Equilibria and Dynamics
of
Supply Chain Network Competition
with
Information Asymmetry in Quality and Minimum Quality Standards

Anna Nagurney[1,2] and Dong Li[1]
[1]Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts, Amherst, Massachusetts 01003
and
[2]School of Business, Economics and Law
University of Gothenburg, Gothenburg, Sweden

November 2013; updated April 2014
Computational Management Science 11(3), (2014), pp 285-315

Abstract

In this paper, we construct a supply chain network model with information asymmetry in product quality. The competing, profit-maximizing firms with, possibly, multiple manufacturing plants, which may be located on-shore or off-shore, are aware of the quality of the product that they produce but consumers, at the demand markets, only know the average quality. Such a framework is relevant to products ranging from certain foods to pharmaceuticals. We propose both an equilibrium model and its dynamic counterpart and demonstrate how minimum quality standards can be incorporated. Qualitative results as well as an algorithm are presented, along with convergence results. The numerical examples, accompanied by sensitivity analysis, reveal interesting results and insights for firms, consumers, as well as policy-makers, who impose the minimum quality standards.

Keywords: supply chains, game theory, quality competition, information asymmetry, networks, variational inequalities, projected dynamical systems, minimum quality standards

1. Introduction

Supply chain networks have transformed the manner in which goods are produced, transported, and consumed around the globe and have created more choices and options for consumers during different seasons. At the same time, given the distances that may be involved as well as the types of products that are consumed, there may be information asymmetry associated with knowledge about the quality of the products. For example, producers in many different industries may be aware of their product quality whereas consumers at the demand markets may only be aware of the average quality. Such information asymmetry in quality results in products being, in effect, homogeneous at demand markets since there is no differentiation by brands or labels (see Baltzer (2012)).

Information asymmetry becomes increasingly complex when manufacturers (producers) have, at their disposal, multiple manufacturing plants, which may be on-shore or off-shore, with the ability to monitor the quality in the latter sometimes challenging. Indeed, major issues and quality problems associated with distinct manufacturing plants and products ranging from food to pharmaceuticals have been the focus of increasing attention (cf. Gray, Roth, and Leiblein (2011), Thomas (2012), McDonald (2013), Hogenau (2013), Masoumi, Yu, and Nagurney (2012), and Yu and Nagurney (2013)). Akerlof (1970) utilized used automobiles, with those of inferior quality referred to as *lemons*, as an illustrative example in his classic work on information asymmetry in quality, which has stimulated research in this domain. Baltzer (2012) further noted that firms producing the product have control over the quality but consumers may be unable to observe the level of quality as in the case not only with respect to the safety of cars but also the level of microbiological contaminants in food as well as chemical residues in toys.

To-date, markets with asymmetric information in terms of product quality have been studied by many notable economists, including Akerlof (1970), already noted above, Spence (1973, 1975), and Stiglitz (1987), all of whom shared the Nobel Prize in Economic Sciences. Leland (1979) further argued that such markets may benefit from minimum quality standards. However, information asymmetry in a supply chain network context with a focus on production, as well as transportation, has been minimally explored research-wise. Hence, in this paper, we develop both static and dynamic competitive supply chain network models with information asymmetry in quality. We consider multiple profit-maximizing firms, which are spatially separated, and may have multiple plants at their disposal. The firms are involved in the production of a product, and compete in multiple demand markets in a Cournot-Nash manner in product shipments and product quality levels. In addition, we demonstrate how minimum quality standards can be incorporated into the framework, which has wide relevance for policy-making and regulation (see, e.g., Giraud-Heraud and Soler (2006) and Smith (2009)). We consider imperfect competition in the form of a supply chain network oligopoly and note that Baltzer (2012) considered two firms involved in Bertrand competition with specific underlying functional forms. Our model, in contrast, assumes Cournot-Nash competition in both quantities and quality levels, is network-based, is not limited to two firms, among other distinctions.

Recent examples of supply chain network models include: the optimization models for supply chain network design of Nagurney (2010a) and of Nagurney and Nagurney (2012), which focused on medical nuclear supply chains, and the game theory models for fashion supply chain networks with competition by Nagurney and Yu (2012); for food by Yu and Nagurney (2013), and for pharmaceuticals by Masoumi, Yu, and Nagurney (2012), among others. See also the models in the books by Nagurney (2006) and Nagurney et al. (2013b). However, quality and information asymmetry were not considered in any of these models. Furthermore, in our quality competition supply chain research (cf. Nagurney and Li (2013) and Nagurney, Li, and Nagurney (2013)), as well as that in a service-oriented Internet (see Nagurney and Wolf (2013), Nagurney et al. (2013a, 2014), and Saberi, Nagurney, and Wolf (2013)), we assume that consumers can always differentiate among the products of different firms and that no quality information asymmetry exists.

Specifically, in this paper, information asymmetry in quality is considered, which occurs between the firms, producing the product, and the consumers at the demand markets. Firms are aware of the quality of the product produced at each of their manufacturing plants, with different manufacturing plants owned by the same firm having, possibly, different quality levels. However, the quality levels perceived by consumers at the demand markets are the average quality levels of the products (see also Akerlof (1970) and Leland (1979)). Information asymmetry between produced and perceived quality levels and quality uncertainty were widely discussed in Wankhade and Dabade (2010), but no supply chain network models were established.

We now provide a literature review of supply chain models with information asymmetry. The value, effects, and/or incentives of information and information sharing were recently studied in Corbett, Zhou, and Tang (2004), Mishra, Raghunathan, and Yue (2009), Thomas, Warsing, and Zhang (2009), Ren et al. (2010), and Esmaeili and Zeephongsekul (2010). All of these studies were based on a single supplier and a single buyer supply chain. An extensive review of the early literature on information sharing can be found in Chen (2003).

In addition, a significant literature on information asymmetry focuses on supply chain contracting problems; for an early review, see Cachon (2003). Hasija, Pinker, and Shumsky (2008) examined contracts for a call center outsourcing problem with information asymmetry in worker productivity. Xu et al. (2010) investigated a contract setting problem of a manufacturer who has one prime supplier and one urgent supplier. Lee and Yang (2013) studied supply chain contracting problems involving one retailer and two suppliers. Examples of recent quantity discount contracting models with information asymmetry are given in Burnetas, Gilbertm, and Smith (2007) and Zhou (2008). Except where noted otherwise, all the above models are based on two entity supply chain "networks," and the asymmetric information considered is mostly in terms of demand and cost.

The novelty of the contributions in this paper consists in the following:

1. Both static (equilibrium) and dynamic versions of supply chain network competition are captured under information asymmetry in quality with and without minimum quality standards using, respectively, variational inequality theory and projected dynamical systems theory.

2. Firms have, as their strategic variables, both the shipment amounts produced at their manufacturing plants as well as the quality levels. The information asymmetry lies in that the firms know the quality of the products produced at their plants but consumers at the demand markets are only aware of the average quality since the consumers cannot distinguish among the producers.

3. Quality is associated not only with the manufacturing plants but also tracked through the transportation process, which is assumed to preserve (at the appropriate cost) the product quality.

4. We do not impose any specific functional forms on the production cost, transportation cost, and demand price functions nor do we limit ourselves to only one or two manufacturers, manufacturing plants, or demand markets. We, nevertheless, assume that the firms compete in an oligopolistic manner (cf. Tirole (1988)).

5. Both theoretical results, in the form of existence and uniqueness results as well as stability analysis, and an effective and efficient algorithmic scheme are provided with convergence for the latter. We also provide solutions to numerical examples, accompanied by sensitivity analyses, to illustrate the generality and usefulness of the models for firms, for consumers, as well as for policy-makers.

The paper is organized as follows. In Section 2, we present both the static models (without and with minimum quality standards), along with their variational inequality formulations, as well as the dynamic version using projected dynamical systems theory. In Section 3, we then provide qualitative properties of the equilibrium solutions and establish that the set of stationary points of our projected dynamical systems formulation coincides with the set of solutions to the corresponding variational inequality problem. In Section 4, we describe the algorithmic scheme, which yields closed form expressions in product shipments and quality levels at each iteration, and establish convergence. In Section 5, we provide numerical examples and conduct sensitivity analyses, which yield valuable insights for firms, consumers, and policy-makers. In Section 6, we summarize the results and present our conclusions.

2. Supply Chain Network Competition with Information Asymmetry in Quality

In this Section, we construct the supply chain network equilibrium model in which the firms compete in product quantities and quality levels and there is information asymmetry in quality. We first consider the case without minimum quality standards and them demonstrate how standards can be incorporated. We follow with the development of the dynamic counterpart of the latter, which contains the former as a special case. The static equilibrium model(s) are given in Section 2.1 and the dynamic version in Section 2.2.

2.1 The Equilibrium Model Without and With Minimum Quality Standards

We first present the model without minimum quality standards and then show how it can be extended to include minimum quality standards, which are useful policy instruments in practice.

We consider I firms, with a typical firm denoted by i, which compete with one another in a noncooperative Cournot-Nash (Cournot (1838), Nash (1950, 1951)) manner in the production and distribution of the product. Each firm i has, at its disposal, n_i manufacturing plants. The firms determine the quantities to produce at each of their manufacturing plants and the quantities to ship to the n_R demand markets. They also control the quality level of the product at each of their manufacturing plants. Information asymmetry occurs in that the firms are aware of the quality levels of the product produced at each of their manufacturing plants but the consumers are only aware of the *average* quality levels of the product at the demand markets.

We consider the supply chain network topology depicted in Figure 1. The top nodes correspond to the firms, the middle nodes to the manufacturing plants, and the bottom nodes to the common demand markets. We assume that the demand at each demand market is positive; otherwise, the demand market (node) will be removed from the supply chain network.

In Figure 1, the first set of links connecting the top two tiers of nodes corresponds to the process of manufacturing at each of the manufacturing plants of firm i; $i = 1, \ldots, I$. Such plants are denoted by $M_i^1, \ldots, M_i^{n_i}$, respectively, for firm i, with a typical one denoted by M_i^j; $j = 1, \ldots, n_i$. The manufacturing plants may be located not only in different regions of a country but also in different countries. The next set of links connecting the two bottom tiers of the supply chain network corresponds to transportation links joining the manufacturing plants with the demand markets, with a typical demand market denoted by R_k; $k = 1, \ldots, n_R$.

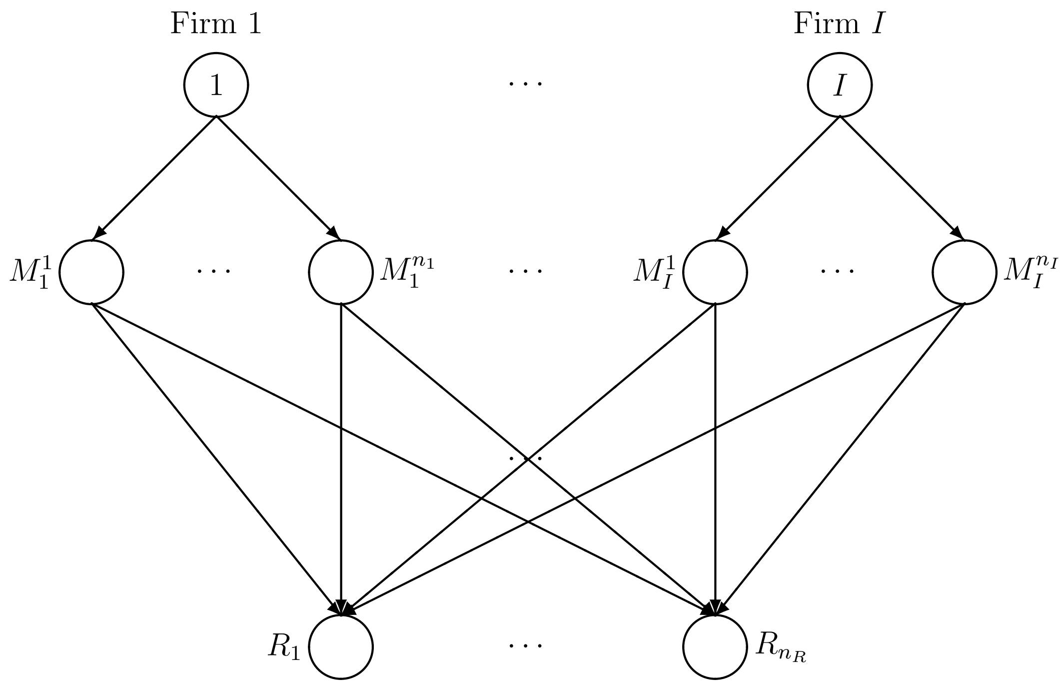

Figure 1: The Supply Chain Network Topology

The nonnegative product amount produced at firm i's manufacturing plant M_i^j and shipped to demand market R_k is denoted by Q_{ijk}; $i = 1, \ldots, I$; $j = 1, \ldots, n_i$; $k = 1 \ldots, n_R$. For each firm i, we group its Q_{ijk}s into the vector $Q_i \in R_+^{n_i n_R}$, and then group all such vectors for all firms into the vector $Q \in R_+^{\sum_{i=1}^{I} n_i n_R}$.

We denote the nonnegative production output of firm i's manufacturing plant M_i^j by s_{ij}. The demand for the product at demand market R_k is denoted by d_k; $k = 1, \ldots, n_R$, and the quality level or, simply, quality, of the product produced by firm i's manufacturing plant M_i^j is denoted by q_{ij}. Note that different manufacturing plants owned by a firm may have different quality levels. This is highly reasonable since, for example, different plants may have different resources available in terms of skilled labor and facilities as well as labor expertise and even infrastructure.

The output at firm i's manufacturing plant M_i^j and the demand for the product at each demand market R_k must satisfy, respectively, the conservation of flow equations (1) and (2):

$$s_{ij} = \sum_{k=1}^{n_R} Q_{ijk}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i, \tag{1}$$

$$d_k = \sum_{i=1}^{I} \sum_{j=1}^{n_i} Q_{ijk}, \quad k = 1, \ldots, n_R. \tag{2}$$

Hence, the output produced at firm i's manufacturing plant M_i^j is equal to the sum of the amounts shipped to the demand markets, and the quantity consumed at a demand market is equal to the sum of the amounts shipped by the firms to that demand market. We group all s_{ij}s into the vector $s \in R_+^{\sum_{i=1}^{I} n_i}$ and all d_ks into the vector $d \in R_+^{n_R}$. For each firm i, we group its own plant quality levels into the vector $q_i \in R_+^{n_i}$ and then group all such vectors for all firms into the vector $q \in R_+^{\sum_{i=1}^{I} n_i}$.

The product shipments must be nonnegative, that is:

$$Q_{ijk} \geq 0, \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_R, \tag{3}$$

In addition, as in Nagurney and Li (2013), Nagurney, Li, and Nagurney (2013), Nagurney and Wolf (2013), and Nagurney et al. (2013a, 2014), we define and quantify quality as the quality conformance level, that is, the degree to which a specific product conforms to a design or specification (Gilmore (1974), Juran and Gryna (1988)). The quality levels cannot be lower than 0; thus,

$$q_{ij} \geq 0, \quad i = 1, \ldots, I; j = 1, \ldots, n_i. \tag{4}$$

The production cost at firm i's manufacturing plant M_i^j is denoted by f_{ij}. We allow for the general situation where f_{ij} may depend upon the entire production pattern and the entire vector of quality levels, that is,

$$f_{ij} = f_{ij}(s, q), \quad i = 1, \ldots, I; j = 1, \ldots, n_i. \tag{5a}$$

In view of (1), we can define the plant manufacturing cost functions $\hat{f}_{ij}$; $i = 1, \ldots, I; j = 1, \ldots, n_i$, in shipment quantities and quality levels, that is,

$$\hat{f}_{ij} = \hat{f}_{ij}(Q, q) \equiv f_{ij}(s, q). \tag{5b}$$

Let $\hat{c}_{ijk}$ denote the total transportation cost associated with shipping the product produced at firm i's manufacturing plant M_i^j to demand market R_k, assuming quality preservation, that is,

$$\hat{c}_{ijk} = \hat{c}_{ijk}(Q, q), \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_R. \tag{6}$$

Note that, according to (6), the transportation cost is such that the quality of the product is not degraded as it undergoes the shipment process. Transportation cost functions in both quantities and quality levels were utilized by Nagurney and Wolf (2013), Saberi, Nagurney, and Wolf (2013), and Nagurney et al. (2014) but in the context of Internet applications and not supply chains.

Since the products of the I firms are, in effect, homogeneous, common resources and technologies may be utilized in the processes of manufacturing and transportation. In this model, in order to capture the competition for resources and technologies on the supply side, we allow for general production cost functions, which may depend on the vectors s and q (cf. (5a)), and general transportation cost functions, which may depend on the vectors Q and q (cf. (6)). Such general production and transportation cost functions in both quantities and quality levels are utilized, for the first time, in a supply chain context in our model. The production cost functions (5a) and the transportation functions (6) are assumed to be convex and continuously differentiable.

The consumers' perception of the quality of the product, which may come from different firms, is for the *average* quality level, $\hat{q}_k$; $k = 1, \ldots, n_R$, where

$$\hat{q}_k = \frac{\sum_{i=1}^{I} \sum_{j=1}^{n_i} Q_{ijk} q_{ij}}{d_k}, \quad k = 1, \ldots, n_R \tag{7}$$

with the average (perceived) quality levels grouped into the vector $\hat{q} \in R_+^{n_R}$. A variant of (7) was utilized in Nagurney, Li, and Nagurney (2013) to assess the average quality level of pharmaceuticals in the case of outsourcing, but the demands were assumed to be fixed and known and there was only a single manufacturer with multiple plants but multiple firms to outsource to. Here, in contrast, we have elastic demands which are price and average quality level sensitive, as we discuss below. Moreover, in the new model(s) we explicitly allow for distinct quality levels associated with individual plants of a firm, rather than with outsourcing.

The demand price at demand market R_k is denoted by ρ_k. We allow the demand price function at a demand market to depend, in general, upon the entire demand pattern, as well as on the average quality levels at all the demand markets, that is,

$$\rho_k = \rho_k(d, \hat{q}), \quad k = 1, \ldots, n_R. \tag{8a}$$

Each demand price function is, typically, assumed to be monotonically decreasing in product quantity but increasing in terms of the average product quality. Demand functions that are functions of the prices and the average quality levels were also used by Akerlof (1970).

Therein the producers, in the form of a supply market, are aware of their product quality levels (cf. (5a)), while consumers at the demand markets are aware only of the average quality levels. However, Akerlof (1970) did not consider multiple manufacturing plants, transportation, and multiple demand markets. Moreover, he did not model the profit-maximizing behavior of individual, competing firms, as we do here.

In light of (2) and (7), we can define the demand price function $\hat{\rho}_k; k = 1, \ldots, n_R$, in quantities and quality levels of the firms, so that

$$\hat{\rho}_k = \hat{\rho}_k(Q, q) \equiv \rho_k(d, \hat{q}), \quad k = 1, \ldots, n_R. \tag{8b}$$

We assume that the demand price functions (8a) and (8b) are continuous and continuously differentiable.

The strategic variables of firm i are its product shipments $\{Q_i\}$ and its quality levels q_i. The profit/utility U_i of firm i; $i = 1, \ldots, I$, is given by:

$$U_i = \sum_{k=1}^{n_R} \rho_k(d, \hat{q}) \sum_{j=1}^{n_i} Q_{ijk} - \sum_{j=1}^{n_i} f_{ij}(s, q) - \sum_{k=1}^{n_R} \sum_{j=1}^{n_i} \hat{c}_{ijk}(Q, q), \tag{9a}$$

which is the difference between its total revenue and its total costs (production and transportation). By making use of (5b) and (8b), (9a) is equivalent to

$$U_i = \sum_{k=1}^{n_R} \hat{\rho}_k(Q, q) \sum_{j=1}^{n_i} Q_{ijk} - \sum_{j=1}^{n_i} \hat{f}_{ij}(Q, q) - \sum_{k=1}^{n_R} \sum_{j=1}^{n_i} \hat{c}_{ijk}(Q, q). \tag{9b}$$

In view of (1) - (9b), we may write the profit as a function solely of the product shipment pattern and quality levels, that is,

$$U = U(Q, q), \tag{10}$$

where U is the I-dimensional vector with components: $\{U_1, \ldots, U_I\}$.

Let K^i denote the feasible set corresponding to firm i, where $K^i \equiv \{(Q_i, q_i) | Q_i \geq 0, \text{ and } q_i \geq 0\}$ and define $K \equiv \prod_{i=1}^{I} K^i$.

We consider Cournot-Nash competition, in which the I firms produce and deliver their product in a noncooperative fashion, each one trying to maximize its own profit. We seek to determine a nonnegative product shipment and quality level pattern $(Q^*, q^*) \in K$ for which the I firms will be in a state of equilibrium as defined below.

Definition 1: A Supply Chain Network Cournot-Nash Equilibrium with Information Asymmetry in Quality

A product shipment and quality level pattern $(Q^*, q^*) \in K$ *is said to constitute a supply chain network Cournot-Nash equilibrium with information asymmetry in quality if for each firm* i*;* $i = 1, \ldots, I$,

$$U_i(Q_i^*, q_i^*, \hat{Q}_i^*, \hat{q}_i^*) \geq U_i(Q_i, q_i, \hat{Q}_i^*, \hat{q}_i^*), \quad \forall (Q_i, q_i) \in K^i, \tag{11}$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*) \quad \textit{and} \quad \hat{q}_i^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_I^*).$$

According to (11), an equilibrium is established if no firm can unilaterally improve upon its profits by selecting an alternative vector of product shipments and quality level of its product.

Variational Inequality Formulations

We now present alternative variational inequality formulations of the above supply chain network Cournot-Nash equilibrium in the following theorem.

Theorem 1: Variational Inequality Formulations

Assume that for each firm i *the profit function* $U_i(Q, q)$ *is concave with respect to the variables in* Q_i *and* q_i*, and is continuous and continuously differentiable. Then the product shipment and quality pattern* $(Q^*, q^*) \in K$ *is a supply chain network Cournot-Nash equilibrium with quality information asymmetry according to Definition 1 if and only if it satisfies the variational inequality*

$$-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q_{ijk}^*) - \sum_{i=1}^{I}\sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q_{ij}^*) \geq 0, \quad \forall (Q, q) \in K, \tag{12}$$

that is,

$$\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R} \left[-\hat{\rho}_k(Q^*, q^*) - \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_l(Q^*, q^*)}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q_{ihl}^* + \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^*, q^*)}{\partial Q_{ijk}} + \sum_{h=1}^{n_i}\sum_{l=1}^{n_R} \frac{\partial \hat{c}_{ihl}(Q^*, q^*)}{\partial Q_{ijk}} \right]$$
$$\times (Q_{ijk} - Q_{ijk}^*)$$
$$+\sum_{i=1}^{I}\sum_{j=1}^{n_i} \left[-\sum_{k=1}^{n_R} \frac{\partial \hat{\rho}_k(Q^*, q^*)}{\partial q_{ij}} \sum_{h=1}^{n_i} Q_{ihk}^* + \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^*, q^*)}{\partial q_{ij}} + \sum_{h=1}^{n_i}\sum_{k=1}^{n_R} \frac{\partial \hat{c}_{ihk}(Q^*, q^*)}{\partial q_{ij}} \right]$$

$$\times(q_{ij} - q^*_{ij}) \geq 0, \quad \forall (Q,q) \in K; \tag{13}$$

equivalently, $(d^*, s^*, Q^*, q^*) \in K^1$ *is an equilibrium production, shipment, and quality level pattern if and only if it satisfies the variational inequality*

$$\sum_{k=1}^{n_R} [-\rho_k(d^*, \hat{q}^*)] \times (d_k - d^*_k) + \sum_{i=1}^{I}\sum_{j=1}^{n_i} \left[\sum_{h=1}^{n_i} \frac{\partial f_{ih}(s^*, q^*)}{\partial s_{ij}}\right] \times (s_{ij} - s^*_{ij})$$

$$+ \sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R} \left[-\sum_{l=1}^{n_R} \frac{\partial \rho_l(d^*, \hat{q}^*)}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q^*_{ihl} + \sum_{h=1}^{n_i}\sum_{l=1}^{n_R} \frac{\partial \hat{c}_{ihl}(Q^*, q^*)}{\partial Q_{ijk}}\right] \times (Q_{ijk} - Q^*_{ijk})$$

$$+ \sum_{i=1}^{I}\sum_{j=1}^{n_i} \left[-\sum_{k=1}^{n_R} \frac{\partial \rho_k(Q^*, \hat{q}^*)}{\partial q_{ij}} \sum_{h=1}^{n_i} Q^*_{ihk} + \sum_{h=1}^{n_i} \frac{\partial f_{ih}(s^*, q^*)}{\partial q_{ij}} + \sum_{h=1}^{n_i}\sum_{k=1}^{n_R} \frac{\partial \hat{c}_{ihk}(Q^*, q^*)}{\partial q_{ij}}\right]$$

$$\times(q_{ij} - q^*_{ij}) \geq 0, \qquad \forall (d, s, Q, q) \in K^1, \tag{14}$$

where $K^1 \equiv \{(d, s, Q, q) | \, Q \geq 0, \, q \geq 0,$ *and (1), (2), and (7) hold*$\}$.

Proof: (12) follows directly from Gabay and Moulin (1980) and Dafermos and Nagurney (1987). For firm i's manufacturing plant M_i^j; $i = 1, \ldots, I$; $j = 1, \ldots, n_i$ and demand market R_k; $k = 1, \ldots, n_R$, we have:

$$-\frac{\partial U_i(Q,q)}{\partial Q_{ijk}} = -\frac{\partial [\sum_{l=1}^{n_R} \hat{\rho}_l(Q,q) \sum_{h=1}^{n_i} Q_{ihl} - \sum_{h=1}^{n_i} \hat{f}_{ih}(Q,q) - \sum_{h=1}^{n_i}\sum_{l=1}^{n_R} \hat{c}_{ihl}(Q,q)]}{\partial Q_{ijk}}$$

$$= -\sum_{l=1}^{n_R} \frac{\partial [\hat{\rho}_l(Q,q) \sum_{h=1}^{n_i} Q_{ihl}]}{\partial Q_{ijk}} + \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q,q)}{\partial Q_{ijk}} + \sum_{h=1}^{n_i}\sum_{l=1}^{n_R} \frac{\partial \hat{c}_{ihl}(Q,q)}{\partial Q_{ijk}}$$

$$= -\hat{\rho}_k(Q,q) - \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_l(Q,q)}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q_{ihl} + \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q,q)}{\partial Q_{ijk}} + \sum_{h=1}^{n_i}\sum_{l=1}^{n_R} \frac{\partial \hat{c}_{ihl}(Q,q)}{\partial Q_{ijk}}. \tag{15}$$

Also, for firm i's manufacturing plant M_i^j; $i = 1, \ldots, I$; $j = 1, \ldots, n_i$, we have:

$$-\frac{\partial U_i(Q,q)}{\partial q_{ij}} = -\frac{\partial [\sum_{k=1}^{n_R} \hat{\rho}_k(Q,q) \sum_{h=1}^{n_i} Q_{ihk} - \sum_{h=1}^{n_i} \hat{f}_{ih}(Q,q) - \sum_{h=1}^{n_i}\sum_{k=1}^{n_R} \hat{c}_{ihk}(Q,q)]}{\partial q_{ij}}$$

$$= -\sum_{k=1}^{n_R} \frac{\partial \hat{\rho}_k(Q,q)}{\partial q_{ij}} \sum_{h=1}^{n_i} Q_{ihk} + \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q,q)}{\partial q_{ij}} + \sum_{h=1}^{n_i}\sum_{k=1}^{n_R} \frac{\partial \hat{c}_{ihk}(Q,q)}{\partial q_{ij}}. \tag{16}$$

Thus, variational inequality (13) is immediate. In addition, by re-expressing the production cost functions and the demand price functions in (15) and (16) as in (5b) and (8b) and using the conservation of flow equations (1) and (2) and $\frac{\partial f_{ih}(s,q)}{\partial Q_{ijk}} = \frac{\partial f_{ih}(s,q)}{\partial s_{ij}} \frac{\partial s_{ij}}{\partial Q_{ijk}}$, the equivalence of variational inequalities (13) and (14) holds true.□

For additional background on the variational inequality problem, we refer the reader to the book by Nagurney (1999).

We now describe an extension of the above framework that incorporates minimum quality standards. The effectiveness of the imposition of minimum quality standards on quality has been studied in economics with or without information asymmetry (see Leland (1979), Shapiro (1983), Besanko, Donnenfeld, and White (1988), Ronnen (1991), and Lutz and Lutz (2010)). In this paper, we integrate our framework with minimum quality standards and the framework without, and present the equilibrium conditions of both through a unified variational inequality formulation.

We retain the above notation and firm behavior and constraints but now we impose nonnegative lower bounds on the quality levels at the manufacturing plants, denoted by $\underline{q}_{ij}$; $i = 1, \ldots, I$; $j = 1, \ldots, n_i$ so that (4) is replaced by:

$$q_{ij} \geq \underline{q}_{ij} \quad i = 1, \ldots, I; j = 1, \ldots, n_i \tag{17}$$

with the understanding that, if the lower bounds are all identically equal to zero, then (17) collapses to (4) and, if the lower bounds are positive, then they represent minimum quality standards.

We define a new feasible set $K^2 \equiv \{(Q, q) | Q \geq 0 \, \text{and} \, (17) \, \text{holds}\}$. Then the following Corollary is immediate.

Corollary 1: Variational Inequality Formulations with Minimum Quality Standards

Assume that for each firm i the profit function $U_i(Q, q)$ is concave with respect to the variables in Q_i and q_i, and is continuous and continuously differentiable. Then the product shipment and quality pattern $(Q^*, q^*) \in K^2$ is a supply chain network Cournot-Nash equilibrium with quality information asymmetry in the presence of minimum quality standards if and only if it satisfies the variational inequality

$$-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q^*_{ijk}) - \sum_{i=1}^{I}\sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q^*_{ij}) \geq 0, \quad \forall (Q, q) \in K^2, \tag{18}$$

that is,

$$\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R} \left[-\hat{\rho}_k(Q^*, q^*) - \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_l(Q^*, q^*)}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q^*_{ihl} + \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^*, q^*)}{\partial Q_{ijk}} + \sum_{h=1}^{n_i}\sum_{l=1}^{n_R} \frac{\partial \hat{c}_{ihl}(Q^*, q^*)}{\partial Q_{ijk}} \right]$$

$$\times(Q_{ijk}-Q^*_{ijk})$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{n_i}\left[-\sum_{k=1}^{n_R}\frac{\partial\hat{\rho}_k(Q^*,q^*)}{\partial q_{ij}}\sum_{h=1}^{n_i}Q^*_{ihk}+\sum_{h=1}^{n_i}\frac{\partial\hat{f}_{ih}(Q^*,q^*)}{\partial q_{ij}}+\sum_{h=1}^{n_i}\sum_{k=1}^{n_R}\frac{\partial\hat{c}_{ihk}(Q^*,q^*)}{\partial q_{ij}}\right]$$

$$\times(q_{ij}-q^*_{ij})\geq 0,\qquad \forall(Q,q)\in K^2. \tag{19}$$

Variational inequality (19) contains variational inequality (13) as a special case when the minimum quality standards are all zero, and it will play a crucial role in the next section when we describe the underlying dynamics associated with the firms' adjustment processes in product shipments and quality levels until an equilibrium point, equivalently, a stationary point, is achieved.

We now put variational inequality (19) into standard form (cf. Nagurney (1999)): determine $X^*\in\mathcal{K}$ where X is a vector in R^N, $F(X)$ is a continuous function such that $F(X): X\mapsto\mathcal{K}\subset R^N$, and

$$\langle F(X^*),X-X^*\rangle\geq 0,\quad \forall X\in\mathcal{K}, \tag{20}$$

where $\langle\cdot,\cdot\rangle$ is the inner product in the N-dimensional Euclidean space, and $\mathcal{K}$ is closed and convex. We define the vector $X\equiv(Q,q)$ and the vector $F(X)\equiv(F^1(X),F^2(X))$. Also, here $N=\sum_{i=1}^{I}n_in_R+\sum_{i=1}^{I}n_i$. $F^1(X)$ consists of components $F^1_{ijk}=-\frac{\partial U_i(Q,q)}{\partial Q_{ijk}}$; $i=1,\ldots,I; j=1,\ldots,n_i; k=1,\ldots,n_R$, and $F^2(X)$ consist of components $F^2_{ij}=-\frac{\partial U_i(Q,q)}{\partial q_{ij}}$; $i=1,\ldots,I; j=1,\ldots,n_i$. In addition, we define the feasible set $\mathcal{K}\equiv K^2$. Hence, (19) can be put into standard form (20).

2.2 The Dynamic Model

We now describe the underlying dynamics for the evolution of product shipments and quality levels under information asymmetry in quality until the equilibrium satisfying variational inequality (19) is achieved. We identify the dynamic adjustment processes for the evolution of the firm's product shipments and quality levels. In Section 4, we provide a an algorithm, which is a discrete-time version of the continuous-time adjustment processes introduced below.

Observe that, for a current vector of product shipments and quality levels at time t, $X(t)=(Q(t),q(t))$, $-F^1_{ijk}(X(t))=\frac{\partial U_i(Q(t),q(t))}{\partial Q_{ijk}}$ is the marginal utility (profit) of firm i with respect to the volume produced at its manufacturing plant j and distributed to demand market k. $-F^2_{ij}(X(t))=\frac{\partial U_i(Q(t),q(t))}{\partial q_{ij}}$ is firm i's marginal utility with respect to the quality level of its manufacturing plant j. In this framework, the rate of change of the product shipment between firm i's manufacturing plant j and demand market k is in proportion to $-F^1_{ij}(X)$, as long as the product shipment Q_{ijk} is positive.

Namely, when $Q_{ijk} > 0$,

$$\dot{Q}_{ijk} = \frac{\partial U_i(Q,q)}{\partial Q_{ijk}}, \tag{21}$$

where $\dot{Q}_{ijk}$ denotes the rate of change of Q_{ijk}. However, when $Q_{ijk} = 0$, the nonnegativity condition (3) forces the product shipment Q_{ijk} to remain zero when $\frac{\partial U_i(Q,q)}{\partial Q_{ijk}} \leq 0$. Hence, we are only guaranteed of having possible increases of the shipment, that is, when $Q_{ijk} = 0$,

$$\dot{Q}_{ijk} = \max\{0, \frac{\partial U_i(Q,q)}{\partial Q_{ijk}}\}. \tag{22}$$

We may write (21) and (22) concisely as:

$$\dot{Q}_{ijk} = \begin{cases} \frac{\partial U_i(Q,q)}{\partial Q_{ijk}}, & \text{if} \quad Q_{ijk} > 0 \\ \max\{0, \frac{\partial U_i(Q,q)}{\partial Q_{ijk}}\}, & \text{if} \quad Q_{ijk} = 0. \end{cases} \tag{23}$$

As for the quality levels, when $q_{ij} > \underline{q}_{ij}$, then

$$\dot{q}_{ij} = \frac{\partial U_i(Q,q)}{\partial q_{ij}}, \tag{24}$$

where $\dot{q}_{ij}$ denotes the rate of change of q_{ij}; when $q_{ij} = \underline{q}_{ij}$,

$$\dot{q}_{ij} = \max\{\underline{q}_{ij}, \frac{\partial U_i(Q,q)}{\partial q_{ij}}\}, \tag{25}$$

since q_i cannot be lower than $\underline{q}_{ij}$ according to the feasible set $\mathcal{K} = K^2$.

Combining (24) and (25), we may write:

$$\dot{q}_{ij} = \begin{cases} \frac{\partial U_i(Q,q)}{\partial q_{ij}}, & \text{if} \quad q_{ij} > \underline{q}_{ij} \\ \max\{\underline{q}_{ij}, \frac{\partial U_i(Q,q)}{\partial q_{ij}}\}, & \text{if} \quad q_{ij} = \underline{q}_{ij}. \end{cases} \tag{26}$$

Applying (23) to all firm and manufacturing plant pairs (i,j); $i = 1, \ldots, I; j = 1, \ldots, n_i$ and all demand markets k; $k = 1, \ldots, n_R$, and then applying (26) to all firm and manufacturing plant pairs (i,j); $i = 1, \ldots, I; j = 1, \ldots, n_i$, and combining the resultants, yields the following pertinent ordinary differential equation (ODE) for the adjustment processes of the product shipments and quality levels, in vector form:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \tag{27}$$

where, since $\mathcal{K}$ is a convex polyhedron, according to Dupuis and Nagurney (1993), $\Pi_{\mathcal{K}}(X, -F(X))$ is the projection, with respect to $\mathcal{K}$, of the vector $-F(X)$ at X defined as

$$\Pi_{\mathcal{K}}(X, -F(X)) = \lim_{\delta \to 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta} \tag{28}$$

with $P_{\mathcal{K}}$ denoting the projection map:

$$P(X) = \operatorname{argmin}_{z \in \mathcal{K}} \|X - z\|, \tag{29}$$

and where $\|\cdot\| = \langle x, x\rangle$ and $-F(X) = \nabla U(Q, q)$, where $\nabla U(Q, q)$ is the vector of marginal utilities as described above.

We now further interpret ODE (27) in the context of the supply chain network competition model with information asymmetry in quality. First, observe that ODE (27) guarantees that the product shipments are always nonnegative and the quality levels never go below the minimum quality standards. In addition, ODE (27) states that the rate of change of the product shipments and the quality levels is greatest when the firm's marginal utilities are greatest. If the marginal utility of a firm with respect to its quality level is positive, then the firm will increase its quality level; if it is negative, then it will decrease the quality level, and the quality levels will also never be outside their lower bounds. A similar adjustment behavior holds for the firms in terms of their product shipments. This type of behavior is rational from an economic standpoint. Therefore, ODE (27) corresponds to reasonable continuous adjustment processes for the supply chain network competition model with information asymmetry in quality.

Since ODE (27) is nonstandard due to its discontinuous right-hand side, we further discuss the existence and uniqueness of (27). Dupuis and Nagurney (1993) constructed the fundamental theory with regards to existence and uniqueness of projected dynamical systems as defined by (27). We cite the following theorem from that paper.

Theorem 2

X^* solves the variational inequality problem (20) if and only if it is a stationary point of the ODE (27), that is,

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)). \tag{30}$$

This theorem demonstrates that the necessary and sufficient condition for a product shipment and quality level pattern $X^* = (Q^*, q^*)$ to be a supply chain network equilibrium with information asymmetry in quality, according to Definition 1, is that $X^* = (Q^*, q^*)$ is a stationary point of the adjustment processes defined by ODE (27), that is, X^* is the point at which $\dot{X} = 0$.

3. Qualitative Properties

We now investigate whether, and, under what conditions, the dynamic adjustment processes defined by (27) approach a Cournot-Nash equilibrium. Recall that Lipschitz continuity of $F(X)$ (cf. Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) guarantees the existence of a unique solution to (31) below, where we have that $X^0(t)$ satisfies ODE (27) with initial shipment and quality level pattern (Q^0, q^0). In other words, $X^0(t)$ solves the initial value problem (IVP)

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X^0, \tag{31}$$

with $X^0(0) = X^0$. For convenience, sometimes we will write $X^0 \cdot t$ for $X^0(t)$.

We know that, if the utility functions are twice differentiable and the Jacobian matrix of $F(X)$, denoted by $\nabla F(X)$, is positive-definite, then the corresponding $F(X)$ is strictly monotone, and the solution to variational inequality (20) is unique, if it exists.

Assumption 1

Suppose that in the supply chain network model with information asymmetry in quality there exists a sufficiently large M, such that for any (i, j, k),

$$\frac{\partial U_i(Q, q)}{\partial Q_{ijk}} < 0, \tag{32}$$

for all shipment patterns Q with $Q_{ijk} \geq M$ and that there exists a sufficiently large $\bar{M}$, such that for any (i, j),

$$\frac{\partial U_i(Q, q)}{\partial q_{ij}} < 0, \tag{33}$$

for all quality level patterns q with $q_{ij} \geq \bar{M} \geq \underline{q}_{ij}$.

We now give an existence result and a uniqueness result, the proof of which follows from the basic theory of variational inequalities (cf. Nagurney (1999)).

Proposition 1

Any supply chain network problem with information asymmetry in quality that satisfies Assumption 1 possesses at least one equilibrium shipment and quality level pattern satisfying variational inequality (19) (or (20)).

Proof: The proof follows from Proposition 1 in Zhang and Nagurney (1995). □

Proposition 2

Suppose that F is strictly monotone at any equilibrium point of the variational inequality problem defined in (20). Then it has at most one equilibrium point.

In addition, an existence and uniqueness result is presented in the following, the proof of which follows from the basic theory of variational inequalities (cf. Nagurney (1999)).

Theorem 3

Suppose that F is strongly monotone. Then there exists a unique solution to variational inequality (20); equivalently, to variational inequality (19).

The following theorem summarizes the stability properties of the utility gradient processes, under various monotonicity conditions on the marginal utilities.

Theorem 4

(i). *If $F(X)$ is monotone, then every supply chain network equilibrium with information asymmetry, X^*, provided its existence, is a global monotone attractor for the projected dynamical system. If $F(X)$ is locally monotone at X^*, then it is a monotone attractor for the projected dynamical system.*

(ii). *If $F(X)$ is strictly monotone, then there exists at most one supply chain network equilibrium with information asymmetry in quality, X^*. Furthermore, given existence, the unique equilibrium is a strictly global monotone attractor for the projected dynamical system. If $F(X)$ is locally strictly monotone at X^*, then it is a strictly monotone attractor for the projected dynamical system.*

(iii). *If $F(X)$ is strongly monotone, then the unique supply chain network equilibrium with information asymmetry in quality, which is guaranteed to exist, is also globally exponentially stable for the projected dynamical system. If $F(X)$ is locally strongly monotone at X^*, then it is exponentially stable.*

Proof: The stability assertions follow from Theorems 1.25, 1.26, and 1.27 in Nagurney (1999); see also Nagurney and Zhang (1996). $\square$

4. Algorithm

As mentioned in Section 3, the projected dynamical system yields continuous-time adjustment processes. For computational purposes, a discrete-time algorithm, which serves as an approximation to the continuous-time trajectories, will be introduced in this section.

The algorithm that we employed for the computation of the solution for supply chain network model with information asymmetry in quality is the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, recall that at iteration τ of the Euler method (see also Nagurney and Zhang (1996)), one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{34}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (20).

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme as well as various applications to the solutions of other game theory models can be found in Nagurney, Dupuis, and Zhang (1994), Nagurney (2010b), Nagurney and Yu (2012), and Nagurney, Li, and Nagurney (2013).

Explicit Formulae for the Euler Method Applied to the Supply Chain Network Competition Model with Information Asymmetry in Quality

The Euler method yields, at each iteration, explicit formulae for the computation of the product shipments and quality levels. In particular, we have the following closed form expressions for the product shipments for $i = 1, \ldots, I$; $j = 1, \ldots, n_i$; $k = 1, \ldots, n_R$:

$$Q_{ijk}^{\tau+1} = \max\{0, Q_{ijk}^{\tau} + a_{\tau}(\hat{\rho}_k(Q^{\tau}, q^{\tau}) + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_l(Q^{\tau}, q^{\tau})}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q_{ihl}^{\tau} - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^{\tau}, q^{\tau})}{\partial Q_{ijk}}$$

$$- \sum_{h=1}^{n_i} \sum_{l=1}^{n_R} \frac{\partial \hat{c}_{ihl}(Q^{\tau}, q^{\tau})}{\partial Q_{ijk}}\} \tag{35}$$

and the following closed form expressions for the quality levels for $i = 1, \ldots, I$; $j = 1, \ldots, n_i$:

$$q_{ij}^{\tau+1} = \max\{\underline{q}_{ij}, q_{ij}^{\tau} + a_{\tau}(\sum_{k=1}^{n_R} \frac{\partial \hat{\rho}_k(Q^{\tau}, q^{\tau})}{\partial q_{ij}} \sum_{h=1}^{n_i} Q_{ihk}^{\tau} - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^{\tau}, q^{\tau})}{\partial q_{ij}} - \sum_{h=1}^{n_i} \sum_{k=1}^{n_R} \frac{\partial \hat{c}_{ihk}(Q^{\tau}, q^{\tau})}{\partial q_{ij}}\}. \tag{36}$$

We now provide the convergence result. The proof follows using similar arguments as those in Theorem 5.8 in Nagurney and Zhang (1996).

Theorem 5

In the supply chain network model with information asymmetry in quality, let $F(X) = -\nabla U(Q,q)$*, where we group all* U_i*;* $i = 1, \ldots, I$*, into the vector* $U(Q,q)$*, be strictly monotone at any equilibrium shipment pattern and quality levels and assume that Assumption 1 is satisfied. Furthermore, assume that* F *is uniformly Lipschitz continuous. Then there exists a unique equilibrium product shipment and quality level pattern* $(Q^*, q^*) \in \mathcal{K}^2$*, and any sequence generated by the Euler method as given by (34) above, with explicit formulae at each iteration given by (35) and (36), where* $\{a_\tau\}$ *satisfies* $\sum_{\tau=0}^{\infty} a_\tau = \infty$*,* $a_\tau > 0$*,* $a_\tau \to 0$*, as* $\tau \to \infty$ *converges to* (Q^*, q^*)*.*

5. Numerical Examples

In this Section, we present numerical supply chain network examples with information asymmetry in quality, which we solve via the Euler method, as described in Section 4. We provide a spectrum of examples, accompanied by sensitivity analysis. We implemented the Euler method using Matlab on a Lenovo E46A. The convergence tolerance is 10^{-6}, so that the algorithm is deemed to have converged when the absolute value of the difference between each successive product shipment and quality level is less than or equal to 10^{-6}. The sequence $\{a_\tau\}$ is set to: $.3\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$. We initialize the algorithm by setting the product shipments equal to 20 and the quality levels equal to 0.

Example 1

The supply chain network topology of Example 1 is given in Figure 2. There are two firms, both of which have a single manufacturing plant and serve the same demand market R_1. The data are as follows.

The production cost functions at the manufacturing plants, M_1^1 and M_2^1, are:

$$\hat{f}_{11}(Q_{111}, q_{11}) = 0.8Q_{111}^2 + 0.5Q_{111} + 0.25Q_{111}q_{11} + 0.5q_{11}^2, \tag{37}$$

$$\hat{f}_{21}(Q_{211}, q_{21}) = Q_{211}^2 + 0.8Q_{211} + 0.3Q_{211}q_{21} + 0.65q_{21}^2. \tag{38}$$

The total transportation cost functions from the plants to the demand market R_1 are:

$$\hat{c}_{111}(Q_{111}, q_{11}) = 1.2Q_{111}^2 + Q_{111} + 0.25Q_{211} + 0.25q_{11}^2, \tag{39}$$

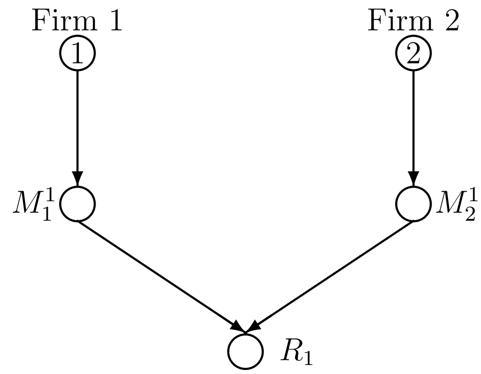

Figure 2: The Supply Chain Network Topology for Example 1

$$\hat{c}_{211}(Q_{211}, q_{21}) = Q_{211}^2 + Q_{211} + 0.35Q_{111} + 0.3q_{21}^2. \tag{40}$$

The demand price function at the demand market R_1 is:

$$\hat{\rho}_1(Q, \hat{q}) = 2250 - (Q_{111} + Q_{211}) + 0.8\hat{q}_1, \tag{41}$$

with the average quality expression given by:

$$\hat{q}_1 = \frac{Q_{111}q_{11} + Q_{211}q_{21}}{Q_{111} + Q_{211}}. \tag{42}$$

Also, we have that there are no positive imposed minimum quality standards, so that:

$$\underline{q}_{11} = \underline{q}_{21} = 0.$$

The Euler method converges in 437 iterations and yields the following equilibrium solution. The equilibrium product shipments are:

$$Q_{111}^* = 323.42, \quad Q_{211}^* = 322.72,$$

with the equilibrium demand at the demand market being, hence, $d_1^* = 646.14$.

The equilibrium quality levels are:

$$q_{11}^* = 32.43, \quad q_{21}^* = 16.91,$$

with the average quality level at R_1, $\hat{q}_1$, being 24.68.

The incurred demand market price at the equilibrium is:

$$\hat{\rho}_1 = 1623.60.$$

The profits of the firms are, respectively, 311,926.68 and 313,070.55.

In terms of qualitative analysis, the Jacobian matrix of $F(X) = -\nabla U(Q, q)$, denoted by $J(Q_{111}, Q_{211}, q_{11}, q_{21})$, for this problem and evaluated at the equilibrium point $X^* = (Q^*_{111}, Q^*_{211}, q^*_{11}, q^*_{21})$ is:

$$J(Q_{111}, Q_{211}, q_{11}, q_{21}) = \begin{pmatrix} 5.99 & 1.01 & -0.35 & -0.20 \\ 0.99 & 6.01 & -0.20 & -0.30 \\ -0.35 & 2.00 & 1.50 & 0 \\ 0.20 & -0.30 & 0 & 1.90 \end{pmatrix}.$$

The eigenvalues of $\frac{1}{2}(J + J^T)$ are: 1.47, 1.88, 5.03, and 7.02, and are all positive. Thus, the equilibrium solution is unique, and the conditions for convergence of the algorithm are also satisfied (cf. Theorem 5). Moreover, according to Theorem 4, the equilibrium solution X^* to this example is a strictly monotone attractor and it is also exponentially stable.

Sensitivity Analysis

We conducted sensitivity analysis by varying $\underline{q}_{11}$ and $\underline{q}_{21}$ beginning with their values set at 0 and increasing them to reflect the imposition of minimum quality standards set to 200, 400, 600, 800, and 1000. We display the results of this sensitivity analysis in Figures 3 and 4.

As the imposed minimum quality standard of a firm increases, its equilibrium quality level increases (cf. Figures 3.c and 3.d), which results in increasing production and transportation costs for the firm. Thus, in order to alleviate increasing costs, its equilibrium shipment quantity decreases as does its profit (cf. Figures 4.b and 4.c). However, due to competition, its competitor's product shipment increases or at least remains the same (cf. Figures 3.a and 3.b).

Moreover, since consumers at the demand market do not differentiate between the products from different firms, and there is information asymmetry in quality between the firms (sellers) and the consumers (buyers) at the demand market, the average quality level at the demand market, as well as the price, which is determined by the quality levels of both firms (cf. (41) and (42)), is for both firms' products. Firms prefer a higher average quality, since, at the same demand level, a higher average quality results in a higher price of the product. However, once a firm increases its own quality level, of course, the average quality level and, hence, the price increases, but its total cost will also increase due to the higher quality. Furthermore, the price increase is not only for the firm's own product, but also for its competitor's product. If a firm increases its own quality, both the firm and its competitor would get the benefits of the price increase, but only the firm itself would pay for the quality

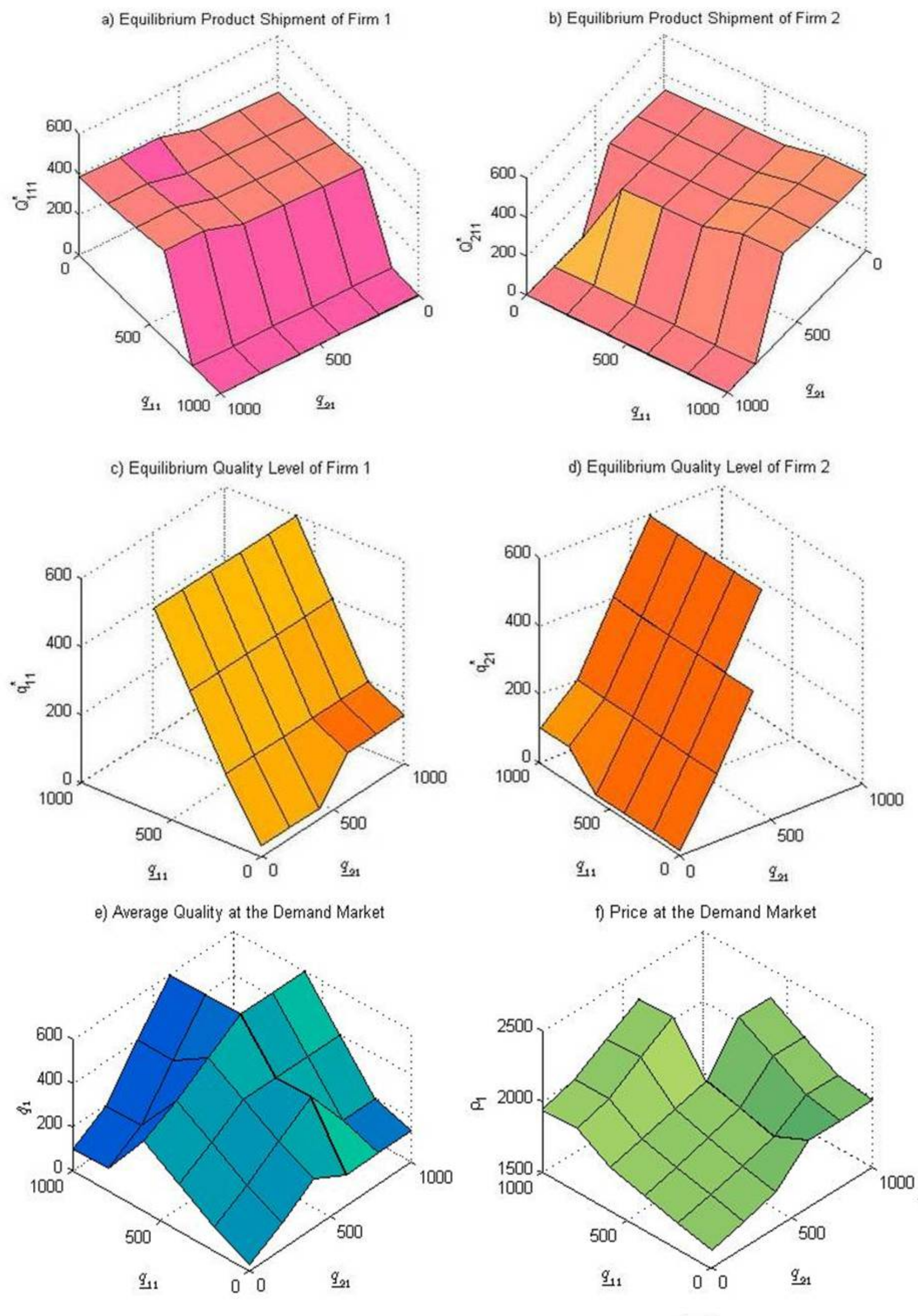

Figure 3: Equilibrium Product Shipments, Equilibrium Quality Levels, Average Quality at the Demand Market, and Price at the Demand Market as $\underline{q}_{11}$ and $\underline{q}_{21}$ Vary in Example 1

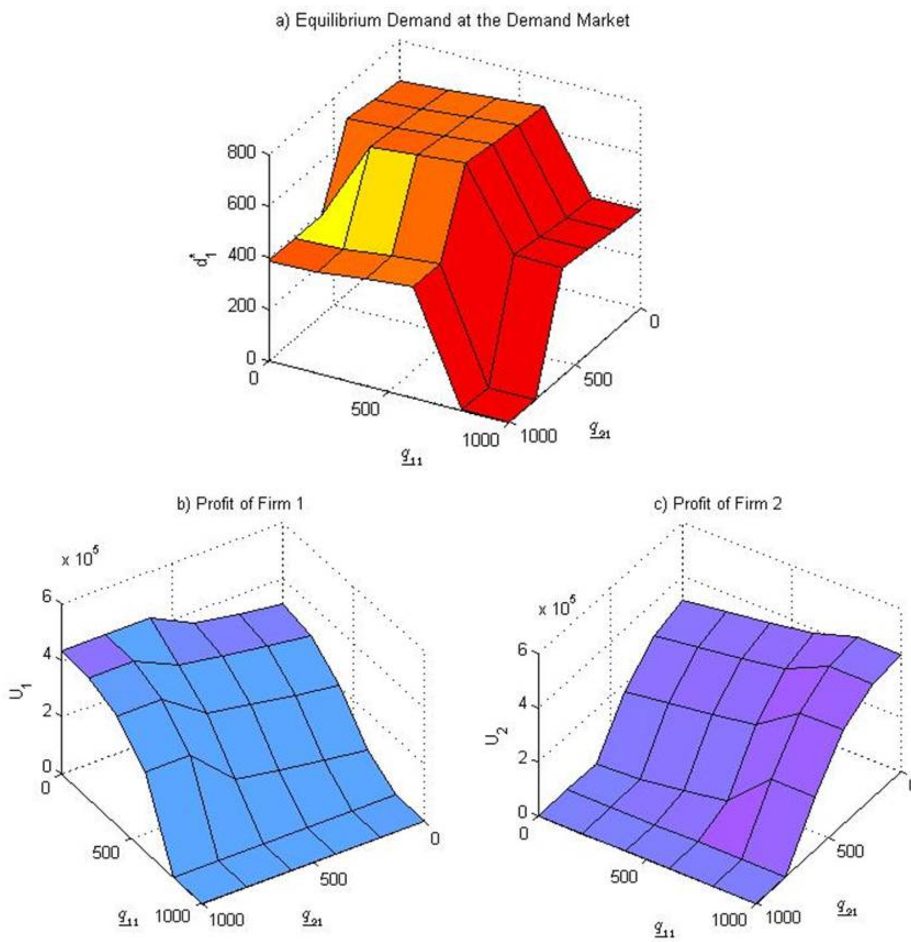

Figure 4: Demand at R_1 and the Profits of the Firms as $\underline{q}_{11}$ and $\underline{q}_{21}$ Vary in Example 1

improvement. Thus, a firm prefers a free ride, that is, it prefers that the other firm improve its product quality and, hence, the price, rather than have it increase its own quality.

Consequently, a firm may not be willing to increase its quality levels, while the other firm is, unless it is beneficial both cost-wise and profit-wise. This explains why, as the minimum quality standard of one firm increases, its competitor's quality level increases slightly or remains the same (cf. Figures 3.c and 3.d).

When there is an enforced higher minimum quality standard imposed on a firm's plant(s), the firm is forced to achieve a higher quality level, which may bring its own profit down but raise the competitor's profit (cf. Figures 4.b and 4.c), even though the latter firm may actually face a lower minimum quality standard. When the minimum quality standard of a firm increases to a very high value, but that of its competitor is low, the former firm will not be able to afford the high associated cost with decreasing profit, and, hence, it will produce no product for the demand market and will be forced to leave the market.

The above results and discussion indicate the same result, but in a much more general supply chain network context, as found in Ronnen (1991), who, in speaking about minimum quality standards, on page 492, noted that: "low-quality sellers can be better off ... and high-quality sellers are worse off." Also the computational results support the statement on page 490 in Akerlof (1970) that "good cars may be driven out of the market by lemons." Moreover, our results also show that the lower the competitor's quality level, the more harmful the competitor is to the firm with the high minimum quality standard, as shown in Figures 4.b and 4.c. The implications of the sensitivity analysis for policy-makers are clear – the imposition of a one-sided quality standard can have a negative impact on the firm in one's region (or country). Moreover, policy-makers, who are concerned about the products at particular demand markets, should prevent firms located in regions with very low minimum quality standards from entering the market; otherwise, they may not only bring the average quality level at the demand market(s) down and hurt the consumers, but such products may also harm the profits of the other firms with much higher quality levels and even drive them out of the market.

Therefore, it would be beneficial and fair for both firms and consumers if the policy-makers at the same or different regions or even countries could impose the same or at least similar minimum quality standards on plants serving the same demand market(s). In addition, the minimum quality standards should be such that they will not negatively impact either the high quality firms' survival or the consumers at the demand market(s).

Example 2

Example 2 is built from Example 1. In Example 2, there is an additional manufacturing plant available for each of the two firms, and we assume that the new plant for each firm has the same associated data as its original one. This would represent a scenario in which each firm builds an identical plant in proximity to its original one. Thus, the forms of the production cost functions associated with the new plants, M_1^2 and M_2^2, and the total transportation cost functions associated with the new links to R_1 are the same as those for their counterparts in Example 1 (but depend on the corresponding variables). This example has the topology given in Figure 5.

The data associated with the new plants are as below.

The production cost functions at the new manufacturing plants, M_1^2 and M_2^2, are:

$$\hat{f}_{12}(Q_{121}, q_{12}) = 0.8Q_{121}^2 + 0.5Q_{121} + 0.25Q_{121}q_{12} + 0.5q_{12}^2, \tag{43}$$

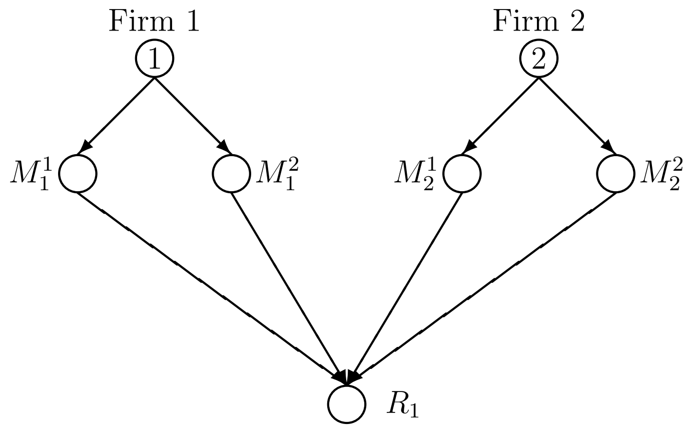

Figure 5: The Supply Chain Network Topology for Examples 2 and 3

$$\hat{f}_{22}(Q_{221}, q_{22}) = Q_{221}^2 + 0.8Q_{221} + 0.3Q_{221}q_{22} + 0.65q_{22}^2. \tag{44}$$

The total transportation cost functions on the new links are:

$$\hat{c}_{121}(Q_{121}, q_{12}) = 1.2Q_{121}^2 + Q_{121} + 0.25Q_{221} + 0.25q_{12}^2, \tag{45}$$

$$\hat{c}_{221}(Q_{221}, q_{22}) = Q_{221}^2 + Q_{221} + 0.35Q_{121} + 0.3q_{22}^2. \tag{46}$$

The demand price function retains its functional form, but with the new potential shipments added so that:

$$\hat{\rho}_1 = 2250 - (Q_{111} + Q_{211} + Q_{121} + Q_{221}) + 0.8\hat{q}_1, \tag{47}$$

with the average quality at R_1 expressed as:

$$\hat{q}_1 = \frac{Q_{111}q_{11} + Q_{211}q_{21} + Q_{121}q_{12} + Q_{221}q_{22}}{Q_{111} + Q_{211} + Q_{121} + Q_{211}}. \tag{48}$$

Also, at the new manufacturing plants we have that, as in the original ones:

$$\underline{q}_{12} = \underline{q}_{22} = 0.$$

The Euler method converges in 408 iterations to the following equilibrium solution. The equilibrium product shipments are:

$$Q_{111}^* = 225.96, \quad Q_{121}^* = 225.96, \quad Q_{211}^* = 225.54, \quad Q_{221}^* = 225.54.$$

The equilibrium demand at R_1 is, hence, $d_1^* = 903$.

The equilibrium quality levels are:

$$q_{11}^* = 22.65, \quad q_{12}^* = 22.65, \quad q_{21}^* = 11.83, \quad q_{22}^* = 11.83,$$

with the average quality level, $\hat{q}_1$, now equal to 17.24. Note that the average quality level has dropped precipitously from its value of 24.68 in Example 1.

The incurred demand market price at R_1 is:

$$\hat{\rho}_1 = 1,360.78.$$

The profits of the firms are, respectively, 406,615.47 and 407,514.97.

We now discuss the results. Since, for each firm, its new manufacturing plant and the original one are assumed to be identical, the equilibrium product shipments and the quality levels associated with the two plants are identical for each firm.

The availability of an additional manufacturing plant for each firm leads to the following results. First, the total cost of manufacturing and transporting the same amount of products is now less than in Example 1 for each firm, which can be verified by substituting $Q_{111}+Q_{121}$ for Q_{111} and $Q_{211}+Q_{221}$ for Q_{211} in (37) - (40) and comparing the total cost of each firm in Example 1 with that in Example 2. Hence, although the product shipments produced by the same manufacturing plant decrease in comparison to the associated values in Example 1, the total amount supplied by each firm increases, as does the total demand. The strategy of building an identical plant at the same location as the original one appears to be cost-wise and profitable for the firms; however, at the expense of a decrease in the average quality level at the demand market, as reflected in the results for Example 2. Policy-makers may wish to take note of this.

The Jacobian matrix of $F(X) = -\nabla U(Q,q)$, $J(Q_{111}, Q_{121}, Q_{211}, Q_{221}, q_{11}, q_{12}, q_{21}, q_{22})$, and evaluated at X^* for Example 2, is

$$J(Q_{111}, Q_{121}, Q_{211}, Q_{221}, q_{11}, q_{12}, q_{21}, q_{22})$$

$$= \begin{pmatrix} 5.99 & 1.99 & 1.00 & 1.00 & -0.25 & -0.10 & -0.10 & -0.10 \\ 1.00 & 6.00 & 1.00 & 1.00 & -0.10 & -0.25 & -0.10 & -0.10 \\ 1.00 & 1.00 & 6.00 & 2.01 & -0.10 & -0.10 & -0.20 & -0.10 \\ 1.00 & 1.00 & 2.00 & 6.00 & -0.10 & -0.10 & -0.10 & -0.20 \\ -0.25 & -0.10 & 0.10 & 0.10 & 1.50 & 0 & 0 & 0 \\ -0.10 & -0.25 & 0.10 & 0.10 & 0 & 1.50 & 0 & 0 \\ 0.10 & 0.10 & -0.20 & -0.10 & 0 & 0 & 1.90 & 0 \\ 0.10 & 0.10 & -0.10 & -0.20 & 0 & 0 & 0 & 1.90 \end{pmatrix}.$$

We note that the Jacobian matrix for this example is strictly diagonally dominant, which guarantees its positive-definiteness. Thus, the equilibrium solution X^* is unique, the conditions for convergence of the algorithm are also satisfied, and the equilibrium solution is a strictly monotone attractor. Moreover, X^* is exponentially stable.

Example 3

Example 3 is constructed from Example 2, but now the new plant for Firm 1, M_1^2, is located in a country where the production cost is much lower but the total transportation cost to the demand market R_1 is higher, in comparison to the data in Example 2. In addition, the location of the second plant of Firm 2, M_2^2, also changes, resulting in both a higher production cost and a higher transportation cost to R_1. Thus, the new manufacturing plants for each firm now have different associated cost functions as given below.

The production cost functions of the new plants, M_1^2 and M_2^2, are:

$$\hat{f}_{12}(Q_{121}, q_{12}) = 0.3Q_{121}^2 + 0.1Q_{121} + 0.3Q_{121}q_{12} + 0.4q_{12}^2,$$

$$\hat{f}_{22}(Q_{221}, q_{22}) = 1.2Q_{221}^2 + 0.5Q_{221} + 0.3Q_{221}q_{22} + 0.5q_{22}^2.$$

The total transportation cost functions on the new links are now:

$$\hat{c}_{121}(Q_{121}, q_{12}) = 1.8Q_{121}^2 + Q_{121} + 0.25Q_{221} + 0.25q_{12}^2,$$

$$\hat{c}_{221}(Q_{221}, q_{22}) = 1.5Q_{221}^2 + 0.8Q_{221} + 0.3Q_{121} + 0.3q_{22}^2.$$

The Euler method converges in 498 iterations, yielding the equilibrium product shipments:

$$Q_{111}^* = 232.86, \quad Q_{121}^* = 221.39, \quad Q_{211}^* = 240.82, \quad Q_{221}^* = 178.45,$$

with an equilibrium demand $d_1^* = 873.52$. The equilibrium quality levels are:

$$q_{11}^* = 25.77, \quad q_{12}^* = 19.76, \quad q_{21}^* = 10.64, \quad q_{22}^* = 9.37,$$

with the average quality level at R_1, $\hat{q}_1$, equal to 16.73. The incurred demand market price is

$$\hat{\rho}_1 = 1,389.86.$$

The profits of the firms are, respectively, 415,706.05 and 378,496.95,

Although the production cost of Firm 1's foreign plant, M_1^2, is lower than that of the original plant, M_1^1, because of the high transportation cost to the demand market, the quantity produced at and shipped from M_1^2 decreases, in comparison to the value in Example 2. In addition, because of the higher manufacturing cost at Firm 2's foreign plant, M_2^2, the total supply of the product from Firm 2 now decreases. The other results are: the demand at demand market R_1 decreases and the average quality there decreases slightly.

The Jacobian matrix of $F(X) = -\nabla U(Q, q)$ at equilibrium, denoted by $J(Q_{111}, Q_{121}, Q_{211}, Q_{221}, q_{11}, q_{12}, q_{21}, q_{22})$, for this example, is

$$J(Q_{111}, Q_{121}, Q_{211}, Q_{221}, q_{11}, q_{12}, q_{21}, q_{22})$$

$$= \begin{pmatrix} 5.99 & 1.99 & 1.01 & 1.01 & -0.27 & -0.10 & -0.11 & -0.08 \\ 1.99 & 6.20 & 1.00 & 1.00 & -0.10 & -0.21 & -0.11 & -0.08 \\ 0.99 & 1.00 & 6.01 & 2.01 & -0.11 & -0.11 & -0.20 & -0.08 \\ 0.99 & 1.00 & 2.01 & 7.41 & -0.11 & -0.11 & -0.11 & -0.17 \\ -0.27 & -0.10 & 0.11 & 0.11 & 1.50 & 0 & 0 & 0 \\ -0.10 & -0.21 & 0.11 & 0.11 & 0 & 1.30 & 0 & 0 \\ 0.11 & 0.11 & -0.20 & -0.11 & 0 & 0 & 1.90 & 0 \\ 0.08 & 0.08 & -0.08 & -0.17 & 0 & 0 & 0 & 1.60 \end{pmatrix}.$$

This Jacobian matrix is strictly diagonally dominant, and, hence, it is positive-definite. Thus, the uniqueness of the computed equilibrium is guaranteed. Also, the conditions for convergence of the algorithm are satisfied. The equilibrium solution for Example 3 has the same qualitative properties as the solution to Example 2.

Example 4

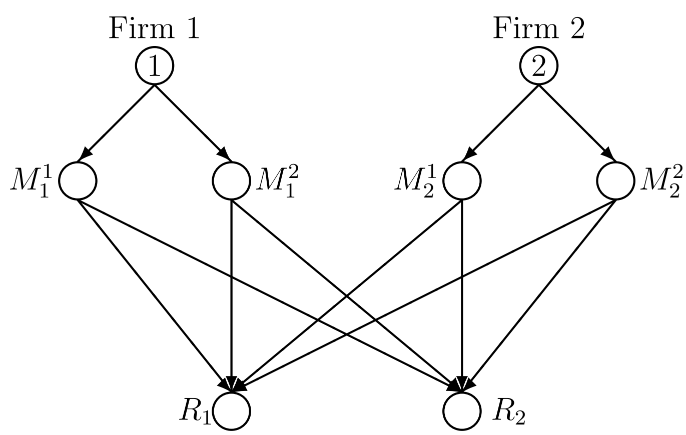

Figure 6: The Supply Chain Network Topology for Example 4

Example 4 considers the following scenario. Please refer to Figure 6 for the supply chain network topology for this example. There is a new demand market, R_2, added to Example 3, which is located closer to both firms' manufacturing plants than the original demand market

R_1. The total transportation cost functions for transporting the product to R_2 for both firms, respectively, are:

$$\hat{c}_{112}(Q_{112}, q_{11}) = 0.8Q_{112}^2 + Q_{112} + 0.2Q_{212} + 0.05q_{11}^2, \tag{49}$$

$$\hat{c}_{122}(Q_{122}, q_{12}) = 0.75Q_{122}^2 + Q_{122} + 0.25Q_{222} + 0.03q_{12}^2, \tag{50}$$

$$\hat{c}_{212}(Q_{212}, q_{21}) = 0.6Q_{212}^2 + Q_{212} + 0.3Q_{112} + 0.02q_{21}^2, \tag{51}$$

$$\hat{c}_{222}(Q_{222}, q_{22}) = 0.5Q_{222}^2 + 0.8Q_{222} + 0.25Q_{122} + 0.05q_{22}^2. \tag{52}$$

The production cost functions at the manufacturing plants have the same functional forms as in Example 3, but now they include the additional shipments to the new demand market, R_2, that is:

$$\hat{f}_{12}(Q_{121}, Q_{122}, q_{12}) = 0.3(Q_{121} + Q_{122})^2 + 0.1(Q_{121} + Q_{122}) + 0.3(Q_{121} + Q_{122})q_{12} + 0.4q_{12}^2,$$

$$\hat{f}_{22}(Q_{221}, Q_{222}, q_{22}) = 1.2(Q_{221} + Q_{222})^2 + 0.5(Q_{221} + Q_{222}) + 0.3(Q_{221} + Q_{222})q_{22} + 0.5q_{22}^2.$$

$$\hat{f}_{11}(Q_{111}, Q_{112}, q_{11}) = 0.8(Q_{111} + Q_{112})^2 + 0.5(Q_{111} + Q_{112}) + 0.25(Q_{111} + Q_{112})q_{11} + 0.5q_{11}^2,$$

$$\hat{f}_{21}(Q_{211}, Q_{212}, q_{21}) = (Q_{211} + Q_{212})^2 + 0.8(Q_{211} + Q_{212}) + 0.3(Q_{211} + Q_{212})q_{21} + 0.65q_{21}^2.$$

In addition, consumers at the new demand market R_2 are more sensitive to the quality of the product than consumers at the original demand market R_1. The demand price functions for both the demand markets are, respectively:

$$\hat{\rho}_1 = 2250 - (Q_{111} + Q_{211} + Q_{121} + Q_{221}) + 0.8\hat{q}_1,$$

$$\hat{\rho}_2 = 2250 - (Q_{112} + Q_{122} + Q_{212} + Q_{222}) + 0.9\hat{q}_2,$$

where

$$\hat{q}_1 = \frac{Q_{111}q_{11} + Q_{211}q_{21} + Q_{121}q_{12} + Q_{221}q_{22}}{Q_{111} + Q_{211} + Q_{121} + Q_{211}},$$

and

$$\hat{q}_2 = \frac{Q_{112}q_{11} + Q_{212}q_{21} + Q_{122}q_{12} + Q_{222}q_{22}}{Q_{112} + Q_{212} + Q_{122} + Q_{222}}.$$

The Euler method converges in 597 iterations, and the equilibrium solution is as below. The equilibrium product shipments are:

$$Q_{111}^* = 208.70, \quad Q_{121}^* = 211.82, \quad Q_{211}^* = 203.90, \quad Q_{221}^* = 129.79,$$

$$Q_{112}^* = 165.39, \quad Q_{122}^* = 352.11, \quad Q_{212}^* = 182.30, \quad Q_{222}^* = 200.05.$$

The equilibrium demand at the two demand markets is now $d_1^* = 754.21$ and $d_2^* = 899.85$.

The equilibrium quality levels are:

$$q_{11}^* = 53.23, \quad q_{12}^* = 79.08, \quad q_{21}^* = 13.41, \quad q_{22}^* = 13.82.$$

The value of $\hat{q}_1$ is 42.94 and that of $\hat{q}_2$ is 46.52.

The incurred demand market prices are:

$$\hat{\rho}_1 = 1,530.15, \quad \hat{\rho}_2 = 1,392.03.$$

The profits of the firms are, respectively, 882,342.15 and 651,715.83.

Due to the addition of R_2, which has associated lower transportation costs, each firm ships more product to demand market R_2 than to R_1, and, at the same time, some of the previous demand at R_1 is shifted to R_2. Hence, the total demand $d_1 + d_2$ is now 88.76% larger than the total demand d_1 in Example 2.

In addition, Firm 1 is the one with larger market shares, and is able to achieve higher profit by attaining higher quality levels. Thus, as the total demand increases, the quality levels of Firm 1 increase significantly. However, since it is not cost-wise for Firm 2 to do so, due to its higher costs and lower market shares, Firm 2 prefers a "free ride" from Firm 1 with its quality levels basically remaining the same. The average quality levels, nevertheless, increase substantially anyway, which leads to the increase in the prices and both firms' profits.

The Jacobian matrix of $-\nabla U(Q,q)$, for Example 4, evaluated at the equilibrium, and denoted by $J(Q_{111}, Q_{121}, Q_{211}, Q_{221}, Q_{112}, Q_{122}, Q_{212}, Q_{222}, q_{11}, q_{12}, q_{21}, q_{22})$, is

$$J(Q_{111}, Q_{121}, Q_{211}, Q_{221}, Q_{112}, Q_{122}, Q_{212}, Q_{222}, q_{11}, q_{12}, q_{21}, q_{22})$$

$$= \begin{pmatrix}
5.99 & 1.98 & 1.02 & 1.02 & 1.60 & 0 & 0 & 0 & -0.29 & -0.10 & -0.10 & -0.06 \\
1.98 & 6.17 & 1.04 & 1.04 & 0 & 0.60 & 0 & 0 & -0.10 & -0.25 & -0.10 & -0.06 \\
0.98 & 0.96 & 6.03 & 2.03 & 0 & 0 & 2.00 & 0 & -0.12 & -0.13 & -0.17 & -0.08 \\
0.98 & 0.96 & 2.03 & 7.43 & 0 & 0 & 0 & 2.40 & -0.12 & -0.13 & -0.12 & -0.13 \\
1.60 & 0 & 0 & 0 & 5.19 & 1.98 & 1.02 & 1.02 & -0.34 & -0.15 & -0.08 & -0.09 \\
0 & 0.60 & 0 & 0 & 1.98 & 4.07 & 1.03 & 1.03 & -0.07 & -0.37 & -0.08 & -0.09 \\
0 & 0 & 2.00 & 0 & 0.98 & 0.97 & 5.24 & 2.04 & -0.10 & -0.20 & -0.19 & -0.12 \\
0 & 0 & 0 & 2.40 & 0.98 & 0.97 & 2.04 & 5.44 & -0.10 & -0.20 & -0.10 & -0.20 \\
-0.29 & -0.10 & 0.12 & 0.12 & -0.34 & -0.07 & 0.10 & 0.10 & 1.60 & 0 & 0 & 0 \\
-0.10 & -0.25 & 0.13 & 0.13 & -0.15 & -0.37 & 0.20 & 0.20 & 0 & 1.36 & 0 & 0 \\
0.10 & 0.10 & -0.17 & -0.12 & 0.08 & 0.08 & -0.19 & -0.10 & 0 & 0 & 1.94 & 0 \\
0.06 & 0.06 & -0.08 & -0.13 & 0.09 & 0.09 & -0.12 & -0.20 & 0 & 0 & 0 & 1.70
\end{pmatrix}.$$

The eigenvalues of $\frac{1}{2}(J + J^T)$ are all positive and are: 1.29, 1.55, 1.66, 1.71, 1.93, 2.04, 3.76, 4.73, 6.14, 7.55, 8.01, and 11.78. Therefore, both the uniqueness of the equilibrium solution and the conditions for convergence of the algorithm are guaranteed. The equilibrium solution to Example 4 is a strictly monotone attractor and is exponentially stable.

Sensitivity Analysis

We now explore the impact of the firms' proximity to the second demand market R_2. We multiply the coefficient of the second Q_{ijk} term, that is, the linear one, in each of the transportation cost functions $\hat{c}_{ijk}$ (49) – (52) by a positive factor β, but retain the other transportation cost functions as in Example 4. We vary β from 0 to 50, 100, 150, 200, 250, 300, and 350. The results are reported in Figure 7.

As β increases, that is, as R_2 is located farther, the transportation costs to R_2 increase. In order to decrease their total costs and increase their profits, firms ship less of the product to R_2 while their shipments to R_1 increase, as shown in Figure 7.a. In addition, at the same time, firms cannot afford higher quality as the total costs of both firms increase, so the average quality levels at both demand markets decrease, as indicated in Figure 7.b. Due to the changes in the demands and the average quality levels, the price at R_1 decreases, but that at R_2 increases, and the profits of both firms decrease, as in Figures 7.c and 7.d. When $\beta = 350$, demand market R_2 will be removed from the supply chain network, due to the demand there dropping to zero. Thus, when $\beta = 350$, the results of Example 4 are the same as those for Example 3.

The numerical examples in this Section, along with the sensitivity analysis results, reveal the type of questions that can be explored and addressed through computations. Moreover, the analyses demonstrate the impacts of minimum quality standards even "across borders" as well as the importance of the location of manufacturing plants vis a vis the demand markets. The insights gained from the numerical examples are useful to firms, to consumers at demand markets, as well as to policy-makers.

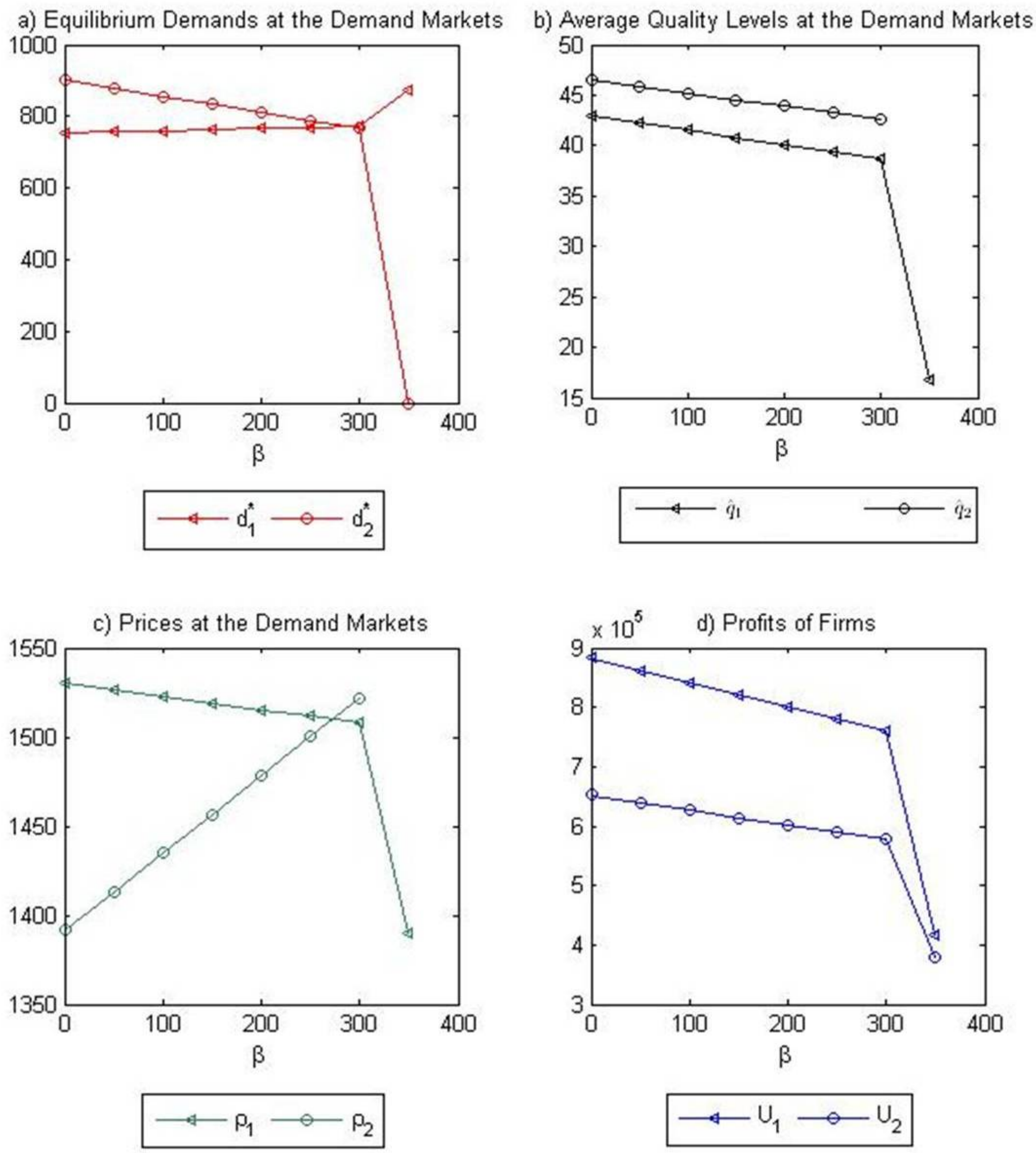

Figure 7: The Equilibrium Demands, Average Quality Levels, Prices at the Demand Markets, and the Profits of the Firms as β Varies in Example 4

6. Summary and Conclusions

In this paper, we developed a rigorous framework for the modeling, analysis, and computation of solutions to competitive supply chain network problems in static and dynamic settings in which there is information asymmetry in quality. We also demonstrated how our framework can capture the inclusion of policy interventions in the form of minimum quality standards.

This research adds to the literature on information asymmetry with imperfect competition, which has only recently garnered attention, and which has focused on analytical results for stylized problems. It also contributes to the literature on supply chains with quality competition and reveals the spectrum of insights that can be obtained through computations, supported by theoretical analysis. Finally, it contributes to the integration of economics with operations research and the management sciences.

In future research, we plan on exploring issues and applications of information asymmetry in quality in various imperfectly competitive environments, including those arising in healthcare settings. We also intend to assess the value of product differentiation for both producers and consumers alike and the role that minimum quality standards can play in such settings.

Acknowledgments

This research was supported, in part, by the National Science Foundation (NSF) grant CISE #1111276, for the NeTS: Large: Collaborative Research: Network Innovation Through Choice project awarded to the University of Massachusetts Amherst. This support is gratefully acknowledged.

The first author also thanks the organizers of the 10th International Conference on Computational Management Science held on Montreal, Quebec, Canada, May 1-3, 2013 for discussions with participants that led to this work.

The authors also thank the Guest Editors of this special volume.

This paper is dedicated to Professor Berc Rustem to thank him for his highest of standards, dedication, leadership, and vision as a scholar and as the Editor of the journal *Computational Management Science* over many years.

References

Akerlof, G.A., 1970. The market for 'lemons': Quality uncertainty and the market mechanism. Quarterly Journal of Economics, 84(3), 488-500.

Baltzer, K., 2012. Standards vs. labels with imperfect competition and asymmetric information. Economics Letters, 114(1), 61-63.

Besanko, D., Donnenfeld, S., White, L.J., 1988. The multiproduct firm, quality choice, and regulation. The Journal of Industrial Economics, 36(4), 411-429.

Burnetas, A., Gilbert. S.M., Smith, C.E., 2007. Quantity discounts in single-period supply contracts with asymmetric demand information. IIE Transactions, 39(5), 465-479.

Cachon, G. 2003. Supply chain coordination with contracts. In: Graves, S., de Kok, F. (Eds.), Handbooks in Operations Research and Management Science: Supply Chain Management, Volume 11 of Handbooks in OR & MS. North Holland, Amsterdam, The Netherlands, 229-339.

Chen, F., 2003. Information sharing and supply chain coordination. In: Graves, S., de Kok, F. (Eds.), Handbooks in Operations Research and Management Science: Supply Chain Management, Volume 11 of Handbooks in OR & MS. North Holland, Amsterdam, The Netherlands, 341-421.

Corbett, C., Zhou, D., Tang, C., 2004. Designing supply contract: Contract type and asymmetric information. Management Science, 50(4), 550-559.

Cournot, A.A., 1838. Researches into the Mathematical Principles of the Theory of Wealth, English translation. MacMillan, London, England.

Dafermos, S., Nagurney, A., 1987. Oligopolistic and competitive behavior of spatially separated markets. Regional Science and Urban Economics, 17, 245-254.

Dupuis, P., Nagurney, A., 1993. Dynamical systems and variational inequalities. Annals of Operations Research, 44, 9-42.

Esmaeili, M., Zeephongsekul, P., 2010. Seller-buyer models of supply chain management with an asymmetric information structure. International Journal of Production Economics, 123(1), 146-154.

Gabay, D., Moulin, H., 1980. On the uniqueness and stability of Nash equilibria in noncooperative games. In: Bensoussan, A., Kleindorfer, P., Tapiero, C.S. (Eds.), Applied Stochastic Control in Econometrics and Management Science, North-Holland, Amsterdam, The Netherlands, 271-294.

Gilmore, H.L., 1974. Product conformance cost. Quality Progress, 7(5), 16-19.

Giraud-Heraud, E., Soler, L.-G., 2006. Retailers' supply chain, product differentiation and quality standards. In: Ondersteijn, C.J.M., Wijnands, J.H.M., Huirne, R.B.M., van Kooten, O. (Eds.), Quantifying the Agri-Food Supply Chain. Springer, The Netherlands, 67-83.

Gray, J.V., Roth, A.V., Leiblein, M.V., 2011. Quality risk in offshore manufacturing: Evidence from the pharmaceutical industry. Journal of Operations Management, 29(7-8), 737-752.

Hasija, S., Pinker, E.J., Shumsky, R.A., 2008. Call center outsourcing contracts under information asymmetry. Management Science, 54(4), 793-807.

Hogenau, B., 2013. U.S. manufacturers gain edge over foreign competition with quality and sustainability initiatives. Environmental Protection;
http://eponline.com/articles/2013/01/11/us-manufacturers-gain-edge-over-foreign-competition-with-quality-and-sustainability-initiatives.aspx

Juran, J.M., Gryna, F.M., 1988. Quality Control Handbook, 4th edition. McGraw-Hill, New York.

Lee, C.Y., Yang, R., 2013. Supply chain contracting with competing suppliers under asymmetric information. IIE Transactions, 45(1), 25-52.

Leland, H.E., 1979. Quacks, lemons, and licensing: A theory of minimum quality standards. Journal of Political Economy, 87(6), 1328-1346.

Lutz, M.B., Lutz S., 2010. Pre-emption, predation, and minimum quality standards. International Economic Journal, 24(1), 111-123.

Mishra, B.K., Raghunathan, S., Yue, X., 2009. Demand forecast sharing in supply chains. Production and Operations Management, 18(2), 152-166.

Masoumi, A.H., Yu, M., Nagurney, A., 2012. A supply chain generalized network oligopoly model for pharmaceuticals under brand differentiation and perishability. Transportation

Research E, 48, 762-780.

McDonald, B., 2013. A fresh look at a dirty problem. Food Quality & Safety Magazine, February/March;
http://www.foodquality.com/details/article/834893/A_Fresh_Look_at_a_Dirty_Problem.html

Nagurney, A., 1999. Network Economics: A Variational Inequality Approach, second and revised edition. Kluwer Academic Publishers, Boston, Massachusetts.

Nagurney, A., 2006. Supply Chain Network Economics: Dynamics of Prices, Flows, and Profits. Edward Elgar Publishing, Cheltenham, England.

Nagurney, A., 2010a. Optimal supply chain network design and redesign at minimal total cost and with demand satisfaction. International Journal of Production Economics, 128(1), 200-208.

Nagurney, A., 2010b. Formulation and analysis of horizontal mergers among oligopolistic firms with insights into the merger paradox: A supply chain network perspective. Computational Management Science, 7, 377-410.

Nagurney, A., Dupuis, P., Zhang, D., 1994. A dynamical systems approach for network oligopolies and variational inequalities. Annals of Regional Science, 28, 263-283.

Nagurney, A., Li, D., 2013. A dynamic network oligopoly model with transportation costs, product differentiation, and quality competition. Computational Economics, in press.

Nagurney, A., Li, D., Nagurney, L.S., 2013. Pharmaceutical supply chain networks with outsourcing under price and quality competition. International Transactions in Operational Research, 20(6), 859-888.

Nagurney, A., Li, D., Wolf, T., Saberi, S., 2013a. A network economic game theory model of a service-oriented Internet with choices and quality competition. Netnomics, 14(1-2), 1-25.

Nagurney, A., Li, D., Saberi S., Wolf, T., 2014. A dynamic network economic model of a service-oriented Internet with price and quality competition. In:Kalyagin, V.A., Pardalos, P.M., Rassias, T.M. (Eds.), Network Models in Economics and Finance, Springer, New York, in press.

Nagurney, A., Nagurney, L.S., 2012. Medical nuclear supply chain design: A tractable network model and computational approach. International Journal of Production Economics, 140(2), 865-874.

Nagurney, A., Wolf, T., 2013. A Cournot-Nash-Bertrand game theory model of a service-oriented Internet with price and quality competition among network transport providers. Computational Management Science, in press.

Nagurney, A., Yu, M., 2012. Sustainable fashion supply chain management under oligopolistic competition and brand differentiation. International Journal of Production Economics, 135, 532-540.

Nagurney, A., Yu, M., Masoumi, A.H., Nagurney, L.S., 2013b. Networks Against Time: Supply Chain Analytics for Perishable Products. Springer Science + Business Media, New York.

Nagurney, A., Zhang, D., 1996. Projected Dynamical Systems and Variational Inequalities with Applications. Kluwer Academic Publishers, Boston, Massachusetts.

Nash, J.F., 1950. Equilibrium points in n-person games. Proceedings of the National Academy of Sciences, USA, 36, 48-49.

Nash, J.F., 1951. Noncooperative games. Annals of Mathematics, 54, 286-298.

Ren, Z.J., Cohen, M.A., Ho, T.H., Terwiesch, C., 2010. Information sharing in a long-term supply chain relationship: The role of customer review strategy. Operations Research, 58(1), 81-93.

Ronnen, M., 1991. Minimum quality standards, fixed costs, and competition. RAND Journal of Economics, 22(4), 490-504.

Saberi, S., Nagurney, A., Wolf, T., 2013. Price and quality competition in a service-oriented Internet. Isenberg School of Management, University of Massachusetts Amherst.

Shapiro, 1983. Premiums for high quality products as returns to reputations. Quarterly Journal of Economics, 98, 659-679.

Smith, G., 2009. Interaction of public and private standards in the food chain. OECD Food, Agriculture and Fisheries Papers, No. 15, OECD Publishing. http://dx.doi.org/10.1787/221282527214

Spence, M., 1973. Job market signaling. Quarterly Journal of Economics, 87(3), 355-374.

Spence, M., 1975. Monopoly, quality, and regulation. The Bell Journal of Economics, 6(2), 417-429.

Stiglitz, J.E., 1987. The causes and consequences of the dependence of quality on price. Journal of Economic Literature, 25(1), 1-48.

Thomas, K., 2013. Lapses in big drug factories as to shortages and danger. The New York Times, October 17.

Thomas, D.J., Warsing, D.P., Zhang, X., 2009. Forecast updating and supplier coordination for complementary component purchases. Production and Operations Management, 18(2), 167-184.

Tirole, J., 1988. The Theory of Industrial Organization. MIT Press, Cambridge, Massachusetts.

Wankhade, L., Dabade, B., 2010. Quality Uncertainty and Perception, Information Asymmetry and Management of Quality Uncertainty and Quality Perception. Springer-Verlag, Berlin, Germany.

Xu, H., Shi, N., Ma, S.H., Lai, K.K., 2010. Contracting with an urgent supplier under cost information asymmetry. European Journal of Operational Research, 206(2), 374-383.

Yu, M., Nagurney, A., 2013. Competitive food supply chain networks with application to fresh produce. European Journal of Operational Research, 224(2), 273-282.

Zhang, D., Nagurney, A., 1995. On the stability of projected dynamical systems. Journal of Optimization Theory and its Applications, 85, 97-124.

Zhou, Y.W., 2007. A comparison of different quantity discount pricing policies in a two-echelon channel with stochastic and asymmetric demand information. European Journal of Operational Research, 181(2), 686-703.