Multicommodity International Agricultural Trade Network Equilibrium: Competition for Limited Production and Transportation Capacity Under Disaster Scenarios with Implications for Food Security

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Abstract: The number of people affected by disasters, including man-made ones, is on the rise globally, with rising food insecurity being one of the most critical impacts. Disasters, both sudden-onset and slow-onset ones, can cause disruptions to the production and transportation of agricultural commodities. Having the tools that can quantitatively assess the changes in agricultural commodity shipment volumes and their prices under disruptions caused by disaster scenarios is of major importance. In this paper, we utilize the theory of variational inequalities as the methodology to construct a multicommodity international agricultural trade network equilibrium model, which contains novel features of capacities on the production and transportation of multiple agricultural commodities to capture competition. The model includes exchange rates and accounts for multiple routes and possibly distinct transportation modes and combinations. Theoretical results are given and an algorithm is proposed. A series of numerical examples, both illustrative and algorithmically solved ones, inspired by Russia's war on Ukraine, highlight the effects of reduced production and transportation capacities on food security in the Middle Eastern and North African (MENA) countries of Lebanon and Egypt. We also include sensitivity analysis results for exchange rates. The solutions reveal insights into the importance of the production and transportation capacities regarding food security, along with having multiple transportation routes that are cost-efficient as well as the importance of the magnitude of exchange rates.

Key words: agriculture, supply chains, network equilibrium, international trade, capacity limits, food security

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1. Introduction

Our planet and people are being faced with immense challenges brought about by disasters ranging from those caused by natural phenomena (earthquakes, tornadoes, volcanic eruptions, tsunamis, hurricanes, floods, droughts, etc.) and those wrought by unnatural phenomena, caused by humans (wars, explosions, terrorist attacks, chemical spills, etc.). Some disasters, such as wildfires, may be caused by lightning (nature) or be man-made due to intentional, accidental, or because of lack of investment in mitigation reasons. Climate change is exacerbating the intensity of many disasters as well as the number of people affected by them. Disasters, both slow-onset and sudden-onset ones, can have an immense impact on nations, regions, businesses, organizations, and individuals and their families with long-lasting repercussions, including the escalation of economic and social costs.

A multiplicity of disasters can also impact food security. Even before Russia launched the full-scale invasion of Ukraine on February 24, 2022, climate change and COVID-19 had already impacted the affordability and accessibility of agri-food products around the globe. With the added disruptions of what has become a de facto war in Europe, according to Belgrave (2022), around 47 million people are estimated to have been added to the more than 276 million who were already facing food insecurity, corresponding to an increase of about 17% in the food-insecure population, mainly in the vulnerable communities of Sub-Saharan Africa and the Middle East. The Food and Agriculture Organization (FAO) of the United Nations (2022) reports that between 20 and 30 percent of the Ukrainian land previously used for cultivating winter crops will probably remain unsown due to the ongoing war. The reduction of lands used to harvest spring crops is projected at about 20%. Further exacerbated by the damaged production and storage facilities and the accessibility to essential inputs, such as seeds, fertilizer, fuel, etc., the decrease in Ukrainian cereal production is estimated at around 40% as compared to the output levels of 2021. In addition, there have been immense challenges in getting the exports of agricultural products out of Ukraine with the blockade of the Black Sea in wartime. The Black Sea Grain Initiative among Ukraine, Turkey, the United Nations, and Russia came into force on August 1, 2022 (cf. UN News (2022a)). The deep-water Black Sea ports of Odesa, Chornomorsk, and Pivdenny were opened for exports. However, although the total capacity of these ports is approximately three million tons per month, the actual exports are below this level due to military risks and slow inspections by the Russian part of the commission in Bosphorus (Ukrainska Pravda (2023)). The initiative, as of mid-March 2023, had been extended twice (Hall (2023)). And now, more recently, as of July 17, 2023, Russia suspended the Initiative, and the controlled transport of Ukrainian grain from several of its Black Sea ports have stopped, imposing additional food security concerns globally (Nichols and Faulconbridge (2023)). For some background on the impacts of the war on agricultural supply chains and food insecurity see Nagurney (2022a).

With the major earthquake and aftershock striking Turkey and Syria in February 2023, the FAO is assessing their effects on the agricultural sector in both countries. Early reports reveal severe disruptions to agricultural infrastructure and production capacity, especially in Syria. The damage spans all parts of the infrastructure, from dams and irrigation systems to roads, markets, storage facilities, and grain processing plants (European Food Agency (2023)). In Turkey, the early estimate is approximately a 1% decrease in its GDP due to the damages sustained as a consequence of the immense natural hazard. Do Rosario (2023) highlights that most afflicted areas are agricultural regions, where other industrial activities are insignificant. As such, a good part of the projected economic damage is solely caused by the disruptions from the earthquake and the aftershock to the agricultural sector. The World Bank (2023) reports that one in five Turkish citizens live off agricultural sector jobs. Even before the earthquake, with the economic downturn in Turkey, an estimated 30% of Turkish citizens fell below the national poverty line (World Food Programme (2022a)). Furthermore, more than 4 million people in northern parts of Syria have already been identified as being food insecure (Mercy Corps (2023)).

Plus, the Horn of Africa is experiencing its worst drought recorded in modern history. The drought has been persistent for five rainfall seasons, and the forecasts indicate yet another poor rainfall season in 2023 (World Meteorological Organization (2023)). As reported by Cassidy (2022), the ongoing dry spell has caused food insecurity for 21 million people in the region in countries such as Somalia, Kenya, and Ethiopia, with more than 3 million of them facing extreme levels of food insecurity, that is, commonly going without eating any food for at least a day. The drought has also caused the displacement of millions of people. According to the UN News (2022b), from July to December 2022, the number of children facing severe food insecurity in the Horn of Africa region doubled from 10 million to more than 20 million. The high food prices caused by COVID-19, climate change, and the shortage of grains due to the ongoing war in Ukraine have further complicated the disastrous situation in the region.

In this paper, we construct a multicommodity international agricultural trade model, which contains novel features of capacities on the production outputs and on the transportation flows of agricultural commodities from supply market countries via different routes to demand market countries. The need for the inclusion of such capacities is based on real-world issues, notably, in the case of disasters, some of which are highlighted above. The model includes exchange rates, multiple agricultural commodities, multiple possible routes between country supply and demand market pairs, and expanded network equilibrium conditions to include the production and transportation bounds, along with the associated Lagrange multipliers. The network equilibrium model allows for supply price, demand price, and unit transportation cost functions to depend on the commodity flow variables, and these functions can be nonlinear and asymmetric. The capacity constraints, along with the generality of the underlying functions, enable the modeling of competition for production and transportation capacity among the commodities.

2. Literature Review and Organization of the Paper

The intellectual scientific foundation of our international trade network modeling framework is based on the classical spatial price equilibrium models of Samuelson (1952) and Takayama and Judge (1964, 1971), but with the application of variational inequality theory (cf. Florian and Los (1982), Dafermos and Nagurney (1984), Nagurney and Aronson (1989), Nagurney (1989, 1999)) to allow for the integration of salient features of relevance to various disaster scenarios and accompanying issues of food insecurity. Spatial price equilibrium models have had wide application to the trade of different agricultural products (see, e.g., Thompson (1989), Bishop, Pratt, and Novakovic (1994), Ruijs et al. (2001), Barraza De La Cruz, Pizzolato, and Barraza de La Cruz (2010)). They have also gathered attention in the context of the quantification of the impacts of various policies such as quotas (e.g., Nagurney, Li, and Nagurney (2014), Nagurney (2022b), Nagurney, Salarpour, and Dong (2022)), tariffs, including tariff-rate quotas (see, for example, Nagurney, Besik, and Dong (2019)) and ad valorem tariffs (see Nagurney, Nicholson, and Bishop (1996)), as well as non-tariff measures in the form of sanitary and phytosanitary measures (Lopez, Rau, and Woltjer (2019)) plus even goal targets (see Nagurney, Thore, and Pan (1996)).

The new model in this paper differs from previous ones in that, although bounds have been imposed on commodity flows in spatial price equilibrium models in order to model either quotas or capacity limits in terms of transportation, such constraints were commodity-specific. In this paper, in contrast, we have bounds across commodities in terms of production in each country as well as bounds across commodities on each route of transportation. The former bounds, which we model as constraints, capture implicitly the available land for planting, which may have been reduced due to war and/or climate-related disasters. The latter bounds, in turn, allow us to model the limits on the total volume of commodities that can be transported on different routes, which can correspond to different modes of transport such as rail, road, or maritime transport, for example. Decreases in available transport capacity, in disaster scenarios, can arise because of compromised infrastructure, heightened risk because of war, lack of labor availability, a reduction in number of available vehicles, etc. The bounds on the total amount of commodities that can be produced in the new model capture competition among the commodities as do the multicommodity supply price functions, which can be nonlinear. In addition, the bounds on the total amount of commodities that can be transported on a given route, in turn, capture competition among the commodities for transport freight services along different routes. Furthermore, the generality of the transportation cost functions, which can be nonlinear and asymmetric allow for a further refinement of competition, plus, what is very important in various disaster scenarios - congestion.

In addition, we include exchange rates in our model. Exchange rates are essential parameters in international trade and associated decision-making. The inclusion of exchange rates in spatial price equilibrium models is very limited (see Devadoss and Sabala (2020), Nagurney et al. (2023)). Furthermore, tying the disruptions that we consider here, due to various disaster scenarios, to impacts on food security in terms of agricultural commodity volumes and prices, plus the impacts on the portfolio of commodities produced and transported, is novel.

The international trade network equilibrium model that we construct in this paper is a perfectly competitive model in contrast to the imperfectly competitive (oligopolistic) models of relevance to agricultural and food supply chains constructed as variational inequlity problems by Yu and Nagurney (2013) and Besik, Nagurney, and Dutta (2023).

The paper is organized as follows. The multicommodity international agricultural trade network equilibrium model is constructed in Section 3, where the equilibrium conditions are stated, and the finite-dimensional variational inequality formulation is established. Several illustrative numerical examples are presented for exposition purposes and to highlight the types of insights that the model yields in terms of the impacts of production and transportation disruptions due to disasters with relevance to food security. In Section 4, an alternative variational inequality formulation is provided, along with qualitative properties of the solution. Here, the Lagrange multipliers of prices associated with the production and transportation capacities are made fully explicit. Theoretical results of existence of a solution and uniqueness are also provided. In Section 5, we detail the algorithmic scheme, which is the modified projection method (cf. Korpelevich (1977)). Because of the variational inequality that we construct with Lagrange multipliers, with the variables being the agricultural commodity trade shipments between countries and the Lagrange multipliers associated with the country production capacity constraints and the transportation route capacity constraints, the resolution of the algorithm, in the context of

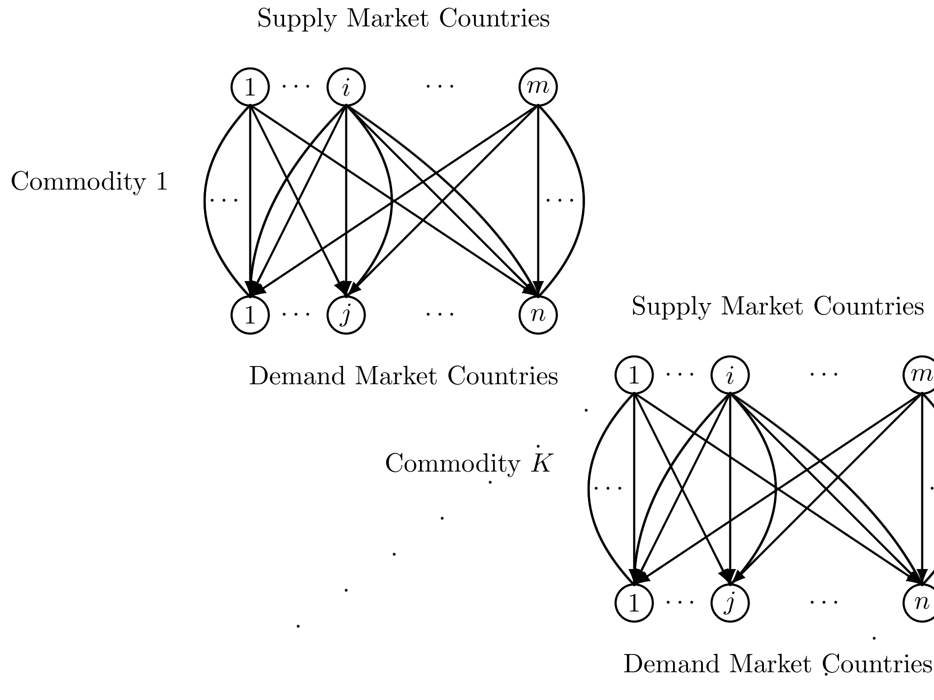

Figure 1: The Multicommodity International Trade Network

the model, which is defined over a feasible set that is the nonnegative orthant, results in closed form expressions for each of the variables at each iteration. We provide the explicit formulae for these for easy implementation. We then apply the algorithm in Section 6 to compute solutions to larger scale agricultural numerical examples drawn from Russia's war on Ukraine. In Section 7, we summarize our results and present our conclusions.

3. The Multicommodity International Agricultural Trade Network Equilibrium Model with Production and Transportation Capacities

In this Section, the multicommodity international agricultural trade network equilibrium model is constructed. The trade network consists of m countries that are supply markets where the agricultural commodities are produced, with a typical country supply market denoted by i, and n countries where the commodities are consumed, with a typical country demand market denoted by j. There are K commodities that are produced and destined for the demand markets. A typical commodity is denoted by k. Joining each pair of country supply and demand markets are transportation routes represented by links as in Figure 1. For simplicity, we assume that there are L transportation routes joining each pair of country supply and demand markets. Associated with each pair of country supply and demand markets (i, j) is an exchange rate e_{ij} for $i = 1, \ldots, m$; $j = 1, \ldots, n$. Of course, if the country of the supply market is the same as the country of the demand market, then the corresponding exchange rate is set equal to 1.

Let Q^{kl}_{ij} denote the amount of commodity k produced at country supply market i and shipped on route l to country demand market j. The commodity flows are grouped into the vector $Q \in R^{KLmn}_{+}$. It is worth noting that a specific route l between a particular pair of country supply and demand markets i and j is not the same as the route with the same superscript l between another pair of country supply and demand markets. For example, Q^{11}_{11} denotes the flow of the first commodity on the first route between country supply market 1 and country demand market 1, while Q_{12}^{11} corresponds to the flow of the first commodity on the first route between country supply market 1 and country demand market 2; in other words, although the superscripts are the same, different routes are represented, since the market pairs are different.

Let s_i^k denote the supply of commodity k produced at country supply market i. All the commodity supplies are grouped into the vector $s \in R_+^{Km}$. The demand for commodity k at country demand market j is denoted by d_j^k, and all the demands are gathered into the vector $d \in R_+^{Kn}$. Note that the products are homogeneous and that the model assumes perfect competition. All vectors are assumed to be column vectors.

The conservation of flow equations are:

$$s_i^k = \sum_{j=1}^{n} \sum_{l=1}^{L} Q_{ij}^{kl}, \quad k = 1, \dots, K; i = 1, \dots, m, \tag{1}$$

$$d_j^k = \sum_{i=1}^{m} \sum_{l=1}^{L} Q_{ij}^{kl}, \quad k = 1, \dots, K; j = 1, \dots, n. \tag{2}$$

Also, all the commodity shipments must be nonnegative; that is:

$$Q_{ij}^{kl} \geq 0, \quad k = 1, \dots, K; l = 1, \dots, L; i = 1, \dots, m; j = 1, \dots, n. \tag{3}$$

The equations in (1) guarantee that the supply of a commodity produced at a country supply market is equal to the shipments of the commodity to all the country demand markets via all the different transportation routes.

According to (2), the demand for each commodity at each country demand market must be equal to the commodity shipments from all the commodity supply markets on all the transportation routes. According to (1), and (2), we assume here that the market clears for each commodity and that there is no excess supply of a commodity and no excess demand.

Let $\bar{Q}_{ij}^l$ denote the transportation capacity of route l between country supply market i and country demand market j, for all l, i, j. Typically, the units of flow for the commodities are in tons, since many of the applications of relevance are for agricultural products.

Hence, the following transportation capacity constraints must be satisfied:

$$\sum_{k=1}^{K} Q_{ij}^{kl} \leq \bar{Q}_{ij}^l, \quad l = 1, \dots, L; i = 1, \dots, m; j = 1, \dots, n. \tag{4}$$

According to (4), the sum of all commodity shipments from country supply market i on route l to country demand market j cannot violate the shipment capacity of that route. Expressions in (4) allow us to capture competition among the commodities for transportation services along particular routes.

Let $\bar{S}_i$ denote the production capacity of country supply market i across all the commodities. The below production capacity constraints must be met:

$$\sum_{k=1}^{K} s_i^k \leq \bar{S}_i, \quad i = 1, \dots, m. \tag{5a}$$

Due to the conservation of flow equations (1), constraints (5a) can take the form:

$$\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{n} Q_{ij}^{kl} \le \bar{S}_i, \quad i = 1, \ldots, m. \tag{5b}$$

According to (5a) or (5b), a country supply market i cannot violate its aggregate production capacity.

The constraints in (5a) (and (5b)), in effect, capture competition among commodities that farmers plant due to a capacity of the aggregate production.

The country supply price functions π_i^k, for all k, i, are:

$$\pi_i^k = \pi_i^k(s), \quad k = 1, \ldots, K; i = 1, \ldots, m. \tag{6a}$$

Due to the conservation of flow equations (1), we may construct country supply price functions $\tilde{\pi}_i^k$, for all k, i, such that:

$$\tilde{\pi}_i^k(Q) \equiv \pi_i^k(s), \quad k = 1, \ldots, K; i = 1, \ldots, m. \tag{6b}$$

According to (6a) or (6b), the supply price of a commodity at a country supply market can, in general, be a function not only of the supply of the commodity in the country (the amount produced) but also of the supplies of other commodities in the country as well as the supplies of the commodities in all other countries.

The demand price of a commodity k in country j, ρ_j^k, in turn, can depend on the entire vector of demands of the commodities in all countries:

$$\rho_j^k = \rho_j^k(d), \quad k = 1, \ldots, K; j = 1, \ldots, n. \tag{7a}$$

Similarly, due to (2), we may construct new country demand price functions $\tilde{\rho}_j^k$, for all k, j, such that:

$$\tilde{\rho}_j^k(Q) \equiv \rho_j^k(d), \quad k = 1, \ldots, K; j = 1, \ldots, n. \tag{7b}$$

The unit transportation cost associated with transporting commodity k from country i to country j on transportation route l is denoted by c_{ij}^{kl} and is as follows:

$$c_{ij}^{kl} = c_{ij}^{kl}(Q), \quad k = 1, \ldots, K; l = 1, \ldots, L; i = 1, \ldots, m; j = 1, \ldots, n. \tag{8}$$

The generality of the above transportation cost functions, where the unit transportation cost can depend on the vector of commodity shipments between all pairs of country supply and demand markets, allows one to further capture competition for transportation services among commodities.

We assume that the supply price, demand price, and unit transportation cost functions are all continuous.

We introduce Lagrange multipliers: λ_{ij}^l, $l = 1, \ldots, L; i = 1, \ldots, m; j = 1, \ldots, n$; μ_i, $i = 1, \ldots, m$, associated with the capacity constraints in (4) and (5b), respectively, and we group these Lagrange multipliers into the vectors $\lambda \in R_+^{Lmn}$ and $\mu \in R_+^m$.

The multicommodity international agricultural trade network equilibrium conditions are now stated.

Definition 1: The Multicommodity International Agricultural Trade Network Equilibrium Conditions Under Limited Production and Transportation Capacity

A multicommodity shipment and Lagrange multiplier pattern $(Q^*, \lambda^*, \mu^*) \in \mathcal{K}^1$*, where* $\mathcal{K}^1 \equiv \{(Q, \lambda, \mu) | (Q, \lambda, \mu) \in R_+^{KLmn+Lmn+m}\}$ *is a multicommodity international agricultural trade network equilibrium with exchange rates, under limited production and transportation capacities, if the following conditions hold: For all commodities* k*;* $k = 1, \ldots, K$*; for all routes* l*;* $l = 1, \ldots, L$*, and for all country supply and demand market pairs:* (i, j)*;* $i = 1, \ldots, m$*;* $j = 1, \ldots, n$*:*

$$(\tilde{\pi}_i^k(Q^*) + c_{ij}^{kl}(Q^*))e_{ij} + \lambda_{ij}^{l*} + \mu_i^* \begin{cases} = \tilde{\rho}_j^k(Q^*), & \textit{if } Q_{ij}^{kl*} > 0, \\ \geq \tilde{\rho}_j^k(Q^*), & \textit{if } Q_{ij}^{kl*} = 0, \end{cases} \tag{9}$$

and for all routes l*;* $l = 1, \ldots, L$*, and all country market pairs* (i, j)*;* $i = 1, \ldots, m$*;* $j = 1, \ldots, n$*:*

$$\lambda_{ij}^{l*} \begin{cases} \geq 0, & \textit{if } \sum_{k=1}^K Q_{ij}^{kl*} = \bar{Q}_{ij}^l, \\ = 0, & \textit{if } \sum_{k=1}^K Q_{ij}^{kl*} < \bar{Q}_{ij}^l, \end{cases} \tag{10}$$

and for all country supply markets i*;* $i = 1, \ldots, m$*:*

$$\mu_i^* \begin{cases} \geq 0, & \textit{if } \sum_{k=1}^K \sum_{l=1}^L \sum_{j=1}^n Q_{ij}^{kl*} = \bar{S}_i, \\ = 0, & \textit{if } \sum_{k=1}^K \sum_{l=1}^L \sum_{j=1}^n Q_{ij}^{kl*} < \bar{S}_i. \end{cases} \tag{11}$$

The multicommodity international agricultural trade network equilibrium conditions (9) through (11) state that, if there is a positive flow of a commodity on a route between a pair of country supply and demand markets, and the route is not at its capacity, and the production at the country supply market is not at its capacity, then the supply price of the commodity at the country supply market plus the unit transportation cost associated with transporting the commodity on the route, multiplied by the exchange rate between the two countries is equal to the demand price of the commodity at the country demand market. On the other hand, if the route is at its capacity, or the production is at its capacity at the country supply market, and the flow of the commodity on a route is positive, then the demand price of the commodity at the country demand market is greater than or equal to the sum of commodity supply price and its unit transportation cost multiplied by the applicable exchange rate, with the sum of the corresponding Lagrange multipliers equal to the nonnegative difference. If the flow of a commodity is equal to zero on a route, then the country demand market price of the commodity is less than or equal to the country supply market price plus the unit transportation cost multiplied by the appropriate exchange rate plus the Lagrange multipliers.

Along with the Lagrange multipliers corresponding to capacity constraints in (10) and (11), the equilibrium conditions (9) expand the classical spatial price equilibrium conditions of Samuelson (1952) and Takayama and Judge (1971) to include exchange rates, and limited transportation and production capacity. Furthermore, the underlying supply price, demand price, and unit transportation cost functions in our model need not be separable (nor symmetric), and the unit transportation cost functions are flow-dependent.

Theorem 1: Variational Inequality Formulation of the Multicommodity International Agricultural Trade Network Equilibrium Conditions Under Limited Production and Transportation Capacity

A multicommodity shipment and Lagrange multiplier pattern $(Q^*, \lambda^*, \mu^*) \in \mathcal{K}^1$ *is a multicommodity international agricultural trade network equilibrium with exchange rates, under limited production and transportation* *capacities, according to Definition 1, if and only if it satisfies the variational inequality:*

$$\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{i=1}^{m}\sum_{j=1}^{n}\Big[(\tilde{\pi}_i^k(Q^*)+c_{ij}^{kl}(Q^*))e_{ij}+\lambda_{ij}^{l*}+\mu_i^*-\tilde{\rho}_j^k(Q^*)\Big]\times(Q_{ij}^{kl}-Q_{ij}^{kl*})$$

$$+\sum_{l=1}^{L}\sum_{i=1}^{m}\sum_{j=1}^{n}\Big[\bar{Q}_{ij}^{l}-\sum_{k=1}^{K}Q_{ij}^{kl*}\Big]\times(\lambda_{ij}^{l}-\lambda_{ij}^{l*})+\sum_{i=1}^{m}\Big[\bar{S}_i-\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{n}Q_{ij}^{kl*}\Big]\times(\mu_i-\mu_i^*)\geq 0,\quad\forall(Q,\lambda,\mu)\in\mathcal{K}^1. \quad (12)$$

Proof: First, we proceed to demonstrate necessity; that is, we show that if $(Q^*,\lambda^*,\mu^*)\in\mathcal{K}^1$ satisfies equilibrium conditions (9) through (11), then it also satisfies variational inequality (12). From the equilibrium conditions, as defined in Definition 1, for an equilibrium commodity shipment and Lagrange multiplier pattern, and for fixed k,l,i,j, we know that:

$$\Big[(\tilde{\pi}_i^k(Q^*)+c_{ij}^{kl}(Q^*))e_{ij}+\lambda_{ij}^{l*}+\mu_i^*-\tilde{\rho}_j^k(Q^*)\Big]\times(Q_{ij}^{kl}-Q_{ij}^{kl*})\geq 0,\quad\forall Q_{ij}^{kl}\geq 0, \quad (13)$$

because if $Q_{ij}^{kl*}>0$, then the left-hand side in (13) preceding the multiplication sign is zero, so (13) holds. Also, if $Q_{ij}^{k*l}=0$, then the left-hand side expression is nonnegative, and (13) holds, since Q_{ij}^{kl} is always greater than or equal to Q_{ij}^{kl*}. Since (13) is true for any k,l,i,j, summation of (13) over these indices gives us:

$$\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{i=1}^{m}\sum_{j=1}^{n}\Big[(\tilde{\pi}_i^k(Q^*)+c_{ij}^{kl}(Q^*))e_{ij}+\lambda_{ij}^{l*}+\mu_i^*-\tilde{\rho}_j^k(Q^*)\Big]\times(Q_{ij}^{kl}-Q_{ij}^{kl*})\geq 0,\forall Q\in R_+^{KLmn}. \quad (14)$$

Plus, from equilibrium conditions (10), we know that, for a fixed l,i,j:

$$\Big[\bar{Q}_{ij}^{l}-\sum_{k=1}^{K}Q_{ij}^{kl*}\Big]\times(\lambda_{ij}^{l}-\lambda_{ij}^{l*})\geq 0,\quad\forall\lambda_{ij}^{l}\geq 0. \quad (15)$$

Again, summing (15) over all indices l,i,j results in:

$$\sum_{l=1}^{L}\sum_{i=1}^{m}\sum_{j=1}^{n}\Big[\bar{Q}_{ij}^{l}-\sum_{k=1}^{K}Q_{ij}^{kl*}\Big]\times(\lambda_{ij}^{l}-\lambda_{ij}^{l*})\geq 0,\quad\forall\lambda\in R_+^{Lmn}. \quad (16)$$

And, from equilibrium conditions (11), we have that, for a fixed i:

$$\Big[\bar{S}_i-\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{n}Q_{ij}^{kl*}\Big]\times(\mu_i-\mu_i^*)\geq 0,\quad\forall\mu_i\geq 0. \quad (17)$$

Summing (17) over all indices i gives us the following:

$$\sum_{i=1}^{m}\Big[\bar{S}_i-\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{n}Q_{ij}^{kl*}\Big]\times(\mu_i-\mu_i^*)\geq 0,\quad\forall\mu\in R_+^{m}. \quad (18)$$

Adding (14), (16), and (18) yields variational inequality (12). Therefore, necessity has been established.

We now establish sufficiency. Setting $\lambda_{ij}^l = \lambda_{ij}^{l*}$ for all l, i, j; $\mu_i = \mu_i^*$ for all i; and $Q_{ij}^{kl} = Q_{ij}^{kl*}$ for all k, l, i, j except for $k = \tilde{k}$, $l = \tilde{l}$, $i = \tilde{i}$, and $j = \tilde{j}$, and plugging the resultants into (12), reduces the variational inequality (12) to:

$$\left[(\tilde{\pi}_{\tilde{i}}^{\tilde{k}}(Q^*) + c_{\tilde{i}\tilde{j}}^{\tilde{k}\tilde{l}}(Q^*))e_{\tilde{i}\tilde{j}} + \lambda_{\tilde{i}\tilde{j}}^{\tilde{l}*} + \mu_{\tilde{i}}^* - \tilde{\rho}_{\tilde{j}}^{\tilde{k}}(Q^*)\right] \times (Q_{\tilde{i}\tilde{j}}^{\tilde{k}\tilde{l}} - Q_{\tilde{i}\tilde{j}}^{\tilde{k}\tilde{l}*}) \geq 0, \quad \forall Q_{\tilde{i}\tilde{j}}^{\tilde{k}\tilde{l}} \geq 0, \tag{19}$$

from which it follows that the multicommodity international trade network equilibrium conditions (9) hold.

Now, setting $Q_{ij}^{kl} = Q_{ij}^{kl*}$ for all k, l, i, j; $\mu_i = \mu_i^*$ for all i; and $\lambda_{ij}^l = \lambda_{ij}^{l*}$ for all l, i, j except for $k = \tilde{k}$; $l = \tilde{l}$, $i = \tilde{i}$, and $j = \tilde{j}$ and substituting the resultant values into (12), reduces (12) to:

$$\left[\bar{Q}_{\tilde{i}\tilde{j}}^{\tilde{l}} - \sum_{k=1}^{K} Q_{\tilde{i}\tilde{j}}^{k\tilde{l}*}\right] \times (\lambda_{\tilde{i}\tilde{j}}^{\tilde{l}} - \lambda_{\tilde{i}\tilde{j}}^{\tilde{l}*}) \geq 0, \quad \forall \lambda_{\tilde{i}\tilde{j}}^{\tilde{l}} \geq 0, \tag{20}$$

from which it follows that the equilibrium conditions (10) must hold.

Similarly, setting $Q_{ij}^{kl} = Q_{ij}^{kl*}$ for all k, l, i, j; $\lambda_{ij}^l = \lambda_{ij}^{l*}$ for all l, i, j; and $\mu_i = \mu_i^*$ for all i except for $k = \tilde{k}$; $l = \tilde{l}$, $i = \tilde{i}$, and $j = \tilde{j}$ and substituting the resultant values into (12), reduces (12) to:

$$\sum_{i=1}^{m}\left[\bar{S}_i - \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{n} Q_{ij}^{kl*}\right] \times (\mu_i - \mu_i^*) \geq 0, \quad \forall \mu \in R_+^m, \tag{21}$$

and, hence, equilibrium conditions (11) must hold. Sufficiency has also been established. □

Variational inequality (12) is now put into standard form (cf. Nagurney (1999)), VI$(F, \mathcal{K})$, where one seeks to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{22}$$

with F being a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, where $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

Specifically, we define $X \equiv (Q, \lambda, \mu)$, $\mathcal{K} \equiv \mathcal{K}^1$, and $\mathcal{N} \equiv KLmn + Lmn + m$. Additionally, $F(X) \equiv (F_1(X), F_2(X), F_3(X))$ where $F_1(X)$ consists of the elements: $\left[(\tilde{\pi}_i^k(Q) + c_{ij}^{kl}(Q))e_{ij} + \lambda_{ij}^l + \mu_i - \tilde{\rho}_j^k(Q)\right], \forall k, l, i, j$, and the components of $F_2(X)$ are: $\left[\bar{Q}_{ij}^l - \sum_{k=1}^K Q_{ij}^{kl}\right], \forall l, i, j$, and $F_3(X)$ is comprised of the elements: $\left[\bar{S}_i - \sum_{k=1}^K \sum_{l=1}^L \sum_{j=1}^n Q_{ij}^{kl}\right], \forall i$.

Clearly, variational inequality (12) can be put into standard form (22).

3.1 Illustrative Examples

In this Subsection, several illustrative examples are provided, the solutions to which can be obtained analytically using the above equilibrium conditions. For simplicity, we assume that the supply market and the demand market are in the same country and, hence, we have that the exchange rate $e_{11} = 1$. In Examples 1 through 4, the production capacity bound $\bar{S}_1$ is very high and is set equal to 100.00. In Example 5, we tighten the bound to 5.00 to reflect a production disruption to agriculture, which reduces the volume of agricultural commodities that can be produced.

Example 1: Two Commodities, a Single Transportation Route with High Transportation Capacity, a Single Country Supply Market, and a Single Country Demand Market

There are two commodities in Example 1. The supply price functions are:

$$\pi_1^1(s) = 5s_1^1 + 5, \quad \pi_1^2(s) = s_1^2 + 5.$$

From these functions, one can see that it is more costly to produce commodity 1 than commodity 2.

The unit transportation cost functions are:

$$c_{11}^{11}(Q) = Q_{11}^{11} + 1, \quad c_{11}^{21}(Q) = Q_{11}^{21} + 2.$$

The second commodity has a higher unit transportation cost than the first commodity.

The demand price functions are:

$$\rho_1^1(d) = -d_1^1 + 20, \quad \rho_1^2(d) = -d_1^2 + 37.$$

Note that the consumers at the country's demand market are willing to pay a higher price for commodity 2 than for commodity 1.

We assume that the transportation capacity on the route is 15.00.

The international trade network equilibrium commodity shipment pattern is:

$$Q_{11}^{11*} = 2.00, \quad Q_{11}^{21*} = 10.00,$$

with the equilibrium Lagrange multiplier $\lambda_{11}^{1*} = 0.00$. In this example, the commodity shipments are below the transportation capacity on the route, and, therefore, the equilibrium Lagrange multiplier is 0.00.

The equilibrium conditions (10) and (11) hold accurately. Indeed, for the first commodity, we have that, in equilibrium, its supply price is 15.00, the unit transportation cost is 3.00, and the demand price is 18.00; for the second commodity, the supply price is 15.00, the unit transportation cost is 12.00, and the demand price is 27.00. Even though the second commodity is more costly to ship, since consumers are willing to pay a higher demand price, it has a flow five times that of the first commodity.

Example 2: Data as in Example 1 but with Reduced Transportation Capacity

Example 2 is constructed from Example 1 and has the same data except that now we consider the situation of disruption to transportation capacity, with $\bar{Q}_{11}^1 = 10.00$.

The new equilibrium commodity shipment and Lagrange multiplier pattern is:

$$Q_{11}^{11*} = 1.40, \quad Q_{11}^{21*} = 8.60, \quad \lambda_{11}^{1*} = 4.20.$$

Since the commodity shipments are now at the capacity of the transportation route, the Lagrange multiplier is positive.

The supply price for commodity 1 is now: 12.00, and that for commodity 2 is: 13.60. The unit transportation cost for commodity 1 is: 2.40, and that for commodity 2 is: 10.60. The demand price for commodity 1 is now:

18.60, whereas the demand price for commodity 2 is: 28.40. Again, the equilibrium conditions hold precisely. Note that, in this example, one must add the value of the Lagrange multiplier to the commodity supply price plus the commodity unit transportation cost to get the demand price for each commodity.

With a more limited transportation capacity, the first commodity loses 30% of its shipment as compared to Example 1, while the second commodity, which has the higher demand price, loses only 14% of its flow in the first example. Now, the flow of the second commodity is more than six times that of commodity 1. Mapping this simple example to the case of Ukraine, the results are in line with the expected decreased major Ukrainian grain harvest in 2022. According to the U.S. Department of Agriculture (2022), the harvest for all major grains, due to Russia's invasion, is expected to drop by around 40%. USDA's report mentions bridges, railways, grain warehouses and silos, roads, and ports as part of the targeted infrastructure by the Russians. The effects of the reduced crop harvest in Ukraine are highlighted in reports by Belgrave (2022) and the Food and Agriculture Organization (FAO) of the United Nations (2022).

Example 3: Data as in Example 1 but with an Added Transportation Route with High Transportation Capacity

Example 3 has the same data as that in Example 1 except that now we add route 2 connecting the pair of country supply and demand markets with the following unit transportation costs for the two commodities:

$$c^{12}_{11}(Q) = Q^{12}_{11} + 5, \quad c^{22}_{11}(Q) = Q^{22}_{11} + 7.$$

Also, we have for the second transportation route the following capacity: $\bar{Q}^2_{11} = 15.00$.

The equilibrium commodity shipment and Lagrange multiplier pattern is:

$$Q^{11*}_{11} = 2.00, \quad Q^{12*}_{11} = 0.00, \quad Q^{21*}_{11} = 8.00, \quad Q^{22*}_{11} = 3.00, \quad \lambda^{1*}_{11} = 0.00, \quad \lambda^{2*}_{11} = 0.00.$$

The supply price for commodity 1, at the equilibrium, is 15.00; the unit transportation cost on route 1 is 3.00, and the demand price is 18.00. The unit cost on transportation route 2 is 5.00, and, hence, there is zero flow of commodity 1 on the second route since the supply price plus unit transportation cost is equal to 20.00, which exceeds the demand price of 18.00.

As for commodity 2, in equilibrium, the supply price is 16.00; the unit transportation cost on route 1 is 10.00 and is also 10.00 on route 2, with the demand price being 26.00. The equilibrium conditions (10) and (11) are satisfied precisely. Note that the second commodity, for which the consumers at the demand market are willing to pay a higher demand price, is shipped through both routes; that is, even via the more costly route 2, while the first commodity has a positive flow only on the cheaper route 1.

Again, inspired by the case of Russia's aggression on Ukraine, the added transportation route could be compared to the internal barge shipments of grains through the Danube River in Ukraine (Reuters (2022a)) or transporting Ukrainian grains via rail (Associated Press (2022a)). Interestingly, although the total supply and the total demand for commodity 1 remain unchanged from those in Example 1, the supply and demand for commodity 2, with the addition of a transportation route, increases from 10.00 to 11.00! This example illustrates the importance of having multiple transportation routes connecting supply markets with demand markets. In addition, we see that the supply price for commodity 2 increases, which benefits producers, while the demand price decreases (as compared to the respective values in Example 1), which benefits consumers.

Example 4: Data as in Example 3 but with Reduced Transportation Capacities

Example 4 has the identical data to that in Example 3 but now the capacities on both transportation routes are reduced, where: $\bar{Q}^1_{11} = 5.00$ and $\bar{Q}^2_{11} = 2.00$. The new equilibrium pattern is:

$$Q^{11*}_{11} = .30, \quad Q^{12*}_{11} = 0.00, \quad Q^{21*}_{11} = 4.70, \quad Q^{22*}_{11} = 2.00, \quad \lambda^{1*}_{11} = 11.90, \quad \lambda^{2*}_{11} = 9.60.$$

For the first commodity, in equilibrium, the supply price is 6.50, and the unit transportation cost is 1.30. The sum of these two values plus the sum of the Lagrange multiplier, which is 11.90, is equal to the demand price of: 19.70. Note that the second route is not used for transporting commodity 1 since, with the unit transportation cost of 5, the supply price plus the unit transportation cost plus the Lagrange multiplier is equal to 21.10, which exceeds the demand price of 19.70. Observe that the transportation capacity on route 1 hits the bound of 5.00 and, therefore, there is a positive associated Lagrange multiplier. There is also a positive Lagrange multiplier associated with the second route since the capacity there is also met.

As for the second commodity, the supply price is 11.70, and the unit transportation cost on the first route is 6.70. The demand price at 30.30 is equal to the sum of this supply price and unit transportation cost plus the Lagrange multiplier of 11.90. As for route 2, the unit transportation cost is 9.00, and the sum of the supply price and unit transportation cost plus the Lagrange multiplier, which is 9.60, again, equals the demand price at 30.30. Note that both routes are used for the transportation of the second commodity.

Again, with the decrease in the capacity levels, most of the transportation capacity of route 1, and all of the now more limited shipment capacity of route 2, is appropriated by the commodity that commands the higher price; that is, commodity 2. Commodity 1 is only shipped via the first route, which has a lower transportation cost, and the high demand price that consumers are willing to pay for the second commodity results in the shipment of the second commodity via both routes, even on the higher-cost route 2. One can observe that, with the tighter capacities on the two routes, commodity 1, on aggregate, has lost 85% of its shipment volume as compared to that in Example 3. In contrast, the second commodity's cumulative flow is reduced by only about 39%. One observes that the results are reiterated in the case of Ukraine's grain exports after Russia's aggression, and its impacts on global food security (World Food Programme (2022b)).

We remark that in Examples 1 through 4, since $\bar{S}^1 = 100.00$, $\mu^*_1 = 0.00$.

Example 5: Data as in Example 4 but with Reduced Production Capacity

Example 5 has the identical data to the data in Example 4 except that now we consider a big disruption to the production capacity with $\bar{S}_1 = 5.00$. All the commodity shipments, in equilibrium, are now equal to 0.00, except that $Q^{21*}_{11} = 5.00$. Since the commodity production at the supply market is also equal to 5.00, the Lagrange multiplier μ^*_1 is positive and is equal to 15.00. All other Lagrange multipliers are equal to 0.00. The supply price of the second commodity at the supply market is now equal to 10.00; the unit transportation cost to the demand market is equal to 7.00, with the demand price at the country demand market now equal to 32.00.

4. Alternative Variational Inequality Formulation and Qualitative Properties

We now provide an alternative variational inequality to the one in (12) followed by some qualitative properties. Having alternative variational inequalities enables the application of different algorithms for computational purposes. Here, the alternative variational inequality yields deeper theoretical insights. Specifically, the alternative variational inequality makes use of the KKT system associated with it, which reveals the equilibrium conditions (9) through (11). For background on such an approach, see Tong and Xiao (2006), and for an application to multiclass international migration problems, see Passacantando and Raciti (2022).

Theorem 2: Alternative Variational Inequality Formulation

The solution to the variational inequality problem: determine $Q^{**} \in \mathcal{K}^2$, such that

$$\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{i=1}^{m}\sum_{j=1}^{n}[(\tilde{\pi}_i^k(Q^{**})+c_{ij}^{kl}(Q^{**}))e_{ij}-\tilde{\rho}_j^k(Q^{**})]\times(Q_{ij}^{kl}-Q_{ij}^{kl**})\geq 0, \quad \forall Q\in K^2, \tag{23}$$

where $\mathcal{K}^2 \equiv \{Q \in R_+^{KLmn} | \sum_{k=1}^{K} Q_{ij}^{kl} \leq \bar{Q}_{ij}^l, \forall l,i,j,$ and $\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{n} Q_{ij}^{kl} \leq \bar{S}_i, \forall i\}$, also satisfies equilibrium conditions (9) through (11), where λ_{ij}^{l**} is the Lagrange multiplier associated with the constraint: $\sum_{k=1}^{K} Q_{ij}^{kl} \leq \bar{Q}_{ij}^l$, $\forall l,i,j$, and μ_i^{**} is the Lagrange multiplier associated with the constraint $\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{n} Q_{ij}^{kl} \leq \bar{S}_i, \forall i$.

Proof: We write

$$f(Q)\equiv\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{i=1}^{m}\sum_{j=1}^{n}[(\tilde{\pi}_i^k(Q^{**})+c_{ij}^{kl}(Q^{**}))e_{ij}-\tilde{\rho}_j^k(Q^{**})]\times Q_{ij}^{kl} \tag{24}$$

and note that

$$f(Q^{**})\leq f(Q), \quad \forall Q\in\mathcal{K}^2; \tag{25}$$

in other words, Q^{**} is a minimum point for f in $\mathcal{K}^2$. We now construct the KKT system for the above minimization problem, which yields, after some algebra:

$$(\tilde{\pi}_i^k(Q^{**})+c_{ij}^{kl}(Q^{**}))e_{ij}-\tilde{\rho}_j^k(Q^{**})+\lambda_{ij}^{l**}+\mu_i^{**}-\alpha_{ij}^{kl**}=0, \quad \forall k,l,i,j, \tag{26}$$

$$Q_{ij}^{kl**}\alpha_{ij}^{kl**}=0, \quad \alpha_{ij}^{kl**}\geq 0, \forall k,l,i,j, \tag{27}$$

$$\lambda_{ij}^{l**}(\sum_{k=1}^{K}Q_{ij}^{kl**}-\bar{Q}_{ij}^l)=0, \quad \lambda_{ij}^{l**}\geq 0, \forall l,i,j, \tag{28}$$

$$\mu_i^{**}(\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{n}Q_{ij}^{kl**}-\bar{S}_i)=0, \quad \mu_i^{**}\geq 0, \forall i, \tag{29}$$

$$Q^{**}\in\mathcal{K}^2. \tag{30}$$

It is clear that the above KKT conditions coincide with equilibrium conditions (9) through (11) and that they are both necessary and sufficient for Q^{**} to be a solution to variational inequality (23). The proof is complete. □

We now put variational inequality (23) into standard form (22). We define $X \equiv (Q)$, $\mathcal{K} \equiv \mathcal{K}^2$, and $\mathcal{N} \equiv KLmn$. Also, $F(X)$ consists of the elements: $\left[(\tilde{\pi}_i^k(Q) + c_{ij}^{kl}(Q))e_{ij} - \tilde{\rho}_j^k(Q)\right], \forall k, l, i, j$. The conclusion follows.

We know from the classical theory of variational inequalities (cf. Kinderlehrer and Stampacchia (1980) and Nagurney (1999)) that existence of a solution X^* to variational inequality (23) is guaranteed, since the underlying feasible set $\mathcal{K}$ is compact and the function $F(X)$, under our imposed assumptions, is continuous. Furthermore, it follows that, if the function $F(X)$ is strictly monotone, then the solution X^* is unique. It is worth noting that the sum of a monotone function and a strictly monotone one is also strictly monotone. Hence, not all the economic functions in either variational inequality (23) or (12) (with the supply price functions and the unit transportation cost functions modified as in the variational inequalities with the exchange rates) need to be strictly monotone for uniqueness of the equilibrium commodity shipment pattern to hold.

Referring back to the equilibrium conditions (9) through (11), one sees that the Lagrange multipliers associated with the capacity constraints of production and transportation can be interpreted as prices/costs. Furthermore, it is clear that, with a unique solution to variational inequality (23), which yields the equilibrium commodity shipment pattern, the same commodity shipment pattern satisfies variational inequality (12). In the next Section, we outline an algorithm for the solution of variational inequality (12), which provides us with both the equilibrium commodity shipments as well as the Lagrange multipliers.

5. The Algorithm

The modified projection method of Korpelevich (1977) is applied to solve a series of numerical examples in Section 6. The convergence of this algorithm is guaranteed if the function $F(X)$ that enters the variational inequality problem (22) is monotone and Lipschitz continuous.

The function $F(X)$ is said to be monotone if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{31}$$

$F(X)$ is Lipschitz continuous, if there exists a Lipschitz constant, $\eta > 0$, such that

$$\|F(X^1) - F(X^2)\| \leq \eta \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{32}$$

The steps of the modified projection method are now recalled for ease of reference, with τ denoting an iteration counter. Then, the closed-form expressions for the commodity shipments and the Lagrange multipliers at each iteration are presented.

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau = 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{33}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{34}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

Because of the structure of the feasible set $\mathcal{K}^1$ underlying the multicommodity international agricultural trade network equilibrium model with production and transportation capacities, the solution of each of the subproblems in (33) and (34) can be obtained via closed-form expressions, which are made explicit below.

Explicit Formulae at Iteration τ for the Multicommodity Shipments in Step 1

The closed-form expressions for the multicommodity shipments for (33) for the solution of variational inequality (12) are:

$$\bar{Q}_{ij}^{kl\tau} = \max\{0, Q_{ij}^{kl\tau-1} + \beta(\tilde{\rho}_j^k(Q^{\tau-1}) - (\tilde{\pi}_i^k(Q^{\tau-1}) + c_{ij}^{kl}(Q^{\tau-1}))e_{ij} - \lambda_{ij}^{l\tau-1} - \mu_i^{\tau-1})\},$$
$$\forall k, l, i, j. \tag{35}$$

Explicit Formulae at Iteration τ for the Transportation Capacity Lagrange Multipliers in Step 1

The closed-form expressions for the transportation capacity Lagrange multipliers for (33) for variational inequality (12) are:

$$\bar{\lambda}_{ij}^{l\tau} = \max\{0, \lambda_{ij}^{l\tau-1} + \beta(\sum_{k=1}^{K} Q_{ij}^{kl\tau-1} - \bar{Q}_{ij}^l)\}, \quad \forall l, i, j. \tag{36}$$

Explicit Formulae at Iteration τ for the Production Capacity Lagrange Multipliers in Step 1

The closed-form expressions for the production capacity Lagrange multipliers for (33) for variational inequality (12) are:

$$\bar{\mu}_i^\tau = \max\{0, \mu_i^{\tau-1} + \beta(\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{n} Q_{ij}^{kl\tau-1} - \bar{S}_i)\}, \quad \forall i. \tag{37}$$

The explicit formulae for the variables in (34) in Step 2 easily follow.

6. Numerical Examples Focused on Disruptions to Agricultural Trade During Russia's War on Ukraine.

We now present a series of numerical examples solved using the modified projection method detailed in the previous section. The examples consist of two commodities, wheat and corn, exported from Ukraine to Lebanon and Egypt. Ukraine, prior to the full-scale invasion on February 24, 2022, used its Black Sea ports to export almost all of its grains. Lebanon and Egypt are MENA (Middle East and North Africa) countries and are

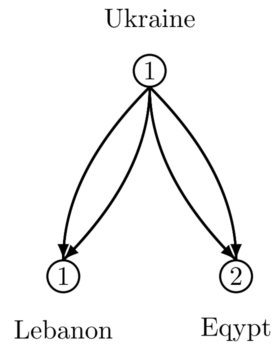

Figure 2: The International Trade Network for the Examples

highly dependent on imports of grains from the Black Sea region. The choice of the supply and demand country markets is such that the examples can provide insights into food security issues as a result of the ongoing war in Ukraine. Additionally, the examples have implications for the war-driven logistical challenges of the export of grain from the Black Sea region, the impacts of the reduced availability of lands for agriculture, the resultant congestion in the transportation routes in Ukraine, the competition of the two commodities over transportation routes, as well as for arable land (and even labor as well as fertilizer, etc.) on the supply side.

The network topology for the examples is shown in Figure 2. On the supply side, node 1 represents Ukraine. On the demand side, Lebanon is denoted by node 1, and node 2 represents Egypt. There are two routes from Ukraine to each of the demand country markets, with the first route representing the export through a Black Sea port in Ukraine, such as the port of Odesa, and the second route denoting the transportation of grains via barge, rail, or truck through the western borders of Ukraine to Romania, and then from a Romanian port on the Black Sea, such as the port of Costanza.

The functions in the numerical examples in this Section are constructed based on reported data on supply prices, demand prices, transportation costs, and commodity flows publicly available on the web. For reports on supply prices in Ukraine, see, for example, Arhirova (2022), Associated Press (2022b), Balmforth and Polityuk (2022), Brower (2022), and Martyshev, Nivievskyi and Bogonos (2023). For reports on the demand prices in Lebanon and Egypt, one can refer to Andrée (2022), Breisinger et al. (2022), El Safety (2022), Galal (2022), Hamdan (2022), Nivievskyi (2022), and Rose (2022). For information on transportation costs in Ukraine and freight rates from Black Sea ports, please refer to Belikova (2022a,b), Nivievskyi (2022), and Pratt (2022). For reports on agricultural commodity flows to Lebanon and Egypt and shipment flows out of Ukraine, see, e.g., Hamdan (2022), IndexMundi (2022a,b), Martyshev, Nivievskyi and Bogonos (2023), and TrendEconomy (2022a,b). Furthermore, the website https://ukragroconsult.com/en/ provides regular reports on supply prices and freight rates in Ukraine and premium data on daily grain shipments from Ukraine by volume and destination. Similar data had been previously utilized to construct the functions used in Nagurney et al. (2023).

It must be noted that the model provides the equilibrium at given exchange rates between the supply and demand country nodes. Accordingly, the exchange rates are assumed to be fixed in each scenario assessed in the below numerical examples. Countries use a floating, fixed, or mixed exchange rate regime (Ghosh and Ostray (2009)). Developed countries often utilize a floating exchange rate regime under which the rates can change multiple times a day through market forces (Erhardt (1977)). Developing countries tend to have a fixed exchange rate regime by pegging their currency to a common currency such as the US dollar (Ghosh and Ostray (2009)). For example, Lebanon started pegging its currency to the US dollar in 1997 but economic turmoil has resulted in the currency losing 90% of its value (Chehayeb (2022), Reuters (2023)). China is an example of a mixed regime with it moving away from a fixed to a more flexible, yet still carefully controlled, exchange rate (Das (2019)). Accordingly, exchange rates are subject to predictable or sudden volatilities; however, any changes to them, whether sudden due to disasters or macroeconomic volatilities or deliberate because of new financial policies at the country level, can be accounted for by solving for the commodity flows and prices at the new equilibrium. Specifically, this is suited to countries implementing a fixed exchange rate during large-scale man-made disasters such as war (e.g., Ukraine) or deep economic crises (e.g., Lebanon).

The local currency codes are UAH for Ukrainian hryvnia, LBP for the Lebanese pound, EGP for the Egyptian pound, and USD for the United States dollar. Here, superscript $k = 1$ denotes wheat, and corn is represented by superscript $k = 2$. The time horizon for each example is one year and the unit for the commodity shipments is tons with prices and costs also associated with a ton of the specific commodity. The modified projection method was implemented in FORTRAN on a Linux system at the University of Massachusetts Amherst. The algorithm was deemed to have converged if the absolute value of each computed variable and two successive iterations differed by no more than 10^{-2}.

Example 6 - Pre-War Scenario

Example 6 considers the pre-war scenario when almost all of the grains in Ukraine were exported through their Black Sea ports. The exchange rates are derived from early January 2022, before the invasion occurs. The exchange rates are:

$$e_{11} = 55.0581, \quad e_{12} = .5714,$$

$$USD/UAH = 27.4619, \quad USD/LBP = 1,512.0000, \quad USD/EGP = 15.7300.$$

The supply price functions for wheat and corn per ton in Ukrainian hryvnia are:

$$\pi_1^1(s) = .000136 s_1^1 + .000068 s_1^2 + 7,001.60, \quad \pi_1^2(s) = .000073 s_1^1 + .000142 s_1^2 + 6,728.20.$$

The unit transportation cost functions for wheat and corn per ton in Ukrainian hryvnia are:

$$c_{11}^{11} = .000556 Q_{11}^{11} + 2,046.80, \; c_{11}^{12} = .007512 Q_{11}^{12} + 10,984.60,$$

$$c_{12}^{11} = .000185 Q_{12}^{11} + 2,046.80, \; c_{12}^{12} = .007312 Q_{12}^{12} + 10,984.60,$$

$$c_{11}^{21} = .005566 Q_{11}^{21} + 2,046.80, \; c_{11}^{22} = .006812 Q_{11}^{22} + 10,984.60,$$

$$c_{12}^{21} = .001259 Q_{12}^{21} + 2,046.80, \; c_{12}^{22} = .007012 Q_{12}^{22} + 10,984.60.$$

The demand price functions for wheat and corn in local currencies are:

$$\rho_1^1(d) = -.15 d_1^1 + 602,344.00, \quad \rho_1^2(d) = -.68 d_1^2 + 574,560.00,$$

$$\rho_2^1(d) = -.000475 d_2^1 + 6,290.00, \quad \rho_2^2(d) = -.000758 d_2^2 + 5,980.00.$$

The supply capacity, in tons, in Ukraine is: $\bar{S}_1 = 5,000,000.00$.

The transportation capacities, in tons, over routes are:

$$\bar{Q}^1_{11} = 5,000,000.00, \quad \bar{Q}^2_{11} = 500,000.00, \quad \bar{Q}^1_{12} = 5,000,000.00, \quad \bar{Q}^2_{12} = 500,000.00.$$

These capacities are derived based on the fact that Ukraine can, at most, export around 10% of its grains without its Black Sea ports (BBC (2022)).

The modified projection method yields the following equilibrium commodity shipment pattern:

$$Q^{11*}_{11} = 477,085.5938, \quad Q^{11*}_{12} = 1,605,672.5000, \quad Q^{12*}_{11} = 0.0000, \quad Q^{12*}_{12} = 0.0000,$$

$$Q^{21*}_{11} = 79,128.0781, \quad Q^{21*}_{12} = 560,130.3750, \quad Q^{22*}_{11} = 0.0000, \quad Q^{22*}_{12} = 0.0000.$$

This commodity flow pattern is quite close to Ukraine's actual wheat and corn exports to Lebanon and Egypt in 2021 and the projected amounts in 2022, with the assumption that the invasion would have never occurred. Lebanon, on the average, imports more than 70% of its wheat and about 20% of its corn from Ukraine, while these percentages for Egypt are, on the average, 25%, and 5%, for wheat and corn, respectively (IndexMundi (2022a,b), TrendEconomy (2022a,b)). Hamdan (2022) reports that Ukraine's wheat exports to Lebanon were at 520,000 tons in 2021, and an even greater amount of exports was expected for 2022.

The equilibrium commodity supplies are: $s^{1*}_1 = 2,082,758.1250, \quad s^{2*}_1 = 639,258.4375$.

The equilibrium commodity demands are:

$$d^{1*}_1 = 477,085.5938, \quad d^{2*}_1 = 79,128.0781, \quad d^{1*}_2 = 1,605,672.5000, \quad d^{2*}_2 = 560,130.3750.$$

The incurred supply prices in Ukraine in hryvnia at the equilibrium are:

$$\pi^1_1(s^*) = 7,328.3252 = \$266.8542, \quad \pi^2_1(s^*) = 6,971.0166 = \$253.8432.$$

Pre-war, Ukrainian farmers could earn close to \$270 per ton for wheat and corn (Associated Press (2022b)), which is very close to the reported supply prices in this example. The results are also quite close to the prices in January 2022, as reported by Martyshev, Nivievskyi, and Bogonos (2023).

The incurred demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho^1_1(d^*) = 530,781.1875 = \$351.0457, \quad \rho^2_1(d^*) = 520,752.9063 = \$344.4132,$$

whereas the corresponding demand prices in Egypt in Egyptian pounds are:

$$\rho^1_2(d^*) = 5,527.3057 = \$351.3862, \quad \rho^2_2(d^*) = 5,555.4214 = \$353.1736.$$

We observe that the resultant demand prices in Lebanon and Egypt resemble the prices reported pre-war (see Breisinger et al. (2022), El Safty (2022), Galal (2022), Hamdan (2022)).

All the Lagrange multipliers were equal to: 0.0000, since the production and transportation capacities exceeded the corresponding flows. Note that only the maritime routes have positive commodity flows.

Example 7: Early Period Post Full-Scale Invasion of February 24, 2022

We now consider the following disaster scenario. It is the early period after the full-scale invasion but before the Black Sea Grain Initiative. During this period, the Black Sea routes were mined as well as blockaded by the Russians. Hence, the capacity of these maritime routes was greatly reduced to essentially zero.

Example 7 has identical data to that in Example 6, except that the maritime route links are no longer available. We retain the same superscripts and subscripts as in Example 6 but note that for each pair of supply and demand country market pairs, there is only route 2 available for the transport of the wheat and corn to Lebanon and Egypt.

The modified projection method results in the following equilibrium commodity shipment pattern:

$$Q_{11}^{12*} = 216,433.1406, \quad Q_{12}^{12*} = 500,000.0000, \quad Q_{11}^{22*} = 0.0000, \quad Q_{12}^{22*} = 0.0000.$$

Observe that now, with the cheaper maritime routes blockaded and no longer functional, the more expensive alternative routes are in use. Nivievskyi (2022) reports that, after the start of the war, the transportation cost of grains inside Ukraine reached an unprecedented level at around \$200. The alternative routes are used for the export of the first commodity, that is, wheat, but not for the second commodity, that is, corn. Lebanon and Egypt rely heavily on wheat as their main source of nutrition; however, corn is mostly used to feed animals. As such, given the importance of wheat to food security in both countries, Ukrainian wheat keeps on being imported, even with the high transportation costs associated with the alternative routes, while the importing of corn stops. Given that Egypt has a population of about twenty times that of Lebanon, naturally, its wheat import is such that the full capacity of the alternative route is used. On the other hand, the commodity flow of wheat to Lebanon does not even reach the low capacity of the alternative route, which is due to the high cost of transportation.

The equilibrium commodity supplies are: $s_1^{1*} = 716,432.1875, \quad s_1^{2*} = 0.0000.$

The equilibrium commodity demands are:

$$d_1^{1*} = 216,433.1406, \quad d_1^{2*} = 0.0000, \quad d_2^{1*} = 500,000.0000, \quad d_2^{2*} = 0.0000.$$

All Lagrange multipliers are equal to 0.0000 except that $\lambda_{12}^{2*} = 468.4277$.

The incurred supply prices in Ukraine in hryvnia at the equilibrium are:

$$\pi_1^1(s^*) = 7,099.0347 = \$258.5048, \quad \pi_1^2(s^*) = 6,780.4995 = \$246.9056.$$

Note that the supply prices are lower than in Example 6. Ukrainian farmers are essentially selling their wheat at lower prices to compensate for the higher cost of transportation after the start of the full-scale invasion. As a matter of fact, in the later months after the start of the war, the supply price for wheat of the Ukrainian farmers went as low as less than \$100 (Arhirova (2022), Balmforth and Polityuk (2022), Brower (2022)). This example is very early, right after the start of the war; therefore, the supply prices are just starting to go down.

The incurred demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho_1^1(d^*) = 569,879.0000 = \$376.9041, \quad \rho_1^2(d^*) = 6,052.5005 = \$384.7743,$$

whereas the corresponding demand prices in Egypt in Egyptian pounds are:

$$\rho_2^1(d^*) = 574,560.0000 = \$380, \quad \rho_2^2(d^*) = 5,980.0000 = \$380.1652.$$

We observe that the demand prices in both demand country markets are now higher than in Example 6. In later months, close to the establishment of the Black Sea Grain Initiative, demand prices in Lebanon and Egypt reached high levels, even around \$500 (Hernandez (2022), Rose (2022)); here, however, the markets are just starting to react to the war in terms of higher prices and the associated supply and transportation challenges.

Example 8: Black Sea Grain Initiative in Place

In this example, we consider the scenario that the Black Sea Grain Initiative is in place (beginning in August). The exchange rates are derived from late August. The exchange rates are:

$$e_{11} = 41.3469, \quad e_{12} = .5236,$$

$$USD/UAH = 36.5686, \quad USD/LBP = 1,512.0000, \quad USD/EGP = 19.1500.$$

The supply price functions for wheat and corn per ton in Ukrainian hryvnia are:

$$\pi_1^1(s) = .000136s_1^1 + .000068s_1^2 + 3,364.60, \quad \pi_1^2(s) = .000073s_1^1 + .000142s_1^2 + 4,022.50.$$

Given the damages to production inputs and available arable land in Ukraine and the war-induced Ukrainian farmers' low share of the earnings, the supply price functions are updated accordingly to account for these factors.

The unit transportation cost functions for wheat and corn per ton in Ukrainian hryvnia are:

$$c_{11}^{11} = .000556Q_{11}^{11} + 13,867.90, \quad c_{11}^{12} = .007512Q_{11}^{12} + 15,591.10,$$

$$c_{12}^{11} = .000185Q_{12}^{11} + 13,867.90, \quad c_{12}^{12} = .007312Q_{12}^{12} + 15,591.10,$$

$$c_{11}^{21} = .005566Q_{11}^{21} + 13,867.90, \quad c_{11}^{22} = .006812Q_{11}^{22} + 15,591.10,$$

$$c_{12}^{21} = .001259Q_{12}^{21} + 13,867.90, \quad c_{12}^{22} = .007012Q_{12}^{22} + 15,591.10.$$

The transportation cost functions are updated from the previous examples to highlight the war-induced unprecedented high transportation costs.

The demand price functions for wheat and corn per ton in local currencies are:

$$\rho_1^1(d) = -.15d_1^1 + 796,162.50, \quad \rho_1^2(d) = -.68d_1^2 + 718,256.40,$$

$$\rho_2^1(d) = -.000475d_2^1 + 10,000.60, \quad \rho_2^2(d) = -.000758d_2^2 + 9,900.50.$$

Both Lebanon and Egypt, during this period, were facing a severe food security crisis (Khoury (2021), Hernandez (2022)). Both countries are dependent on the flow of Ukrainian grains to meet their populations' nutrition and caloric demands. As such, the war and the reduction of the supply of Ukrainian grains induced high demand prices in these demand country markets. These changes in prices are reflected in the updated demand price functions.

In this example, the maritime routes have the original capacity, as in Example 6 (although there are still slowdowns in processing, etc.). Still, due to mining and the destruction of agricultural land in Ukraine during the war, we have the supply capacity now reduced, with $\bar{S}_1 = 1,000,000.00$.

The modified projection method yields the following equilibrium commodity shipment pattern:

$$Q_{11}^{11*} = 477,651.1563, \quad Q_{12}^{11} = 552,348.4375, \quad Q_{11}^{12*} = 0.0000, \quad Q_{12}^{12*} = 0.0000,$$

$$Q_{11}^{21*} = 0.0000, \quad Q_{12}^{21*} = 0.0000, \quad Q_{11}^{22*} = 0.0000, \quad Q_{12}^{22*} = 0.0000.$$

Once again, only efficient maritime routes are used for the transport of grains, and alternative routes are not utilized. This highlights the importance of the Black Sea Grain Initiative in facilitating the transport of grains from Ukraine, even with limited supply capacity. The wheat commodity flows are improved compared to in Example 7, especially in the case of Lebanon, which has food security implications for the food crisis in both demand country markets. Furthermore, as in Example 7, no corn is produced, which is, again, due to the high dependency of our network's demand country markets, that is, Lebanon and Egypt, on Ukrainian wheat and the war-induced limited supply capacity of Ukraine.

The equilibrium commodity supplies are: $s_1^{1*} = 999,999.6250, \quad s_1^{2*} = 0.0000.$

The equilibrium commodity demands are:

$$d_1^{1*} = 447,651.1563, \quad d_1^{2*} = 0.0000, \quad d_2^{1*} = 552,348.4375, \quad d_2^{2*} = 0.0000.$$

The computed equilibrium Lagrange multipliers are all equal to 0.0000 except that $\mu_1^{1*} = 591.6817$ since, essentially, the supply output of commodities is at the capacity $\bar{S}_1^1 = 1,000,000.00$.

The incurred supply prices in Ukraine in hryvnia at the equilibrium are:

$$\pi_1^1(s^*) = 3,500.6001 = \$95.7269, \quad \pi_1^2(s^*) = 4,095.5000 = \$111.9949.$$

Observe that the share of Ukrainian farmers is less than \$100, as mentioned in the previous example, even with the establishment of the Black Sea Grain Initiative and the facilitation of the transport of grains from Ukrainian Black Sea ports. This could be traced back to transportation costs remaining high even after the Initiative. In other words, due to the war and its associated risks, the transportation costs remain high, even through the transportation corridor provided by the Initiative.

The incurred demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho_1^1(d^*) = 729,014.8125 = \$482.1526, \quad \rho_1^2(d^*) = 718,256.3750 = \$475.0372,$$

whereas the corresponding demand prices in Egypt in Egyptian pounds are:

$$\rho_2^1(d^*) = 9,738.2344 = \$508.5239, \quad \rho_2^2(d^*) = 9,900.5000 = \$516.9973.$$

Even though the transportation capacity limitations are raised, one can see that, again, because of the high transportation costs and limited supply capacity, the demand prices remain at high levels.

Example 9: Black Sea Grain Initiative in Place, Transportation Costs as Pre-Invasion, Supply Production Capacity as in Example 7

Example 9 explores the impact of transportation costs reverting to the pre-February 24, 2022 level. The rest of the data remains as in Example 7, where recall that $\bar{S}_1 = 1,000,000.00$. Hence, this scenario considers changes in supply price and demand functions from those in Example 6, different exchange rates than those in Example 6, plus a reduction in production capacity, due to mining, etc., in wartime. This example helps to reveal the importance of transportation and a reduction in associated costs on the equilibrium pattern.

The modified projection method now yields the following equilibrium commodity shipment pattern:

$$Q_{11}^{11*} = 935,264.3750, \quad Q_{12}^{11*} = 0.0000, \quad Q_{11}^{12*} = 0.0000, \quad Q_{12}^{12*} = 0.0000,$$

$$Q_{11}^{21*} = 64,735.4296, \quad Q_{12}^{21*} = 0.0000, \quad Q_{11}^{22*} = 0.0000, \quad Q_{12}^{22*} = 0.0000.$$

Again, only the efficient maritime routes representing transportation through Ukrainian Black Sea ports are in use. Note that Lebanon and Egypt are essentially competing for the limited supply capacity of Ukrainian grains, as both countries are stricken by a food crisis. In this case, Lebanon is appropriating almost all of this limited production capacity, importing all of its reported wheat demand (Hamdan (2022), IndexMundi (2022a), TrendEconomy (2022a)) from Ukraine, while Egypt is shifting towards importing a small amount of Ukrainian corn. As mentioned in Example 6, Lebanon imports, on the average, more than 70% of its wheat demand from Ukraine, while this percentage for Egypt is at around 25%. Accordingly, Lebanon is much more dependent on Ukrainian wheat than Egypt, and the above commodity flow pattern implies this higher dependency.

The equilibrium commodity supplies are: $s_1^{1*} = 935,264.3750, \quad s_1^{2*} = 64,735.4297.$

The equilibrium commodity demands are:

$$d_1^{1*} = 935,264.3750, \quad d_1^{2*} = 64,735.4297, \quad d_2^{1*} = 552,348.4375, \quad d_2^{2*} = 0.0000.$$

The computed equilibrium Lagrange multipliers are all equal to 0.0000 except that $\mu_1^{1*} = 405,189.5000$ since the supply output of commodities is at the capacity $\bar{S}_1^1 = 1,000,000.00$.

The incurred supply prices in Ukraine in hryvnia at the equilibrium are:

$$\pi_1^1(s^*) = 3,496.1982 = \$95.6065, \quad \pi_1^2(s^*) = 4,099.9668 = \$112.1171.$$

Note that the supply prices remain at the low levels observed in Example 8 without significant improvement. It could be a result of the war-induced low supply prices and full storage in Ukraine (Nivievskyi (2022)), at least in the short term. However, it could also highlight the significant impact of the damages to arable lands and production inputs in Ukraine due to the war, which has, in turn, resulted in a low production capacity.

The incurred demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho_1^1(d^*) = 655,872.8750 = \$433.7783, \quad \rho_1^2(d^*) = 674,236.3125 = \$445.9234,$$

whereas the corresponding demand prices in Egypt in Egyptian pounds are:

$$\rho_2^1(d^*) = 10,000.6000 = \$522.2245, \quad \rho_2^2(d^*) = 9,900.5000 = \$516.9973.$$

Observe that Lebanon sees an improvement in the prices of grains, i.e., cheaper demand prices, due to appropriating almost all of Ukraine's limited production capacity. At the same time, the demand prices in Egypt remain at the same high level as in Example 8. In other words, Lebanon makes an improvement on its food security crisis by winning the competition over Ukrainian grains when Egypt is left to deal with its food security concerns as severe as before. Thus, the importance of the Ukrainian grain harvest, and its capacity, for food security in MENA countries is further highlighted.

Example 10: Same Data as That for Example 9 but with Supply Capacity as in Example 6 (Pre Full-Scale Invasion)

Example 9 reveals not only the importance of investing in transportation routes but also the importance of having sufficient capacity for the production of agricultural products with $\mu_1^* = 405,189.5000$. In Example 10, hence, we retain the data as in Example 9, but now we assume that the available supply capacity is as in Example 6. This example helps to illustrate the importance of having all the original land that Ukraine farmed pre-war made again available for critical agricultural commodities. Hence, in this example $\bar{S}_1 = 5,000,000.00$.

The computed equilibrium commodity shipment pattern is now:

$$Q_{11}^{11*} = 3,122,624.0000, \quad Q_{12}^{11} = 1,153,227.8750, \quad Q_{11}^{12*} = 0.0000, \quad Q_{12}^{12*} = 0.0000,$$

$$Q_{11}^{21*} = 487,816.0000, \quad Q_{12}^{21*} = 236,331.5313, \quad Q_{11}^{22*} = 0.0000, \quad Q_{12}^{22*} = 0.0000.$$

Observe that, with the updated supply and demand price functions, transportation cost functions, and exchange rates relevant to the post - Black Sea Grain Initiative period, as in Examples 8 and 9, but with recovered production and transportation capacities as in the pre-war period, all commodity flows on the maritime routes are now positive and increased, with the less efficient alternative routes, again, not utilized. However, in this example, contrary to the pre-war case, Lebanon is doing better in terms of competition for Ukrainian grains, appropriating more of the production capacity of Ukraine for the nutritional and caloric needs of its population. The severity of the food crisis in Lebanon and its higher dependence on Ukrainian grains could be the reason for this shift in commodity shipments. Additionally, the increase in flows could be related to months of little to no grains being shipped to the demand country markets. It should be noted that, in the long term, assuming the full stop of the war and with the severity of food security concerns in these demand country markets ameliorated, the functions would be re-adjusted.

The equilibrium commodity supplies are: $s_1^{1*} = 4,275,852.0000, \quad s_1^{2*} = 724,147.50000.$

The equilibrium commodity demands are:

$$d_1^{1*} = 3,122,624.0000, \quad d_1^{2*} = 487,816.0000, \quad d_2^{1*} = 1,153,227.8750, \quad d_2^{2*} = 236,331.5313.$$

The computed equilibrium Lagrange multipliers are all equal to 0.0000 except that $\mu_1^{1*} = 6,171.1826$.

The incurred supply prices in Ukraine in hryvnia at the equilibrium are:

$$\pi_1^1(s^*) = 3,995.3579 = \$109.2565, \quad \pi_1^2(s^*) = 4,437.4663 = \$121.3463.$$

With the increase in commodity flows, the supply prices are now higher than those in Example 8 but still quite lower than the pre-war prices. Accordingly, farmers are earning more, but still quite less than pre-war.

The incurred demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho_1^1(d^*) = 327,768.8750 = \$216.7783, \quad \rho_1^2(d^*) = 386,541.5000 = \$255.6491,$$

whereas the corresponding demand prices in Egypt in Egyptian pounds are:

$$\rho_2^1(d^*) = 9,452.8164 = \$493.6196, \quad \rho_2^2(d^*) = 9,721.3604 = \$507.6428.$$

We note that, with the recovery of the production capacity, all demand prices are now lower than in Examples 8 and 9. The lower demand prices could translate into improvements in terms of food security issues in Lebanon and Egypt. However, observe that this improvement in the case of Lebanon is much more significant, as the country in this example appropriates a much higher commodity shipment than Egypt.

6.1 Sensitivity Analysis on Exchange Rates

In this Subsection, we conduct sensitivity analysis on exchange rates, and we report the equilibrium supplies of the commodities of wheat and corn and also their demand market prices in the countries of Lebanon and Egypt. The motivation for conducting such a sensitivity analysis is as follows. In international trade, exchange rates directly affect the purchasing power of consumers and the income of producers. For example, Lebanon has been experiencing an acute economic crisis since 2019, resulting in a sharp depreciation of its currency and, as a result, a significant decrease in its population's purchasing power, resulting in a food security crisis in the country (World Food Programme (2023)). Ukraine has (prior to the full-scale invasion) been the cheapest provider of wheat and corn in the world, thanks to its fertile black soil, also known as Chornozem, and its low cost of fertilizer and energy (Wageningen University & Research (2022)). Coupled with the high transportation costs from alternative supply sources, it makes MENA countries, such as Lebanon and Egypt, highly dependent on the import of these grains from Ukraine. Russia's full-scale invasion of Ukraine has impeded the exports of Ukrainian grain to Lebanon, further worsening the economic crisis of this country and deepening its food security issues when the country is competing with other MENA countries dependent on the limited export of Ukrainian grains, e.g., Egypt, to meet the caloric demand of its people. Furthermore, the International Monetary Fund (2022) estimated a loss of at least one-third of Ukraine's GDP in 2022 due to the economic consequences of the full-scale invasion. Although Ukraine's Central Bank has implemented a fixed exchange rate to protect its people's finances since the start of the full-scale invasion (International Monetary Fund (2022)), such a policy could be unsustainable as the war draws on; in fact, Ukraine had to lower the fixed value of hryvnia once in late July 2022 (the exchange rates were checked on the website https://wise.com/us/). Coupled with the lack of capital due to the limited exports since the start of the full scale-invasion (Brower (2022)) and the war-induced higher prices of seed and fertilizer (Jenkins (2022)), a depreciated hryvnia can constrain Ukrainian farmers' ability to produce grains. Our model considers exchange rates, providing policy-makers with insights into the effects of sudden or pre-planned changes to exchange rates due to disasters on agricultural commodities' flows and prices at equilibrium.

We use Example 10 as a baseline. In Figure 3, we report the multicommodity supplies when $e_{12} = .5236$, but e_{11} varies with $e_{11} = 41.3469$ and then e_{11}=45, 50, and 55. The corresponding demand market prices at the equilibrium for these exchange rates are then reported in Figure 4.

Note that, as seen in Figure 3, with the depreciation of the Lebanese pound with respect to Ukrainian hryvnia, that is, higher rates of e_{11}, while keeping e_{12} fixed, the production of wheat in Ukraine decreases, and the supply of Ukrainian corn increases with a sharper slope. In other words, with less demand from Lebanon because of the depreciation of LBP, Ukraine meets the demand for wheat in Egypt and shifts to produce more corn to satisfy the demand for corn in Egypt. It should be noted that, generally, the supply and demand of wheat are more price inelastic than corn; furthermore, when there is a global deficit, resulting in food security concerns, the trade volumes of wheat are even less sensitive to exchange rate fluctuations.

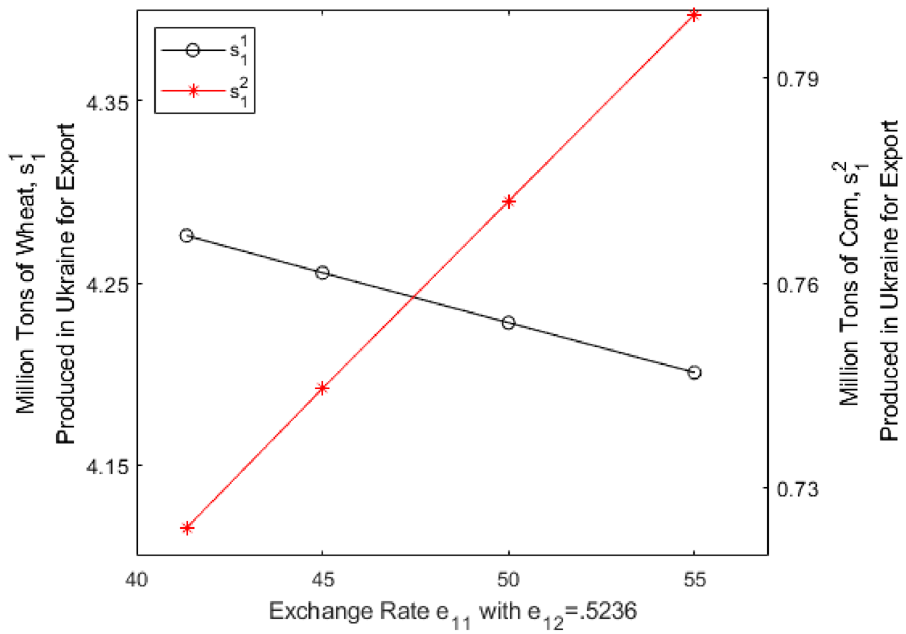

Figure 3: Sensitivity Analysis for Exchange Rates with $e_{12} = .5236$ but with e_{11} Varying: Impact on Commodity Supplies

Looking at Figure 4, one can see that as LBP depreciates and the demand prices of both commodities increase in Lebanon, the demand prices go down in Egypt since Lebanon cannot buy as much grain as before, and Egypt, which is essentially competing with Lebanon for Ukrainian grain, is now appropriating more of the commodity shipments, driving its demand prices down. Also, observe that the decrease in the demand price of wheat in Egypt is sharper than that of corn, which is in line with the importance of wheat as a staple in the country's nutritional and caloric needs of its citizenry. In other words, Egypt will import more wheat than corn, that is, will import a higher commodity flow of wheat than corn, which translates into a sharper decrease in the demand price of wheat.

We then, again, using Example 10 as a baseline, keep $e_{11} = 41.3469$ as in Example 10, but vary e_{12} with $e_{12} = .5236$ and then $e_{12} = .6236$, .7236, and, finally, .8236. The computed equilibrium commodity supplies at these exchange rates are reported in Figure 5, and the equilibrium commodity demand prices in Figure 6.

We observe that, as shown in Figure 5, with the depreciation of the Egyptian pound, with the Lebanese pound fixed, the supply of Ukrainian wheat increases, and the production of Ukrainian corn decreases, both at a decreasing rate. Essentially, with the depreciation of EGP, Egypt cannot afford as much Ukrainian grain as before, and Lebanon, which has a much higher demand for Ukrainian wheat than corn, imports more wheat.

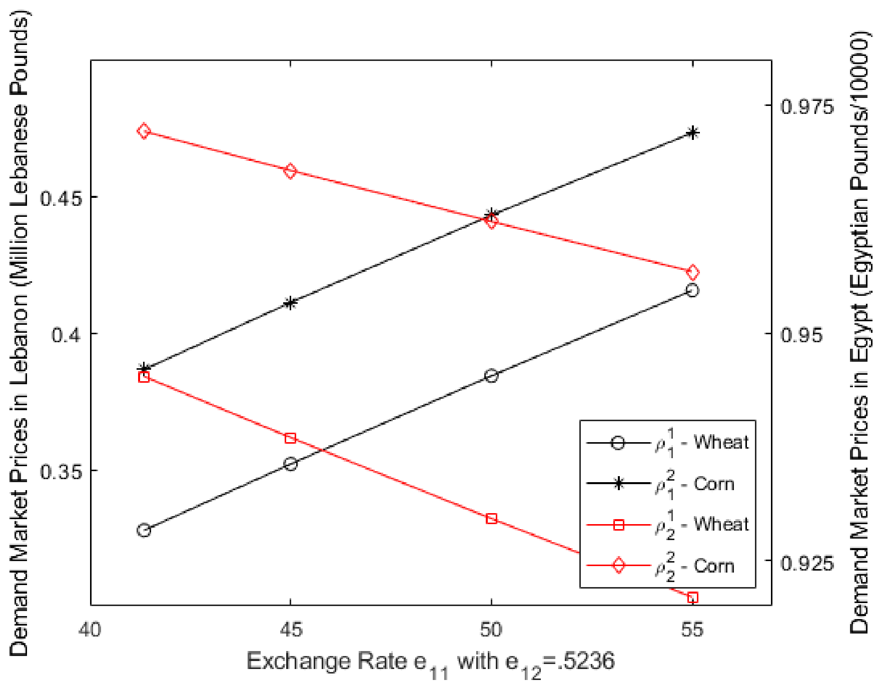

Figure 4: Sensitivity Analysis for Exchange Rates with $e_{12} = .5236$ but with e_{11} Varying: Impact on Commodity Demand Market Prices

However, with the value of EGP going lower, and more wheat commodity flow appropriated by Lebanon, the country's demand for wheat is satisfied; it slowly shifts toward buying more corn, as such, causing a decreasing rate of decrease in the production of Ukrainian corn and a decreasing rate of increase in the production of Ukrainian wheat. Again, it must be noted that wheat has a lower price elasticity compared to corn, and corn plantings could have more variations based on the market conditions.

Note that in Figure 6, with the value of EGP going lower, the demand prices in Lebanon decrease slightly since Lebanon appropriates more commodity flow. At the same time, naturally, the demand price of corn in Egypt goes down, as Egypt cannot afford the previous level of commodity shipment. However, note that the demand price of wheat in Egypt is surprisingly decreasing, albeit at a decreasing rate. The reason for this decrease in the demand price is that Egypt, facing the depreciation of EGP, and given the higher priority of wheat in the country's caloric demand than corn, refrains from importing corn. Hence, at first, Egypt raises its wheat commodity flow by giving up its corn imports, lowering the demand price, although the value of its currency is depreciating. However, the rate of the decrease in the demand price is decreasing as the possible increase in wheat imports is limited, and the depreciation in the value of EGP finally catches up and increases the demand price of wheat. Furthermore, in practice, Egypt's ability to substitute Ukrainian wheat is higher than corn, as Egypt can source its wheat demand from alternative supply country markets, such as Romania, France, and Russia, while the country is far away from alternative sources of corn, e.g., the US and South America. Accordingly, Egypt, in general, is more prone to an increase in the prices of corn than those of wheat.

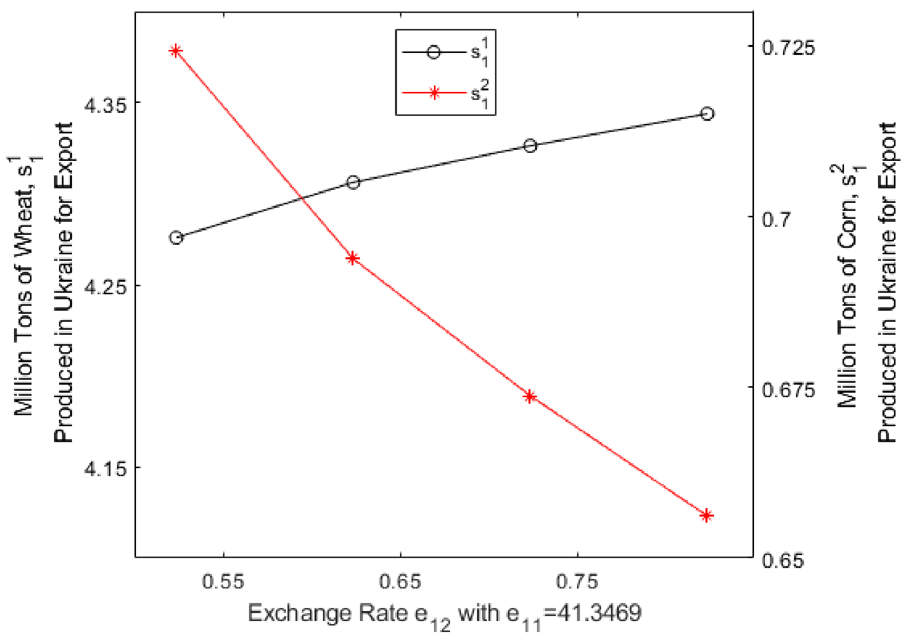

Figure 5: Sensitivity Analysis for Exchange Rates with $e_{11} = 41.3469$ but with e_{12} Varying: Impact on Commodity Supplies

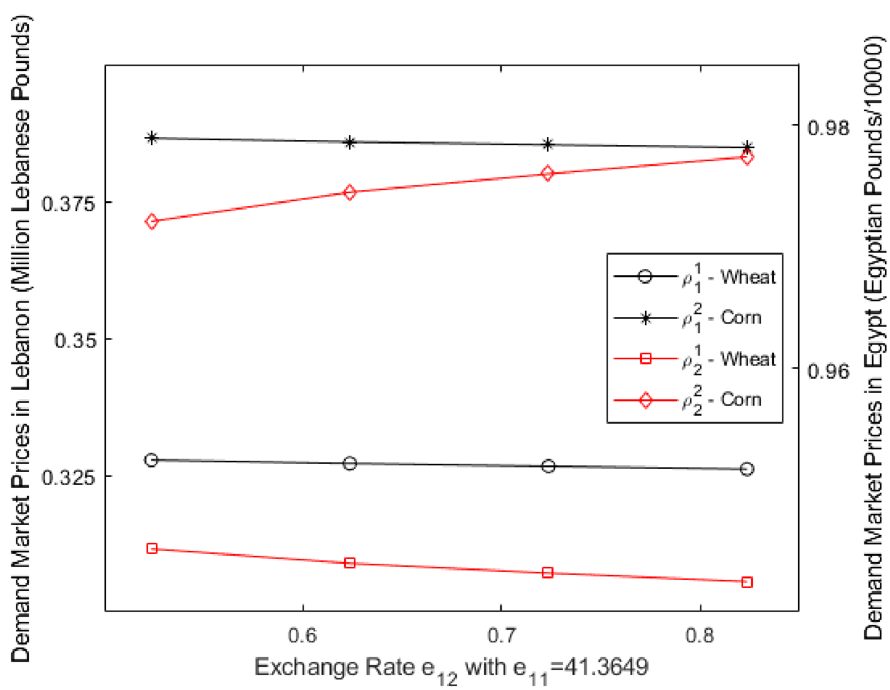

Figure 6: Sensitivity Analysis for Exchange Rates with $e_{11} = 41.3469$ but with e_{12} Varying: Impact on Commodity Demand Market Prices

The numerical examples in this Section consider a single supply market for the grains of wheat and corn - Ukraine. Russia has also, in more recent years, become an exporter of wheat (Handley (2023)). However, because of evidence that Russia has been stealing and cheaply reselling Ukrianian grain in 2022, the data on quantities exported and prices of their grain exports, as the only possible source of competition in the short to mid-term, to such countries as Egypt and Lebanon, are unreliable and, therefore, not used in this study (see Reuters (2022b), Lister and Fylyppov (2022), and Beake, Korenyuk, and Reality Check team (2022)). Nevertheless, our results capture competition for the grains on the demand side, in transportation availability, and also in terms of production capacity under different scenarios.

7. Summary and Conclusions

In this paper, we constructed a general multicommodity international agricultural trade network equilibrium model with production and transportation capacities. The model includes exchange rates, multiple routes from supply country markets to demand country markets, and allows for different modes of transportation. The model enables the quantification of the impacts of decreases in agricultural commodity production and transportation capacities, due to disasters, as well as the magnitude of exchange rates, on the equilibrium commodity production, shipment, demand, and price patterns. The supply market prices are of relevance to farmers and the demand market prices are of concern to consumers. Both supply market and demand market commodity prices are also important to governmental authorities and decision-makers since agricultural commodity trade volumes, along with their prices, affect food security. The generality of the underlying functions that our model can handle provides a broad range of possible applications to different disasters, whether caused by natural or man-made phenomena.

Using the methodological framework of variational inequality theory, we provide alternative formulations of the governing equilibrium conditions, which are new, along with qualitative properties of existence and uniqueness. We also provide a deeper interpretation of the Lagrange multipliers associated with the production/supply constraints, which capture the capacities associated with agricultural commodity production in the countries as well as the Lagrange multipliers associated with the transportation capacities associated with moving the commodities between countries. The proposed algorithm has nice features for implementation and resolves one of our derived variational inequalities into closed form expressions for the commodity shipments and the Lagrange multipliers at each iteration.

A series of numerical examples are then solved to illustrate the multicommodity international agricultural trade network equilibrium model. Several examples are first presented for illustration. These are followed by more general numerical examples inspired by Russia's war on Ukraine and its impact on food security in MENA countries. The model reveals the equilibrium supplies, the transportation volumes, the demands, and the supply and demand prices of the agricultural commodities of wheat and corn from Ukraine to Lebanon and Egypt under different specific disaster scenarios. Different production and transportation capacities from several relevant periods; that is, pre-war, early after the start of the war, before the Black Sea Grain Initiative, and after the Initiative is in place, are used to highlight the changes in the equilibrium supply, commodity shipment, and demand patterns and the supply market and demand market prices.

The numerical results have implications for the Ukrainian government. We see that there is essentially no efficient alternative to the maritime transportation of grains from Ukrainian Black Sea ports, highlighting the importance of extending the Black Sea Grain Initiative during wartime to keep the transportation capacity sufficient not to disrupt the food security of demand country markets. The results confirm how the war has driven the earnings of the Ukrainian farmers, that is, the supply prices, to unprecedented low levels, possibly requiring the Ukrainian government's and global support of Ukrainian farmers for future harvest seasons, given the importance of Ukrainian grain to global food security, especially in the MENA region. On the other hand, knowledge of the changes incommodity shipment patterns and prices helps the governments of the demand country markets, in our case, Lebanon and Egypt, manage the nutrition and caloric demand of their populations, and, hence, the food security of their people. Additionally, the numerical results show the priority of wheat over corn in all scenarios in the demand markets of Lebanon and Egypt, as two countries representative of the MENA region. We find that Lebanon and Egypt compete over the war-induced limited production capacity at war-induced high prices to meet their populations' nutritional and caloric demands. The results demonstrate how the war-induced reduced production capacity in Ukraine intensifies this competition for meeting the fundamental need for food security of the Lebanese and Egyptian people. Additionally, sensitivity analysis on exchange rates reveals how different exchange rates affect the supply and demand prices of the two commodities of wheat and corn at the equilibrium. The economic instability of the demand country markets, in the form of the depreciation of their currencies, lowers their share of the Ukrainian wheat supply, causing food security concerns in these countries. The solutions to the numerical examples show the shift in the percentage of the limited production capacity in Ukraine utilized for producing each of the commodities of wheat and corn as the currency of each of the demand country markets depreciates.

Many possible extensions are promising for future research. It would be interesting to extend the model to an intertemporal trade network equilibrium model with capacities on storage, since storage has been another economic activity disrupted by the war on Ukraine. Also, models can be constructed to include uncertainty in production and transportation. The inclusion of exchange rate uncertainty can also be of interest. Additionally, a model with multiple links in a route, each with its distinct capacities, can be constructed to provide further insights as to more disaggregated impacts of decreasing capacity on specific modes of transportation.

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Declaration of competing interest None.

References

Andrée, B.P.J., 2022. Monthly food price estimates by product and market. World Bank Microdata Library, Washington DC.

Arhirova, H., 2022. Anxiety grows for Ukraine's grain farmers as harvest begins. Associated Press, July 10.

Associated Press, 2022a. Romanian port struggles to handle Flow of Ukrainian grain. Voa - Voice of America English News. June 25.

Associated Press, 2022b. Anxiety grows for Ukraine's farmers. July 10.

Balmforth, T., Polityuk, P., 2022. Ukraine grain storage crisis hits home as farmers harvest new crops. Reuters, July 19.

Barraza De La Cruz, B.C., Pizzolato, N.D., Barraza De La Cruz, A., 2010. An application of the spatial equilibrium model to soybean production in tocantins and neighboring states in Brazil. Pesquisa Operacional, 30(2), 443-464.

Beake, N., Korenyuk, M., Reality Check team, 2022. Tracking where Russia is taking Ukraine's stolen grain. BBC. May 12.

Belgrave, J., 2022. Needs at all-time high even before the war in Ukraine, food crises report says. World Food Programme. May 4.

Belikova, M., 2022a. How much are logistic costs eating into Ukraine's grain trade? May 5.

Belikova, M., 2022b. Drastic freight rate fall in Ukraine's deep sea ports brings no relief for grain trade. December 9.

Besik, D., Nagurney, A., Dutta, P., 2023. An integrated multitiered supply chain network model of competing agricultural firms and processing firms: The case of fresh produce and quality. European Journal of Operational Research, 307(1), 364-381.

Bishop, P., Pratt, J., Novakovic, A., 1994. Using a joint-input, multi-product formulation to improve spatial price equilibrium models. Cornell University, Department of Agricultural Economics Staff Paper no. 94-06, Ithaca, New York.

Breisinger, C., Khouri, N., Glauber, J., Laborde, D., 2022. One of the world's worst economic collapses, now compounded by the Ukraine crisis: What's next for Lebanon? International Food Policy Research Institute, May 6.

Brower, D., 2022. 'They will run out of money': farmers' fight for survival in Ukraine. Financial Times, July 21.

Cassidy, E., 2022. Worst drought on record parches Horn of Africa. NASA Earth Observatory.

Chehayeb, K., 2022. Lebanon announces hike in fixed exchange rate against dollar. Associated Press. September 2022.

Dafermos, S., Nagurney, A., 1984. Sensitivity analysis for the general spatial economic equilibrium problem. Operations Research, 32, 1069-1086.

Das, S., 2019. China's evolving exchange rate regime. International Monetary Fund Working Papers.

Devadoss, S., Sabala, E., 2020. Effects of yuan-dollar exchange rate changes on world cotton markets. Journal of Agriculture and Applied Economics 52(3), 420-439.

Do Rosario, J., 2023. Turkey earthquake could result in loss of up to 1% of country's GDP in 2023. Reuters. February 16.

Erhardt, B., 1977. Floating exchange rates and their problems for the developing countries Intereconomics 12, 29-34.

Ghosh, A. R., Ostray, J. D., 2009. Choosing an exchange rate regime. Finance and Development, 46(4), 38-40.

El Safty, S., 2022. Egypt's private sector wheat imports stall due to dollar shortage. Reuters, October 4.

European Food Agency, 2023. Turkey-Syria earthquake: FAO calls for aid to farmers. February 17.

Florian, M., Los, M., 1982. A new look at static spatial price equilibrium models. Regional Science and Urban Economics, 12, 579-597.

Food and Agriculture Organization, 2022. Note on the impact of the war on food security in Ukraine. July 20.

Galal, S., 2022. Monthly average prices for wheat in Egypt 2019-2022. Statista, December 16.

Hall, B., 2023. Ukrainian Black Sea grain export deal extended. Financial Times. March 18.

Hamdan, H., 2022. Lebanese fear wheat shortage amid Ukrainian crisis. Al-Monitor, March 2.

Handley, L., 2023. Ukraine's corn and wheat exports are set to plummet. Here's what that means for the world's food supply. CNBC, April 20.

Hernandez, B., 2022. Lebanon's economic crisis adds pressure to wheat prices. ETF Database, July 29.

IndexMundi, 2022a. Lebanon wheat imports by year.

IndexMundi, 2022b. Egypt wheat imports by year.

International Monetary Fund, 2002. How Ukraine is managing a war economy. December 22.

Jenkins, S., 2022. How the Russia-Ukraine war helped fuel record fertilizer prices. Federal Reserve Bank of St. Louis. October 4.

Khoury, E., 2021. Lebanon: Unprecedented number of people forced to rely on humanitarian assistance. World Food Program, September 17.

Kinderlehrer, D., Stampacchia, G., 1980. Variational inequlities and their applications. Academic Press, New York.

Korpelevich G.M., 1977. The extragradient method for finding saddle points and other problems. Matekon, 13, 35-49.

Lister, T., Fyllypov, S., 2022. Russian ships carrying stolen Ukrainian grain turned away from Mediterranean ports – but not all of them. CNN, May 12.

Lopez, S.A.I., Rau, M.L., Woltjer, G., 2019. Exploring alternative approaches to estimate the impact of non-tariff measures and further implementation in simulation models. Joint Research Centre (JRC) technical report EUR 29578 EN Publications Office of the European Union, Luxembourg.

Martyshev, M., Nivievskyi, O., Bogonos, M., 2023. Regional war, global consequences: Mounting damages to Ukraine's agriculture and growing challenges for global food security. IFPRI Blog. March 27.

Mercy Corps, 2023. Devastating earthquake kills over 2,000 people in Turkey and Syria, Mercy Corps responding in northwest Syria. February 6.

Nagurney, A., 1989. The formulation and solution of large-scale multicommodity equilibrium problems over space and time. European Journal of Operational Research, 42(2), 166-177.

Nagurney, A., 1999. Network economics: A variational inequality approach, second edition. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nagurney, A., 2022a. Operations Research for the recovery and reconstruction of Ukraine. ORMS Today. September 21.

Nagurney, A., 2022b. Spatial price equilibrium, perishable products, and trade policies in the Covid-19 pandemic. Montes Taurus J. Pure Appl. Math., 4(3), 9-24.

Nagurney, A., Aronson, J., 1989. A general dynamic spatial price network equilibrium model with gains and losses. Networks, 19(7), 751-769.

Nagurney, A., Besik, D., Dong, J., 2019. Tariffs and quotas in world trade: A unified variational inequality framework. European Journal of Operational Research, 275(1), 347- 360.

Nagurney, A., Hassani, D., Nivievskyi, O., Martyshev, P., 2023. Exchange rates and multicommodity international trade: Insights from spatial price equilibrium modeling with policy instruments via variational inequalities. Journal of Global Optimization, 87, 1-30.

Nagurney, A., Li, D., Nagurney, L.S., 2014. Spatial price equilibrium with information asymmetry in quality and minimum quality standards. International Journal of Production Economics, 158, 300-313.

Nagurney, A., Nicholson, B.F., Bishop, P.M., 1996. Spatial price equilibrium models with discriminatory ad valorem tariffs: Formulation and comparative computation via variational inequalities. In: Recent advances in spatial equilibrium modelling, J.C.J.M. Bergh, P. Nijkamp, and P. Rietveld, Editors, Springer, Berlin, Germany, pp. 179-200.

Nagurney, A., Salarpour, M., Dong, J., 2022. Modeling of Covid-19 trade measures on essential products: A multiproduct, multicountry spatial price equilibrium framework. International Transactions in Operational Research, 29(1), 226-258.

Nagurney, A., Thore, S., Pan, J., 1996. Spatial market policy modeling with goal targets. Operations Research, 44(2), 393-406.

Nichols, M., Faulconbridge, G., 2023. Black Sea grain deal expires after Russia quits. Reuters. July 17.

Nivievskyi, O., 2022. Russian War in Ukraine and Global Food Crisis [Webinar]. Public Policy Programs. The Behrend College, Pennsylvania State University Erie.

Passacantando, M., Raciti, F., 2022. A multiclass network international migration model under shared constraints. To appear in the Proceedings of the ODS2022 Conference, Florence, Italy, Springer.

Pratt, S., 2022. Ukrainian farmers struggle as invasion continues. The Western Producer, July 27.

Reuters, 2022a. Grain shipments from Ukraine's Danube ports hit record high in 2022. December 20.

Reuters, 2022b. Ukraine says Russia stole 'several hundred thousand tonnes' of grain. April 30.

Reuters, 2023. Lebanon devalues official exchange rate by 90%. February 1.

Rose, S., 2022. Lebanon faces food crisis with 'no wheat orders since Ukraine war began'. The National, March 16.

Ruijs, A., Schweigman, C., Lutz, C., Sirpe, G., 2001. Cereal trade in developing countries: Stochastic spatial equilibrium models. Technical report, University of Groningen, The Netherlands.

Samuelson, P.A., 1952. Spatial price equilibrium and linear programming. American Economic Review, 42, 283-303.

Takayama, T., Judge, G.G., 1964. An intertemporal price equilibrium model. Journal of Farm Economics, 46, 477-484.

Takayama, T., Judge, G.G., 1971. Spatial and temporal price and allocation models. Amsterdam, The Netherlands: North-Holland.

The World Bank, 2023. Turkiye. Available at: https://data.worldbank.org/country/turkiye

Thompson, R.L., 1989. Spatial and temporal price equilibrium agricultural models. In: Quantitative methods for market oriented economic analysis over space and time. W.C. Labys, T. Takayama, and N. Uri, Editors, Gower Publishing Co. Ltd., Brookfield, Vermont, pp. 49-65.

Tong, X, Xiao, S., 2006. A feasible method for solving the KKT system of variational inequalities. Hokkaido Mathematical Journal, 35, 87-105.

TrendEconomy, 2022a. Annual international trade statistics by country (HS) - Lebanon, November 14. Available at: https://trendeconomy.com/data/h2/Lebanon/1001

TrendEconomy, 2022b. Annual international trade statistics by country (HS) - Egypt, November 14.

Ukrainska Pravda, 2023. Grain Initiative: Russia artificially slows down inspection of ships in Bosphorus.

UN News, 2022a. Black Sea grain exports deal 'a beacon of hope' amid Ukraine war - Guterres, July 22.

UN News, 2022b. Horn of Africa faces most severe drought in more than two generations – UNICEF. December 22.

U.S. Department of Agriculture, 2022. Agricultural marketing service. August.

Wageningen University & Research, 2022. Wheat is not scarce but expensive.

World Food Programme, 2022a. WFP Türkiye country brief. December. World Food Programme, 2022b. War in Ukraine drives global food crisis. June 24.

World Food Programme, 2023. Economic crisis, currency depreciation, and unprecedented increase in food and non-food prices leave 2 million people in need of assistance in Lebanon. January 19.

World Meteorological Organization, 2023. Catastrophic consequences of Horn of Africa drought will continue in 2023. February 17.

Yu, M., Nagurney, A., 2013. Competitive food supply chain networks with application to fresh produce. European Journal of Operational Research, 224, 273-282.