^1_{\xi}, \lambda^1_{\xi})| x^1_{\xi} \text{ satisfies (1)}, x^1_{\xi} \in \mathbb{R}^{n_P}_{+} \text{ and } \lambda^1_{\xi} \in \mathbb{R}^{n_L}_{+} \}.$$

We now state the following result in which we provide variational inequality formulations of the problem in both path flows and in link flows, respectively.

**Theorem 1**

The optimization problem (5), subject to constraints (1) – (3) and (6), is equivalent to the variational inequality problem: determine the vector of optimal path flows and the vector of optimal Lagrange multipliers $(x^1_{\xi \ast}, \lambda^1_{\xi \ast}) \in K^1_{\xi}$, such that:

$$\sum_{k=1}^{n_P} \sum_{p \in P_{\nu_k}} \left[ \frac{\partial \hat{C}_p(x^{1\ast})}{\partial x_p} + \sum_{a \in L} \lambda^1_{\xi \ast} \delta^{ap} \right] \times [x^1_{\xi} - x^{1\ast}_{\xi}]$$
+ \sum_{a \in L} [u_a^{\xi_i} - \sum_{p \in P} x_p^{\xi_i} \delta_{ap}] \times [\lambda_a^{\xi_i} - \lambda_a^{\xi_i*}] \geq 0, \quad \forall (x^{\xi_i}, \lambda^{\xi_i}) \in K^{\xi_i}, \tag{7}

where \( \partial \hat{C}_p(x^{\xi_i}) / \partial x_p \equiv \sum_{a \in L} \partial \hat{c}_a(f^{\xi_i*}) / \partial f_a \delta_{ap} \) for paths \( p \in P_{wk} \); \( k = 1, \ldots, n_R \).

In addition, (7) can be reexpressed in terms of links flows as: determine the vector of optimal link flows and the vector of optimal Lagrange multipliers \((f^{\xi_i*}, \lambda^{\xi_i*}) \in K^{\xi_i} \), such that:

\[
\sum_{a \in L} \left[ \frac{\partial \hat{c}_a(f_a^{\xi_i*})}{\partial f_a} + \lambda_a^{\xi_i*} \right] \times [f_a^{\xi_i} - f_a^{\xi_i*}]
+ \sum_{a \in L} [u_a^{\xi_i} - f_a^{\xi_i*}] \times [\lambda_a^{\xi_i} - \lambda_a^{\xi_i*}] \geq 0, \quad \forall (f^{\xi_i}, \lambda^{\xi_i}) \in K^{\xi_i}, \tag{8}
\]

where \( K^{\xi_i} \equiv \{(f^{\xi_i}, \lambda^{\xi_i}) | \exists x \geq 0, \text{ and (1), (2), and (3) hold, and } \lambda^{\xi_i} \geq 0 \} \).

**Proof:** See Bertsekas and Tsitsiklis (1989) page 287.

Note that both variational inequalities (7) and (8) can be put into standard form (see Nagurney (1993)): determine \( X^* \in K^{\xi_i} \) such that:

\[
\langle F(X^{\xi_i*})^T, X^{\xi_i} - X^{\xi_i*} \rangle \geq 0, \quad \forall X^{\xi_i} \in K^{\xi_i}, \tag{9}
\]

where \( \langle \cdot, \cdot \rangle \) denotes the inner product in \( n \)-dimensional Euclidean space, and \( X, F(X) \), and the feasible set \( K^{\xi_i} \) are defined accordingly.

Specifically, variational inequality (8) can be easily solved using the modified projection method (cf. Korpelevich (1977) and Nagurney (1993)), provided that \( F(X) \) is monotone and Lipschitz continuous and that a solution exists. Monotonicity and Lipschitz continuity of \( F(X) \) can be expected to hold in practice (see also, e.g., Nagurney (1993)). As for existence, we note that the original problem that we are solving is characterized by bounded demands and link flows, and, hence, bounded path flows.

### 2.2 Case II: Demands Cannot be Satisfied Under Disruptions

In this case, disruptions have a more significant impact and the demands cannot all be satisfied. As discussed earlier, the disruption scenario set in this case is \( \Xi^2 \); that is to say, the optimization problem (5) is not feasible anymore. We more fully study the supply chain network performance in this case in Section 3.
3. Performance Measurement of Supply Chain Networks for Critical Needs

Based on the above two cases, we propose a bi-criteria performance measure to evaluate the performance of supply chain networks for critical needs. First, the definition of each performance criterion is given. We then define the bi-criteria supply chain performance measure.

**Definition 1: Performance Measure I: Demands Can be Satisfied**

For a supply chain network $G$ with total cost vector $\hat{c}$, if all the demands can be satisfied, the performance criterion is to assess the supply chain network’s cost-efficiency, which is defined in (10). For disruption scenario $\xi_1^i$, the corresponding network performance measure is:

$$E_{\xi_1^i}(G, \hat{c}, v_{\xi_1^i}) = \frac{TC_{\xi_1^i} - TC^0}{TC^0}, \tag{10}$$

where $TC^0$ is the minimum total cost obtained as the solution to the cost minimization problem (5), subject to constraints (1) – (3), and (6), under no disruptions for the particular supply chain network, as discussed in Section 2.

According to (10), a supply chain network is considered to be more cost-efficient under disruption scenario $\xi_1^i$ if $E_{\xi_1^i}$ is low, which means that a lower total cost increase is needed in order to satisfy the demands.

For a critical needs supply chain network, the most fundamental task is to accommodate all the demands under disruptions since the network delivers lifeline products. Therefore, we propose another performance measure in (11).

**Definition 2: Performance Measure II: Demands Cannot be Satisfied**

For disruption scenario $\xi_2^i$, the corresponding network performance measure is:

$$E_{\xi_2^i}(G, \hat{c}, v_{\xi_2^i}) = \frac{TD_{\xi_2^i} - TSD_{\xi_2^i}}{TD_{\xi_2^i}}, \tag{11}$$

where $TSD_{\xi_2^i}$ is the total satisfied demand and $TD_{\xi_2^i}$ is the total (actual) demand under disruption scenario $\xi_2^i$.

According to definition (11), a critical needs supply chain network fulfills its primary goal well if $E_{\xi_2^i}$ is low since $TD_{\xi_2^i} - TSD_{\xi_2^i}$ is the total unsatisfied demand under disruption scenario $\xi_2^i$. It is easy to see that $E_{\xi_2^i}$ has a lower bound of 0 (not included). Here, our goal is to examine the overall functionality of a critical needs supply chain network. As to the
details of how much of the demand is satisfied (or not) at each demand point is an operational level issue and is not the focus of this paper. Furthermore, in this paper, we assume that the primary goal is to satisfy the demands for the critical needs product. Therefore, we do not penalize the situation when there exists an oversupply of the product.

In particular, when evaluating the network performance, in order to check if the demands can be satisfied under a disruption scenario, we can first solve the maximum flow problem (cf. Ahuja, Magnanti, and Orlin (1993)), which is a classical network optimization problem in operations research. If the demands cannot be met, then $TSD^2_\xi$ is computed and $E^{\xi_2}_i$ is obtained. Otherwise, if all the demands at all the demand points can be satisfied, we can then turn to solving the optimization problem as in Section 2 for the specific supply chain network in order to compute $TC^\xi_1$ and to determine $E^{\xi_1}_i$.

**Definition 3: Bi-Criteria Performance Measurement of a Supply Chain Network for Critical Needs**

The performance, $\mathcal{E}$, of a supply chain network for critical needs under disruption scenario sets $\Xi^1$ and $\Xi^2$ and with associated probabilities $p_{\xi^1_1}$, $p_{\xi^1_2}$, \ldots, $p_{\xi^1_{\omega^1}}$ and $p_{\xi^2_1}$, $p_{\xi^2_2}$, \ldots, $p_{\xi^2_{\omega^2}}$, respectively, is defined as:

$$\mathcal{E} = \epsilon \times (\sum_{i=1}^{\omega^1} \mathcal{E}^{\xi^1}_i p_{\xi^1_i}) + (1 - \epsilon) \times (\sum_{i=1}^{\omega^2} \mathcal{E}^{\xi^2}_i p_{\xi^2_i}),$$

(12)

where $\epsilon$ is the weight associated with the network performance when demands can be satisfied, which has a value between 0 and 1. The higher $\epsilon$ is, the more emphasis is put on the cost efficiency.

As discussed earlier, we believe that the major goal for a supply chain for critical needs is to satisfy demand. This is especially true when the deliverables are lifeline products that have an impact on people’s lives in the affected area. At the same time, and this criterion is especially relevant to funding agencies, the government, as well as to stakeholders and donors, a critical needs supply chain should be evaluated based on its total cost of delivery. The cognizant organization should determine how to weight the two criteria and the above performance measure offers this flexibility while bringing the two criteria into a single metric.

In the next Section, we illustrate the above concepts more fully through explicit numerical examples.
4. Numerical Examples

For completeness, we now provide the explicit form that the steps of the modified projection method take for the solution of the variational inequality (7) governing the supply chain network equilibrium problem under disruptions.

Step 0: Initialization

Set \((x^{\xi_i^0}, \lambda^{\xi_i^0}) \in K^{\xi_i^1}\). Let \(T = 1\) and set \(\alpha\) such that \(0 < \alpha \leq \frac{1}{L}\) where \(L\) is the Lipschitz constant for the problem (cf. Korpelevich (1977) and Nagurney (1993)).

Step 1: Computation

Compute \((\bar{x}^{\xi_i^T}, \bar{\lambda}^{\xi_i^T})\) by solving the variational inequality subproblem:

\[
\begin{align*}
&\sum_{k=1}^{n_R} \sum_{p \in P_{w_k}} \left[ \bar{x}_p^{\xi_i^T} + \alpha \left( \frac{\partial \bar{C}_p(x^{\xi_i^T-1})}{\partial x_p} + \sum_{a \in L} \lambda_a^{\xi_i^T-1} \delta_{ap} - x_p^{\xi_i^{T-1}} \right) \right] \times [x_p^{\xi_i^1} - \bar{x}_p^{\xi_i^T}] \\
&+ \sum_{a \in L} \left[ \bar{\lambda}_a^{\xi_i^T} + \alpha (u_a^{\xi_i^1} - \sum_{p \in P} x_p^{\xi_i^{T-1}} \delta_{ap}) - \lambda_a^{\xi_i^{T-1}} \right] \times [\lambda_a^{\xi_i^1} - \bar{\lambda}_a^{\xi_i^T}] \geq 0, \\
&\forall (x^{\xi_i}, \lambda^{\xi_i}) \in K^{\xi_i^1}.
\end{align*}
\]

Step 2: Adaptation

Compute \((x^{\xi_i^1}, \lambda^{\xi_i^1})\) by solving the variational inequality subproblem:

\[
\begin{align*}
&\sum_{k=1}^{n_R} \sum_{p \in P_{w_k}} \left[ x_p^{\xi_i^1} + \alpha \left( \frac{\partial \bar{C}_p(x^{\xi_i^1})}{\partial x_p} + \sum_{a \in L} \lambda_a^{\xi_i^{T-1}} \delta_{ap} - x_p^{\xi_i^{T-1}} \right) \right] \times [x_p^{\xi_i^1} - x_p^{\xi_i^T}] \\
&+ \sum_{a \in L} \left[ \lambda_a^{\xi_i^1} + \alpha (u_a^{\xi_i^1} - \sum_{p \in P} x_p^{\xi_i^{T-1}} \delta_{ap}) - \lambda_a^{\xi_i^{T-1}} \right] \times [\lambda_a^{\xi_i^1} - \lambda_a^{\xi_i^T}] \geq 0, \\
&\forall (x^{\xi_i}, \lambda^{\xi_i}) \in K^{\xi_i^1}.
\end{align*}
\]

Step 3: Convergence Verification

If \(\max |x_p^{\xi_i^1} - x_p^{\xi_i^{T-1}}| \leq e\) and \(\max |\lambda_a^{\xi_i^1} - \lambda_a^{\xi_i^{T-1}}| \leq e\), for all \(p \in P\) and \(a \in L\), with \(e\) a positive preset tolerance, a prespecified tolerance, then stop; else, set \(T = T + 1\), and return to Step 1.

The general equilibration algorithm (Dafermos and Sparrow (1969)) can be used to obtain the solutions to the quadratic network optimization problems in Steps 1 and 2. We utilized
the modified projection method, embedded with the equilibration algorithm (see also Nagurney (1993)) to compute solutions to the numerical supply chain network examples below. We set $e = .00001$.

The supply chain network topology for all the examples in this Section was as depicted in Figure 2 with the links defined by numbers as in Figure 2. The numerical examples, hence, consisted of an organization faced with 3 manufacturing plants, 2 distribution centers, and had to supply 3 demand points.

In the examples, we use the nonlinear link cost functions to illustrate the proposed performance measure and the algorithm. We believe that the nonlinear functions are more general than the linear ones and therefore, more realistic. In future research, if enough data become available, we can calibrate the cost functions to study the supply chains.

The data for the specific examples along with the solutions are reported in the corresponding tables below. The solutions are reported in link form due to the number of paths.
Table 1: Total Cost Functions, Capacities, and Solution for the Baseline Numerical Example
Under No Disruptions

<table>
<thead>
<tr>
<th>Link $a$</th>
<th>$\hat{c}_a(f_a)$</th>
<th>$u_a^0$</th>
<th>$f_a^{0*}$</th>
<th>$\lambda_a^{0*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$f_1^2 + 2f_1$</td>
<td>10.00</td>
<td>3.12</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>$.5f_2^2 + f_2$</td>
<td>10.00</td>
<td>6.88</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>$.5f_3^2 + f_3$</td>
<td>5.00</td>
<td>5.00</td>
<td>0.93</td>
</tr>
<tr>
<td>4</td>
<td>$1.5f_4^2 + 2f_4$</td>
<td>6.00</td>
<td>1.79</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>$f_5^2 + 3f_5$</td>
<td>4.00</td>
<td>1.33</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>$f_6^2 + 2f_6$</td>
<td>4.00</td>
<td>2.88</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>$.5f_7^2 + 2f_7$</td>
<td>4.00</td>
<td>4.00</td>
<td>0.05</td>
</tr>
<tr>
<td>8</td>
<td>$.5f_8^2 + 2f_8$</td>
<td>4.00</td>
<td>4.00</td>
<td>2.70</td>
</tr>
<tr>
<td>9</td>
<td>$f_9^2 + 5f_9$</td>
<td>4.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>$.5f_{10}^2 + 2f_{10}$</td>
<td>16.00</td>
<td>8.67</td>
<td>0.00</td>
</tr>
<tr>
<td>11</td>
<td>$f_{11}^2 + f_{11}$</td>
<td>10.00</td>
<td>6.33</td>
<td>0.00</td>
</tr>
<tr>
<td>12</td>
<td>$.5f_{12}^2 + 2f_{12}$</td>
<td>2.00</td>
<td>3.76</td>
<td>0.00</td>
</tr>
<tr>
<td>13</td>
<td>$.5f_{13}^2 + 5f_{13}$</td>
<td>4.00</td>
<td>2.14</td>
<td>0.00</td>
</tr>
<tr>
<td>14</td>
<td>$f_{14}^2$</td>
<td>4.00</td>
<td>2.76</td>
<td>0.00</td>
</tr>
<tr>
<td>15</td>
<td>$f_{15}^2 + 2f_{15}$</td>
<td>2.00</td>
<td>1.24</td>
<td>0.00</td>
</tr>
<tr>
<td>16</td>
<td>$.5f_{16}^2 + 3f_{16}$</td>
<td>4.00</td>
<td>2.86</td>
<td>0.00</td>
</tr>
<tr>
<td>17</td>
<td>$.5f_{17}^2 + 2f_{17}$</td>
<td>4.00</td>
<td>2.24</td>
<td>0.00</td>
</tr>
</tbody>
</table>

We assumed that the demand is equal to 5 at each of the three demand points in the situation when there is no disruptions. In this baseline case, we have that $TC^0 = 290.43$. The corresponding link flow solutions and the Lagrangian multipliers are listed in Table 1.

Example Set 1

We assumed that there are three disruption scenarios. In the first scenario, the capacities on the manufacturing links 1 and 2 are disrupted by 50% and the demands remain unchanged. In the second scenario, the capacities on the storage links 10 and 11 are disrupted by 20% and the demands at the demand points 1 and 2 are increased by 20%. Finally, in the third scenario, the capacities on links 12 and 15 are decreased by 50% and the demand at demand point 1 is increased by 100%. The probabilities associated with these three scenarios are: 0.4, 0.3, 0.2, respectively, and the probability of no disruption is 0.1.
Table 2: Total Cost Functions, Capacities, and Solution Under Scenario 1 for Example Sets 1 & 2

<table>
<thead>
<tr>
<th>Link</th>
<th>$\hat{c}_a(f_a)$</th>
<th>$u_a^{\xi_1}$</th>
<th>$f_a^{\xi_1^*}$</th>
<th>$\lambda_a^{\xi_1^*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$f_1^1 + 2f_1$</td>
<td>5.00</td>
<td>5.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>$0.5f_2^2 + f_2$</td>
<td>5.00</td>
<td>5.00</td>
<td>9.15</td>
</tr>
<tr>
<td>3</td>
<td>$0.5f_3^2 + f_3$</td>
<td>5.00</td>
<td>5.00</td>
<td>6.96</td>
</tr>
<tr>
<td>4</td>
<td>$1.5f_4^2 + 2f_4$</td>
<td>6.00</td>
<td>2.51</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>$f_5^2 + 3f_5$</td>
<td>4.00</td>
<td>2.48</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>$f_6^2 + 2f_6$</td>
<td>4.00</td>
<td>2.19</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>$0.5f_7^2 + 2f_7$</td>
<td>4.00</td>
<td>2.81</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>$0.5f_8^2 + 2f_8$</td>
<td>4.00</td>
<td>4.00</td>
<td>2.58</td>
</tr>
<tr>
<td>9</td>
<td>$f_9^2 + 5f_9$</td>
<td>4.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>$0.5f_{10}^2 + 2f_{10}$</td>
<td>16.00</td>
<td>8.70</td>
<td>0.00</td>
</tr>
<tr>
<td>11</td>
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<td>10.00</td>
<td>6.30</td>
<td>0.00</td>
</tr>
<tr>
<td>12</td>
<td>$0.5f_{12}^2 + 2f_{12}$</td>
<td>4.00</td>
<td>3.77</td>
<td>0.00</td>
</tr>
<tr>
<td>13</td>
<td>$0.5f_{13}^2 + 5f_{13}$</td>
<td>4.00</td>
<td>2.15</td>
<td>0.00</td>
</tr>
<tr>
<td>14</td>
<td>$f_{14}^2$</td>
<td>4.00</td>
<td>2.77</td>
<td>0.00</td>
</tr>
<tr>
<td>15</td>
<td>$f_{15}^2 + 2f_{15}$</td>
<td>4.00</td>
<td>1.23</td>
<td>0.00</td>
</tr>
<tr>
<td>16</td>
<td>$0.5f_{16}^2 + 3f_{16}$</td>
<td>4.00</td>
<td>2.85</td>
<td>0.00</td>
</tr>
<tr>
<td>17</td>
<td>$0.5f_{17}^2 + 2f_{17}$</td>
<td>4.00</td>
<td>2.23</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The scenarios 1 and 2 belong to disruption type 1 and scenario 3 is of type 2 according to the discussion in Section 2. After applying the modified projection algorithm, the solutions for link flows and Lagrangian multipliers for scenarios 1 are shown in Table 2 and Table 3 lists the corresponding solutions under scenario 2. We have that $TC^{\xi_1} = 299.02$ and $TC^{\xi_2} = 361.41$. Therefore, according to Definition 1, we have that $E^{\xi_1} = \frac{TC^{\xi_1} - TC^0}{TC^0} = 0.0296$ and $E^{\xi_2} = \frac{TC^{\xi_2} - TC^0}{TC^0} = 0.2444$. After computing the maximum flow, we know that, in the case of the third scenario, the maximum demand that can be satisfied is 14, which leads to a total unsatisfied demand of 6. According to Definition 2, we have that $E^{\xi_2} = \frac{TD^{\xi_2} - TSD^{\xi_2}}{TD^{\xi_2}} = 0.3000$.

We let $\epsilon = 0.2$ to reflect the importance of being able to satisfy demands and we computed the bi-criteria supply chain performance as follows:

$$E = 0.2 \times (0.4 \times E^{\xi_1} + 0.3 \times E^{\xi_2}) + 0.8 \times (0.2 \times E^{\xi_2}) = 0.1290.$$
Table 3: Total Cost Functions, Capacities, and Solution Under Scenario 2 for Example Sets 1 & 2

<table>
<thead>
<tr>
<th>Link</th>
<th>$\hat{c}_a(f_a)$</th>
<th>$u_a^{\xi_1}$</th>
<th>$f_a^{\xi_1*}$</th>
<th>$\lambda_a^{\xi_1*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$f_1^a + 2f_1$</td>
<td>10.00</td>
<td>4.13</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>$0.5f_2^a + f_2$</td>
<td>10.00</td>
<td>7.86</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>$0.5f_3^a + f_3$</td>
<td>5.00</td>
<td>5.00</td>
<td>4.33</td>
</tr>
<tr>
<td>4</td>
<td>$1.5f_4^a + 2f_4$</td>
<td>6.00</td>
<td>2.10</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>$f_5^a + 3f_5$</td>
<td>4.00</td>
<td>2.04</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>$f_6^a + 2f_6$</td>
<td>4.00</td>
<td>3.85</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>$0.5f_7^a + 2f_7$</td>
<td>4.00</td>
<td>4.00</td>
<td>2.49</td>
</tr>
<tr>
<td>8</td>
<td>$0.5f_8^a + f_8$</td>
<td>4.00</td>
<td>4.00</td>
<td>2.24</td>
</tr>
<tr>
<td>9</td>
<td>$f_9^a + 5f_9$</td>
<td>4.00</td>
<td>1.01</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>$0.5f_{10}^a + 2f_{10}$</td>
<td>12.80</td>
<td>9.95</td>
<td>0.00</td>
</tr>
<tr>
<td>11</td>
<td>$f_{11}^a + f_{11}$</td>
<td>8.00</td>
<td>7.05</td>
<td>0.00</td>
</tr>
<tr>
<td>12</td>
<td>$0.5f_{12}^a + 2f_{12}$</td>
<td>4.00</td>
<td>4.00</td>
<td>1.94</td>
</tr>
<tr>
<td>13</td>
<td>$0.5f_{13}^a + 5f_{13}$</td>
<td>4.00</td>
<td>2.97</td>
<td>0.00</td>
</tr>
<tr>
<td>14</td>
<td>$f_{14}^a$</td>
<td>4.00</td>
<td>2.98</td>
<td>0.00</td>
</tr>
<tr>
<td>15</td>
<td>$f_{15}^a + 2f_{15}$</td>
<td>4.00</td>
<td>2.00</td>
<td>0.00</td>
</tr>
<tr>
<td>16</td>
<td>$0.5f_{16}^a + 3f_{16}$</td>
<td>4.00</td>
<td>3.03</td>
<td>0.00</td>
</tr>
<tr>
<td>17</td>
<td>$0.5f_{17}^a + 2f_{17}$</td>
<td>4.00</td>
<td>2.02</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Example Set 2

Next, we assumed that everything is the same as in Example Set 1 above except that under the third scenario above, the disruptions have decreased the demand by 20% at demand point 1 but have increased the demand by 20% at demand point 2. Hence, the solutions under scenarios 1 and 2 are the same as those in Example Set 1. Under scenario 3, it is reasonable to assume that the critical needs demands may move from one point to another under disruptions and it is important for a supply chain to be able to meet the demands in such scenarios. Indeed, those affected may need to be evacuated to other locations, thereby, altering the associated demands. Table 4 shows the corresponding solutions under scenario 3. Under this scenario, we know that all the demands can be satisfied and we have that $TC^{c_3^a} = 295.00$, which means that $E^{c_3^a} = \frac{TC^{c_3^a} - TC^{c_0}}{TC^{c_0}} = 0.0157$. Hence, given the same weight $\epsilon$ as in the First Case, the bi-criteria supply chain performance measure is now:

$$E = 0.2 \times (0.4 \times E^{c_1^a}_1 + 0.3 \times E^{c_2^a}_1 + 0.2 \times E^{c_3^a}_1) + 0.8 \times (0.2 \times E^{c_2^a}_2) = 0.0177.$$  

Although we know that $E^{c_2^a}_2 = 0$, we keep it in the above equation for the sake of consistency with our definition of the supply chain performance measure. The critical needs supply
chain in Example Set 2 performs better than the one in Example Set 1 since the former is more “robust” in terms of satisfying the demands when faced with disruptions, which means the second network deteriorates less under the same set of disruption scenarios.

Table 4: Total Cost Functions, Capacities, and Solution Under Scenario 3 for Example Set 2

<table>
<thead>
<tr>
<th>Link a</th>
<th>( \hat{c}_a(f_a) )</th>
<th>( u^*_a )</th>
<th>( f^*_a )</th>
<th>( \lambda^*_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( f_1^2 + 2f_1 )</td>
<td>10.00</td>
<td>3.19</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>( .5f_2^2 + f_2 )</td>
<td>10.00</td>
<td>6.81</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>( .5f_3^2 + f_3 )</td>
<td>5.00</td>
<td>5.00</td>
<td>1.43</td>
</tr>
<tr>
<td>4</td>
<td>( 1.5f_4^2 + 2f_4 )</td>
<td>6.00</td>
<td>1.67</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>( f_5^2 + 3f_5 )</td>
<td>4.00</td>
<td>1.52</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>( f_6^2 + 2f_6 )</td>
<td>4.00</td>
<td>2.81</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>( .5f_7^2 + 2f_7 )</td>
<td>4.00</td>
<td>4.00</td>
<td>0.63</td>
</tr>
<tr>
<td>8</td>
<td>( .5f_8^2 + 2f_8 )</td>
<td>4.00</td>
<td>4.00</td>
<td>1.98</td>
</tr>
<tr>
<td>9</td>
<td>( f_9^2 + 5f_9 )</td>
<td>4.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>( .5f_{10}^2 + 2f_{10} )</td>
<td>16.00</td>
<td>8.48</td>
<td>0.00</td>
</tr>
<tr>
<td>11</td>
<td>( f_{11}^2 + f_{11} )</td>
<td>10.00</td>
<td>6.52</td>
<td>0.00</td>
</tr>
<tr>
<td>12</td>
<td>( .5f_{12}^2 + 2f_{12} )</td>
<td>2.00</td>
<td>2.00</td>
<td>4.57</td>
</tr>
<tr>
<td>13</td>
<td>( .5f_{13}^2 + 5f_{13} )</td>
<td>4.00</td>
<td>3.29</td>
<td>0.00</td>
</tr>
<tr>
<td>14</td>
<td>( f_{14}^2 )</td>
<td>4.00</td>
<td>3.19</td>
<td>0.00</td>
</tr>
<tr>
<td>15</td>
<td>( f_{15}^2 + 2f_{15} )</td>
<td>2.00</td>
<td>2.00</td>
<td>0.00</td>
</tr>
<tr>
<td>16</td>
<td>( .5f_{16}^2 + 3f_{16} )</td>
<td>4.00</td>
<td>2.71</td>
<td>0.00</td>
</tr>
<tr>
<td>17</td>
<td>( .5f_{17}^2 + 2f_{17} )</td>
<td>4.00</td>
<td>1.81</td>
<td>0.00</td>
</tr>
</tbody>
</table>

5. Summary and Conclusions

In this paper, we developed a supply chain network model for critical needs, which captures disruptions in capacities associated with the various supply chain activities of production, transportation, and storage, as well as those associated with the demands for the product at the various demand points. We showed that the governing optimality conditions can be formulated as a variational inequality problem with nice features for numerical solution.

In addition, we proposed two distinct supply chain network performance measures for critical needs products. The first measure considers disruptions in the link capacities but assumes that the demands for the product can be met. The second measure captures the unsatisfied demand. We then constructed a bi-criteria supply chain network performance measure and used it for the evaluation of distinct supply chain networks. The bi-criteria
measure allows for the comparison of the robustness of different supply chain networks under a spectrum of real-world scenarios. We illustrated the new concepts in this paper with numerical supply chain network examples in which the supply chains were subject to a spectrum of disruptions involving capacity reductions as well as demand changes.

Given that the number of disasters has been growing globally, we expect that the methodological tools introduced in this paper will be applicable in practice in disaster planning and emergency preparedness.

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Reference


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