A Multicommodity Spatial Price Equilibrium Model with Exchange Rates and Non-Tariff Measures for

Agri-Food International Trade

Anna Nagurney* Department of Operations and Information Management Isenberg School of Management University of Massachusetts Amherst, Massachusetts 01003 Elena Besedina Kyiv School of Economics Mykoly Shpaka St. 3 Kyiv, Ukraine 02000 Operations Research Forum 4 (2023), 84

Abstract: International trade of agri-food products is essential for food security and the well-being of societies. Governments have increasingly turned way from tariffs as policy instruments and are, instead, applying non-tariff measures (NTMs). The evaluation of such policies that include: quotas, subsidies, quality standards, as well as sanitary and phytosanitary measures, through the rigorous quantification of their impacts on the flows, prices, as well as quality levels of agricultural commodities, is highly needed, as is the inclusion of exchange rates. In this paper, we construct a multicommodity spatial price equilibrium model with exchange rates for the international trade of agri-food products that includes: quotas, subsidies, quality of the commodities, along with possible deterioriation in transport, minimum quality standards, and ad valorem equivalents to quantify NTMs associated with agricultural products that can affect the health and well-being of humans, animals, and plants. The methodological framework is that of the theory of variational inequalities. The extensive numerical examples are focused on wheat exports from Ukraine to Middle Eastern and North African (MENA) countries during wartime. The numerical examples include sensitivity analysis for ad valorem equivalents and governmental subsidies as well as quality coefficients with important insights for decision makers and policy makers concerned about food security affected by volumes of available agricultural commodities, their quality, as well as their supply and demand prices.

Key words: non-tariff measures, sanitary and phytosanitary measures, spatial price equilibrium, variational inequalities, agriculture and international trade, food security

* Corresponding author email: nagurney@isenberg.umass.edu

1. Introduction

International trade of agri-food products is essential for food security and the well-being of societies. Hence, policies associated with such trade continue to garner great attention from cognizant global organizations, governments, as well as researchers and practitioners. Interestingly, although tariffs, whose impacts are readily captured in a variety of quantitative models, have been decreasing over time in this critical sector of the global economy, non-tariff measures (NTMs), which, as the name implies, include all policies besides tariffs that can impact international trade, continue to be applied. According to the Organization for Economic Cooperation and Development (OECD (2019)), NTMs can be categorized into "technical" and "non-technical" measures with the former including Sanitary and Phytosanitary (SPS) measures, which are of great relevance to agri-food trade and include standards such as quality, and Technical Barriers to Trade (TBT). Sanitary measures are those concerned with human and animal health whereas phytosanitary ones apply to plant health (see World Trade Organization (1998)). In particular, SPS measures aim to protect humans, animals, and/or plants from risks associated with additives, pests, diseases, toxins, etc. TBT measures, in turn, are usually regulations that are imposed on production processes and product characteristics and may also include labeling requirements. An example of a non-technical measure that is relevant to trade in agricultural and food products is that of quotas.

According to Kravchenko et al. (2022), 1947 is identified as the date at which NTMs entered policy discussions as per the creation of the General Agreement on Tariffs and Trade (GATT). The United Nations Conference on Trade and Development (UNCTAD (2010)) provided a compact definition of NTMs stating that "NTMs are policy measures other than ordinary custom tariffs that can potentially have an economic effect on international trade in goods, changing quantities traded, or prices or both." The spectrum of such policies, nevertheless, is expansive. A recent report by the UNCTAD and The World Bank (2017) reports that NTM measures affect most traded goods and most of the products that we encounter in our daily lives since many NTMs, which are regulations, are as relevant to domestic products as to traded ones. The report also emphasizes that many regulatory NTMs are essential for sustainable development since they seek to protect human, animal, and/or plant health as well as the environment. The report documents that NTMs are most prevalent in the agri-food sector and across all regions globally. In addition, TBTs are the most frequently applied NTM sets of policies, impacting 35% of product lines and approximately 65% of world trade. They are followed by SPS measures and export measures. Interestingly, the report states that developed countries are behind the high usage of TBTs, with SPS measures being more evenly distributed. The Food and Agriculture Organization (see FAO (2022)), in its state of agricultural markets 2022 report, in turn, notes that essentially 100% of food and agricultural imports are subject to NTMs in contrast to about 40% of products in the other sectors. The report also emphasizes that, on average, a food product faces eight different NTMs whereas products of all other sectors are subject to just under two NTMs.

In this paper, we take on the challenge of constructing a computable multicommodity spatial price equilibrium framework for agri-food international trade which includes multiple NTMs. The model incorporates quotas, subsidies, and utilizes ad valorem equivalents (AVEs), which are commonly used in practice to quantify the effects of NTMs. As noted in Cadot, Gourdon, and Van Tongeren (2018), the AVE of an NTM is the "proportional rise in the domestic price of the commodity to which it is applied, relative to a counterfactual where it is not applied." Hence, interestingly, AVEs work like an ad valorem tariff (see, e.g., Nagurney, Nicholson, and Bishop (1996)). Furthermore, since various SPS measures focus on food safety and quality, it is imperative to include quality of commodities explicitly. This has only recently been done, as noted by Ghodsi (2023), but, for an imperfectly competitive model with NTMs, rather than for a perfectly competitive one, as is the spatial price equilibrium model in this paper. Here, we also propose minimum quality standards for the various commodities and, importantly, allow for the deterioration in commodity quality as it is transported using a transportation route. In addition, we allow for multiple transportation routes joining an origin country for a commodity with a destination country. Earlier, a single commodity spatial price equilibrium model was constructed by Nagurney, Li, and Nagurney (2014) but there were only single routes joining each pair of origin and destination countries and quality at the origin country was assumed to be preserved until the destination country. Furthermore, there were no subsidies or tariffs (which we also include here, for completeness) considered and no AVEs. Finally, since the new model in this paper is one of international trade, and exchange rates play a very important role in the prices that consumers ultimately pay for commodities, plus exchange rates can also significantly affect the volume of goods exported (and imported), our spatial price equilibrium model also includes explicit exchange rates. There is only limited modeling work done on spatial price equilibrium problems that includes exchange rates (see, e.g., Devadoss and Sabala (2020) and Nagurney et al. (2023)). Our numerical examples focus on Ukraine, which has been under attack by Russia, since the major invasion of February 24, 2022 (see Al-Jazeera (2022)). Ukraine has been a major agricultural exporter and also subject to a spectrum of NTMs (cf. Shepotylo (2022)).

2. Literature Review, Contributions, and Organization of the Paper

Since the focus of this paper is the expansion of modeling frameworks for international trade of agri-food products, in the presence of policies in the form of non-tariff measures, we utilize the methodology of variational inequality theory for the formulation, analysis, and computation of solutions to numerical examples. The theory of variational inequalities allows for the multicommodity spatial price equilibrium model that is constructed here to include more general underlying functions (supply price, demand price, and opportunity cost ones) than have been previously utilized, while also capturing a plethora of NTMs. The literature review, hence, is focused on the various policies included in our model with a specific emphasis on spatial price equilibrium (SPE) models that have also made use of variational inequality theory for their development. We note that SPE models originated in the seminal work of Samuelson (1952) and Takayama and Judge (1964, 1971) and have spurred numerous applications from agriculture to energy. Their relevance to practice has also stimulated the development of more general models and associated theory, including that of variational inequality theory (cf. Florian and Los (1982), Dafermos and Nagurney (1984), Harker (1985), Nagurney, Takayama, and Zhang (1995), Nagurney, Thore, and Pan (1996), Nagurney (1999), Daniele (2004), Nagurney and Besik (2022), and the references therein). In addition, since the international trade model in this paper includes exchange rates, something rarely done explicitly in spatial price equilibrium models, we also highlight some relevant literature on this topic. Recent novel variational inequality models have included ones inspired by various applications to disaster management applications (see, e.g., Nagurney et al. (2020), Nagurney (2021), and Colajanni, Daniele and Sciacca (2022)).

2.1 Quotas

Quotas are an example of quantity-based NTMs, as opposed to price-based ones. Quotas, as noted in Nagurney, Salarpour, and Dong (2022), have been widely applied in the pandemic for many essential products including personal protective equipment (PPEs). Their spatial price equilibrium model is also a multicommodity one and allows for multiple transportation routes between origins and destinations. The authors utilize variational inequality theory but their model does not include exchange rates nor subsidies nor quality of products and minimum quality standards. The new model in this paper also generalizes the model of Nagurney, Li, and Nagurney (2014) to include multiple commodities, exchange rates, quality deterioration of commodities over transportation routes, with more general supply price, demand price, and opportunity cost functions. Plus, in contrast to the above two papers, here we also include AVE constructs that are highly relevant to agri-food international trade. Nagurney, Besik, and Dong (2019) construct a unified framework for world trade using variational inequality theory with a focus on tariff rate quotas. However, no quality aspects are captured and exchange rates are not included and the spatial price equilibrium model is a single commodity one. Nagurney (2022) proposed a variational inequality model of multicommodity spatial price equilibrium with tariffs and quotas which captured perishability of products such as agricultural ones. However, there were no exchange rates nor other NTMs, plus only single routes joining origins with destinations were considered.

2.2 Subsidies

Subsidies are an example of price-based NTMs and they are imposed by governments to reduce, for example, production and other costs as in the case of subsidizing growers of food and agricultural products. In Nagurney (2008), it was noted that unit transaction costs in spatial price equilibrium models, as constructed therein, could include, in addition to transportation costs, tariffs as well as subsidies, but there was no explicit notation provided. Being able to vary subsidies and other NTMs in a modeling framework can yield insights as well as benefits. Devadoss and Sabala (2020), in their spatial price equilibrium model, included subsidies as well as tariffs and also exchange rates. The model, however, was a single commodity one and assumed separable supply price and demand price functions. The subsidies were imposed by governments to support producers. In this paper, in contrast, we allow for subsidies for the various commodities to be imposed on country origin / destination pairs. Effectively, the model allows for pair-specific export subsidies. A negative subsidy would be equivalent to an export tax/duty, which are sometimes also imposed by exporting countries. For example, Ukraine has applied an export tax on sunflower seeds for several years. Of course, if a subsidy is only on an origin country and commodity then the specific term can be changed accordingly. Such subsidies can also be viewed as negative unit tariffs between origin and destination countries. Nagurney et al. (2023) included explicit subsidies in a spatial price equilibrium model with exchange rates but the subsidies are exclusively on the production side. The subsidies in this paper, in turn, can be imposed between country origin and destination pairs. The Nagurney et al. (2023) model introduced exchange rates to spatial price equilibrium models using variational inequality theory along transportation routes. The model, however, did not have any quality constructs nor AVEs and the commodifies were homogeneous. In the new model in this paper the consumers at the demand markets respond not only to the commodity prices but also to their quality and, therefore, we have differentiated (but substitutable) products.

2.3 Ad Valorem Equivalents

According to UNCTAD (2022), quantification of the impacts of NTMs on trade flows can be done by determining how much they add to trade costs through the estimation of ad valorem equivalents (AVEs). As noted earlier, AVEs represent the costs (in terms of percentage) that the presence of NTMs adds to trade and, hence, they play a similar role, interestingly, to that of ad valorem tariffs. Although ad valorem tariffs have been introduced in the context of spatial price equilibrium models using variational inequality theory by Nagurney, Nicholson, and Bishop (1996), ad valorem equivalents for NTMs are introduced here, for the first time, for spatial price equilibrium models using variational inequality theory. Lopez, Rau, and Woltjer (2019) provide additional background on the use of AVEs for the estimation of NTMs, with several examples focused on the food sector. Additional background on AVEs, which would also include compliance costs and associated certificates, as in the case of SPS measures of relevance to foodstuffs, can be found in the report by UNCTAD and The World Bank (2017). For additional background on SPSs, with a focus on agriculture and trade, see the policy brief by Besedina (2015) and the cited references therein. Arita, Beckmann, and Mitchell (2016) report that in a study it was calculated that SPS regulations and TBTs have a large impact on agricultural exports from the United States to the European Union, with the AVEs of such measures ranging from 23% to 102%. Hence, including them in a rigorous theoretical framework is important for both research and practice.

2.4 Quality Constructs and Minimum Quality Standards

Quality of products, including agri-food ones, which are ingested, is a very important characteristic. Quality of agri-food products is also related to nutrition as well as to food safety. Furthermore, insufficient quality can be associated with product perishability and also increased waste and food losses. In terms of spatial price equilibrium models, Nagurney, Li, and Nagurney (2014) were the first to introduce quality into such models, accompanied by minimum quality standards. However, unlike the model in this paper, there was no quality deterioration over transportation routes, which occurs, even under the best conditions, for fresh produce (cf. Besik and Nagurney (2017), Nagurney, Besik, and Yu (2018), and Besik, Nagurney, and Dutta (2023)). Moreover, the model in this paper has NTMs plus exchange rates and multiple commodities. Plus, the new model in this paper handles product differentiation, whereas the majority of SPE models assume homogeneous products. In our model quality is commodity and country specific. The conformity assessment is modeled separately, as the minimum quality standard, that has a similar effect as an SPS will have. This implies that not all products can 'overcome' such a barrier. Such modeling is consistent with evidence that SPS may negatively affect the extensive margin of trade (see Besedina (2015)). Clearly, SPS can also be trade inducing, once (and if) the commodity meets the minimum standard.

Nagurney and Li (2016) provide a series of models, both perfectly competitive and imperfectly competitive ones, in a supply chain context, where quality of products is explicitly considered. Li, Nagurney, and Yu (2018) introduce a spatial price equilibrium model with differentiated products in which consumers learn of product quality with a time delay. Their model is a single commodity one without any imposed policies or exchange rates and there is no quality deterioration over time. Perishability of commodities over space and time using arc multipliers was proposed for a spatial price equilibrium model by Nagurney and Aronson (1988) using variational inequality theory. The use of such multipliers in a variety of supply chain network applications can be found in the book by Nagurney et al. (2013). Here, as in Nagurney, Li, and Nagurney (2014), quality is defined as "the degree to which a specific product conforms to a design or specification."

Assoua et al. (2022) provide a gravity model focusing on trade in agri-food products and highlight the importance of including quality. The paper includes many references to SPS measures and trade of agri-food products, focusing primarily on empirical research. The authors highlight shortcomings associated with gravity models. Dong and Jensen (2007), in an earlier publication, recognize the importance of SPS measures for food safety in international trade, and review the many challenges that China has faced in complying with SPS measures on their agricultural exports. Shepotylo et al. (2022) construct a model to investigate the impacts of non-tariff measures on the productivity of Ukrainian food processing firms. The model is for a single firm and includes minimum quality standards. Shepotylo (2022) also discusses impacts of NTMs on food processing firms in Ukraine and recognizes that NTMs affect the cost of production; however, by setting requirements for the quality and safety of the final goods, they may increase consumer demand.

In this paper, minimum qualoty standards refer only to SPS measures since TBTs do not apply to agro and food procuts, which are the focus of this paper. However, if the model is applied to manufactured goods and their trade flows, then the quality standards can also refer to TBTs.

2.5 Exchange Rates

Devadoss and Sabala (2020) introduced exchange rates into spatial price equilibrium

models but their underlying functions are separable. They also included subsidies and tariffs. Nagurney et al. (2023) generalized their work to include multiple commodities and multiple routes between country pairs, along with novel expressions for exchange rate calculation across trade routes through different countries. Exchange rates affect prices that consumers pay for products as well as the volume of goods traded and, hence, in the context of agri-food products, they play an important role in food security. The spatial price equilibrium model in this paper, therefore, is one of the very few to incorporate exchange rates, and it does so with NTMs quantified with AVEs, and with quotas, specific subsidies, as well as quality, accompanied by possible deterioration of product quality in transport, and the imposition of quality standards.

2.6 Contributions

The novelty of the contributions in this paper are as follows:

1. A spectrum of NTMs is captured in a multicommodity spatial price equilibrium model, allowing for a richer policy investigation framework to assess impacts of various non-tariff measures on agri-food international trade from volumes of commodities traded to prices that consumers will pay.

2. This is the first time that AVEs are used in a multicommodity spatial price equilibrium model to quantify impacts of NTMs using variational inequality theory. AVEs can handle sanitary and phytosanitary measures of high relevance to agri-food international trade with compliance and certificates required and of relevance to food safety.

3. Quality of each of the commodities is a variable, with quality of the commodities decreasing, as appropriate, through the transportation processes. Minimum quality standards can be imposed to assure that SPS measures, including those of relevance to human agri-food safety, are attained for consumers.

4. Product differentiation is captured, whereas, in many spatial price equilibrium models, it is assumed that the products are homogeneous and consumers do not distinguish among the countries of origin. For background on product differentiation, including as in Armington (1969), in spatial price equilibrium models and other models in agricultural trade, see Sarker and Surry (2005).

5. This is one of the very few papers that includes explicit exchange rates in a spatial equilibrium model for international trade. Exchange rates affect commodity trade volumes and prices; consequently, they are important in assessing impacts on food security.

6. More general supply price, demand price, and opportunity cost functions than those that have been used in the existing literature are constructed, allowing for more comprehensive, general applications that address the impacts of NTMs on both the supply and demand sides of multicommodity international trade networks for agri-food products.

7. The numerical examples illustrate the modeling and algorithmic framework constructed in this paper with focus on a very timely major, ongoing global event - that of Russia's war on Ukraine and the impacts on agricultural exports, in particular, on the exports of wheat on MENA countries.

8. The numerical examples quantify (and illustrate) the impacts of various government NTM policies on the volumes of wheat shipments from Ukraine, on the quality of the wheat, as well as on the demand prices that consumers pay and highlight the impacts of the various policies on food security.

2.7 Organization of the Paper

The paper is organized as follows. In addition to the delineated Sections 1 and 2 above, Section 3 presents the model, along with the statement of the governing equilibrium conditions, and the derivation of the variational inequality formulation. Section 4 then outlines an algorithm with nice features for numerical computation. The algorithm, at each iteration, resolves the variational inequality problem into subproblems in the multicommodity flows, the commodity quality levels, and the Lagrange multipliers associated with the quota constraints, which, in turn, can be computed using explicit formulae, which we provide. Section 5 presents a series of numerical examples focusing on agri-food international trade. In particular, the examples consider Ukraine as a producer of wheat with exports to MENA countries of Lebanon and Egypt. NTMs in the form of quality standards, AVEs, subsidies, and quotas are investigated and sensitivity analysis conducted for changes in the AVEs as well as Ukraine's supply price functions with respect to quality of wheat and increases in subsidies to farmers. The insights gained from the numerical examples are useful for policy makers and decision makers. The results are summarized in Section 6 where suggestions for future research are also included.

3. The Multicommodity Agri-Food International Trade Spatial Price Equilibrium Model with Exchange Rates and NTMs

There are m countries in the multicommodity agri-food model with NTMs involved in the production of agricultural commodities, which are transported and consumed in n countries, as depicted in Figure 1. We allow for different transportation routes to connect each origin



Figure 1: The Multicommodity Agri-Food Spatial Price Network for International Trade

country with each destination country, as represented by distinct links. A typical origin country is denoted by i, whereas a typical destination country is denoted by j. There are H commodities, with a typical one denoted by h, and K routes joining each country origin node with each country destination node, with a typical transport route denoted by k.

We now introduce the notation. Let Q_{ij}^{hk} denote the amount of commodity h produced in country i and shipped on route k to country j. We group all the commodity flows into the vector $Q \in R_+^{HKmn}$. Let s_i^h correspond to the supply of commodity h produced in country iand group all the commodity supplies into the vector $s \in R_+^{Hm}$. The demand for commodity h from country i in country j is denoted by d_{ij}^h . Note that, in the model, since we will be capturing quality associated with the different commodities at the destination countries, we have product differentiation. We group the demands into the vector $d \in R_+^{Hmn}$. All vectors are column vectors.

The conservation of flow equations are now presented.

For each commodity h and each origin country i we must have that the supply of the commodity is equal to the shipments out, that is:

$$s_i^h = \sum_{j=1}^n \sum_{k=1}^K Q_{ij}^{hk}, \quad h = 1, \dots, H; i = 1, \dots, m.$$
 (1)

On the demand side, in turn, we must have that, for each commodity h from origin country i at each destination country j, the following holds:

$$d_{ij}^{h} = \sum_{k=1}^{K} Q_{ij}^{hk}, \quad h = 1, \dots, H; i = 1, \dots, m; j = 1, \dots, n;$$
(2)

that is, the demand for each commodity from each country is satisfied by the shipments of the commodity to the destination country. Here, as noted in the Introduction, we are dealing with differentiated (but substitutable) commodities.

Also, all the commodity shipments must be nonnegative, that is:

$$Q_{ij}^{hk} \ge 0, \quad h = 1, \dots, H; k = 1, \dots, K; i = 1, \dots, m; j = 1, \dots, n.$$
 (3)

Since the model is a spatial price equilibrium model for the international trade of commodities, we introduce exchange rates e_{ij} between the origin and destination countries: $i = 1, \ldots, m; j = 1, \ldots, n$, respectively.

Non-Tariff Measures

In the multicommodity spatial price equilibrium model for the international trade, we allow for both technical and non-technical non-tariff measures.

Specifically, we let \bar{Q}_{ij}^h denote the quota, which serves as an upper bound, on commodity h produced in origin country i and destined for destination country j for all h, i, j. Then the constraints to handle such a measure are:

$$\sum_{k=1}^{K} Q_{ij}^{hk} \le \bar{Q}_{ij}^{h}, \quad h = 1, \dots, H; i = 1, \dots, m; j = 1, \dots, n.$$
(4)

In addition, in order to capture various costs associated with NTMs, the OECD and others (cf. Cadot, Gourdon, and van Tongeren (2018) and the references therein) have applied Ad Valorem Equivalents (AVEs) to capture the costs associated with NTMs.

Further, since, as noted in the Introduction, quality standards serve as an NTM and quality standards are especially important when it comes to agri-food products due to safety concerns, we introduce minimum quality standards for the various commodities.

Hence, we allow for the imposition of minimum quality standards on the supply side, such that

$$q_i^h \ge \underline{q}_i^h, \quad h = 1, \dots, H; i = 1, \dots, m, \tag{5}$$

where q_i^h denotes the quality of commodity h produced in country i and \underline{q}_i^h is the minimum quality standard that is imposed by the government of country i on h, which takes on a nonnegative value. We group the quality values into the vector $q \in R_+^{Hm}$.

Originally, non-tariff measures were called non-tariff barriers since the additional costs of such measures were emphasized. Clearly, there may also be demand side effects and including quality and standards can serve as a signal to consumers at the demand markets. We make this more explicit when we define the demand price functions.

Due to the fact that many governments subsidize agriculture, we let sub_{ij}^h denote the unit subsidy provided by country i; i = 1, ..., m, producing commodity h for h = 1, ..., H, which will be shipped to country j; j = 1, ..., n. Of course, a specific case of subsidy is that of sub_i^h , but we believe that the former offers more flexibility on the part of governments in influencing international trade.

Tariff Measures

Since the model that we construct in this paper is general and focuses on agri-food international trade, we also, for completeness, include tariffs. Obviously, if a particular policy trade measure is not applied, then the value of it in the model is adapted accordingly as would be the case if there was no tariff between a pair of origin and destination countries. Also, if the destination country is the same as the origin country, then tariffs are not applied.

In the model, for completeness, hence, we include the possibility of tariffs being imposed with a per unit tariff on commodity h between country i and country j being denoted by t_{ij}^h for $h = 1, \ldots, H$; $i = 1, \ldots, m$; $j = 1, \ldots, n$.

The Supply Price, Demand Price, Unit Transportation Cost, and Opportunity Cost Functions

We now describe the notation for the various functions in the model.

The supply price function for commodity h of country i is π_i^h and, in general, it can depend on the entire vector of supplies (production amounts) of the commodities as well as on the quality levels of the commodities determined in the various countries. Hence, we have that:

$$\pi_i^h = \pi_i^h(s, q), \quad h = 1, \dots, H; i = 1, \dots, m.$$
 (6a)

Due to the commodity supply conservation of flow equations (1), one can define new

supply price functions $\tilde{\pi}_i^h$; $h = 1, \ldots, H$; $i = 1, \ldots, m$, where

$$\tilde{\pi}_i^h(Q,q) \equiv \pi_i^h(s,q). \tag{6b}$$

Note that the quality levels q_i^h ; $h = 1, \ldots, H$; $i = 1, \ldots, m$, are the quality levels of the commodities at the "production" sites in the countries. However, consumers, located at the destination countries, may be a significant distance away and quality of an agri-food product, which may be perishable, would decrease over distance and time (and according to the conditions on the specific transportation route). Consumers, thus, respond to the average quality of a commodity, in terms of the prices that they are willing to pay for the commodity. We now quantify the average quality values as follows. Let β_{ij}^{hk} be the parameter capturing the % retained quality of commodity h transported on k from i to j. Then, the average quality \hat{q}_{ij}^h of commodity h produced at i and destined for j is given by the expression:

$$\hat{q}_{ij}^{h} = \frac{\sum_{k=1}^{K} \beta_{ij}^{hk} q_{i}^{h} Q_{ij}^{hk}}{\sum_{k=1}^{K} Q_{ij}^{hk}}; \quad h = 1, \dots, H; i = 1, \dots, m; j = 1, \dots, n.$$
(7)

We group the average quality values into the vector $\hat{q} \in R^{Hmn}_+$ and assume that there is a positive demand at each country for each commodity; otherwise, the country destination node as well as the links terminating in the node are extracted from the specific bipartite network in Figure 1. We note that Nagurney, Li, and Nagurney (2014) also used a measure of average product quality in their spatial equilibrium model. However, their model was a single commodity one and assumed that quality was preserved in the transportation process. For specific fruits and vegetables, one can readily calculate the β_{ij}^{hk} using results from food science as discussed in Besik and Nagurney (2017) and Nagurney, Besik, and Yu (2018).

The demand price functions, since we are dealing with differentiated commodities, due to possible distinct commodity quality levels, can, in general, depend on the vector of demands for the commodities as well as the vector of average quality levels at the destination countries, such that:

$$\rho_{ij}^{h} = \rho_{ij}^{h}(d,\hat{q}), \quad h = 1, \dots, H; i = 1, \dots, m; j = 1, \dots, n,$$
(8a)

where ρ_{ij}^{h} is the demand price for commodity h from country i in country j.

Due to the demand conservation of flow equations (2), and, in view of (7), one can define equivalent demand price functions $\tilde{\rho}_{ij}^h$; $h = 1, \ldots, H$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, such that:

$$\tilde{\rho}_{ij}^h(Q,q) \equiv \rho_{ij}^h(d,\hat{q}). \tag{8b}$$

Associated with each link k, in turn, joining origin country i with destination country j is a unit transportation cost function for each commodity h, such that:

$$c_{ij}^{hk} = c_{ij}^{hk}(Q), \quad h = 1, \dots, H; k = 1, \dots, K; i = 1, \dots, m; j = 1, \dots, n.$$
 (9)

We utilize opportunity cost functions, which we need to complete the model, since we have both product flow values as well as quality levels in the model. Here, we let OC_i^h denote the opportunity cost associated with commodity h in origin country i, where

$$OC_i^h = OC_i^h(q), \quad h = 1, \dots, H; i = 1, \dots, m.$$
 (10)

Opportunity cost functions are also used in Nagurney, Li, and Nagurney (2014) but they are single commodity ones and, unlike ours in (10), are assumed to be separable.

We are now ready to state the multicommodity spatial price equilibrium conditions for agri-food international trade with NTMs. The model includes NTMs in the form of quotas, subsidies, minimum quality standards, and any technical and nontechnical other NTMs (SPS and TBT ones) where the impact on international trade can be quantified through AVEs.

Definition 1: Multicommodity Spatial Price Equilibrium Conditions for Agri-Food International Trade with NTMs

A multicommodity shipment, quality, and Lagrange multiplier pattern $(Q^*, q^*, \lambda^*) \in \mathcal{K}^1$, where $\mathcal{K}^1 \equiv \{(Q, q, \lambda) | (Q, q, \lambda) \in \mathbb{R}^{HKmn+Hm+Hmn}_+ | q_i^h \geq \underline{q}_i^h, \forall h, i\}$ is a multicommodity spatial price equilibrium for agri-food international trade with NTMs if it satisfies the following conditions: for each commodity $h; h = 1, \ldots, H$, and for each pair of origin and destination countries $(i, j); i = 1, \ldots, m; j = 1, \ldots, n$, and every trade route $k; k = 1, \ldots, K$:

$$\left[(\tilde{\pi}_{i}^{h}(Q^{*},q) + c_{ij}^{hk}(Q^{*}))(1 + AVE_{ij}^{h}) + t_{ij}^{h} - sub_{ij}^{h} \right] e_{ij} + \lambda_{ij}^{h*} \begin{cases} = \tilde{\rho}_{ij}^{h}(Q^{*},q^{*}), & \text{if } Q_{ij}^{hk*} > 0, \\ \ge \rho_{ij}^{h}(Q^{*},q^{*}), & \text{if } Q_{ij}^{hk*} = 0, \end{cases}$$

$$(11)$$

and for each commodity h; h = 1, ..., H and each supply market i; i = 1, ..., m:

$$OC_{i}^{h}(q^{*}) \begin{cases} = \tilde{\pi}_{i}^{h}(Q^{*}, q^{*}), & \text{if } q_{i}^{h*} > \underline{q}_{i}^{h}, \\ \geq \tilde{\pi}_{i}(Q^{*}, q^{*}), & \text{if } q_{i}^{h*} = \underline{q}_{i}^{h}, \end{cases}$$
(12)

plus, for each commodity h; h = 1, ..., H and each pair of origin and destination countries (i, j); i = 1, ..., m; j = 1, ..., n:

$$\lambda_{ij}^{h*} \begin{cases} \geq 0, & \text{if } \sum_{k=1}^{K} Q_{ij}^{hk*} = \bar{Q}_{ij}^{h}, \\ = 0, & \text{if } \sum_{k=1}^{K} Q_{ij}^{hk*} < \bar{Q}_{ij}^{h}. \end{cases}$$
(13)

Equilibrium conditions (11) are a much expanded form of the classical spatial price equilibrium conditions of Samuelson (1952) and Takayama and Judge (1971) to include ad valorem equivalents, tariffs, and subsidies, as well as supply price and demand price dependence on not only commodity quantitities but also on their quality levels, along with exchange rates and the Lagrange multipliers associated with the quota constraints in (13). Equilibrium conditions (12), in turn, are extensions of the analogous Nagurney, Li, and Nagurney (2014) equilibrium conditions to the multicommodity case and with more general opportunity cost functions and supply price functions which can depend on the entire vector of quality levels.

We now state the theorem.

Theorem 1: Variational Inequality Formulation

A multicommodity shipment, quality, and Lagrange multiplier pattern $(Q^*, q^*, \lambda^*) \in \mathcal{K}^1$ is a multicommodity spatial price equilibrium for agri-food international trade with NTMs according to Definition 1 if and only if it satisfies the variational inequality:

$$\sum_{h=1}^{H} \sum_{k=1}^{K} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\left[(\tilde{\pi}_{i}^{h}(Q^{*},q^{*}) + c_{ij}^{hk}(Q^{*}))(1 + AVE_{ij}^{h}) + t_{ij}^{h} - sub_{ij}^{h} \right] e_{ij} + \lambda_{ij}^{h*} - \tilde{\rho}_{ij}^{h}(Q^{*},q^{*})) \right) \\ \times \left(Q_{ij}^{hk} - Q_{ij}^{hk*} \right) + \sum_{h=1}^{H} \sum_{i=1}^{m} (OC_{i}^{h}(q^{*}) - \tilde{\pi}_{i}^{h}(Q^{*},q^{*})) \times (q_{i}^{h} - q_{i}^{h*}) \right) \\ + \sum_{h=1}^{H} \sum_{i=1}^{m} \sum_{j=1}^{n} (\bar{Q}_{ij}^{h} - \sum_{k=1}^{K} Q_{ij}^{hk*}) \times (\lambda_{ij}^{h} - \lambda_{ij}^{h*}) \ge 0, \quad \forall (Q, q, \lambda) \in \mathcal{K}^{1}.$$

$$(14)$$

Proof: We first establish necessity; that is, if $(Q^*, q^*, \lambda^*) \in \mathcal{K}^1$ satisfies the spatial price equilibrium conditions according to Definition 1, then it also satisfies the variational inequality (14). From (11), we know that, for such a commodity shipment, quality, and Lagrange multiplier pattern, and for fixed h, k, i, j, that

$$\left(\left[\left(\tilde{\pi}_{i}^{h}(Q^{*},q^{i})+c_{ij}^{hk}(Q^{*})\right)(1+AVE_{ij}^{h})+t_{ij}^{h}-sub_{ij}^{h}\right]e_{ij}+\lambda_{ij}^{h*}-\tilde{\rho}_{ij}^{h}(Q^{*},q^{*})\right)\times\left(Q_{ij}^{hk}-Q_{ij}^{hk*}\right)\geq0,$$

$$\forall Q_{ij}^{hk}\geq0,$$
(15)

since, if $Q_{ij}^{hk} > 0$, then the left-hand side expression in (15) preceding the \times is zero, so (15) holds. Also, if $Q_{ij}^{hk*} = 0$, then the left-hand side expression before the \times is nonnegative, and,

since Q_{ij}^{hk} is always greater than or equal to Q_{ij}^{hk*} , (15) also holds. But, since (15) holds for every h, k, i, j, summation of (15) over such indices yields:

$$\sum_{h=1}^{H} \sum_{k=1}^{K} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\left[(\tilde{\pi}_{i}^{h}(Q^{*},q) + c_{ij}^{hk}(Q^{*}))(1 + AVE_{ij}^{h}) + t_{ij}^{h} - sub_{ij}^{h} \right] e_{ij} + \lambda_{ij}^{h*} - \tilde{\rho}_{ij}^{h}(Q^{*},q^{*})) \times (Q_{ij}^{hk} - Q_{ij}^{hk*}) \ge 0, \quad \forall Q \in R_{+}^{HKmn}.$$

$$(16)$$

Also, from the multicommodity spatial price equilibrium conditions (12), we know that, for a fixed h and i:

$$(OC_i^h(q^*) - \tilde{\pi}_i^h(Q^*, q^*)) \times (q_i^h - q_i^{h*}), \quad \forall q_i^h \ge \underline{q}_i^h.$$

$$(17)$$

Summing now (16) over all h and over all i yields:

$$\sum_{h=1}^{H} \sum_{i=1}^{m} (OC_i^h(q^*) - \tilde{\pi}_i^h(Q^*, q^*)) \times (q_i^h - q_i^{h*}), \quad \forall q_i^h \ge \underline{q}_i^h, \forall h, i.$$
(18)

From the multicommodity spatial equilibrium conditions (13), in turn, it follows that:

$$(\bar{Q}_{ij}^h - \sum_{k=1}^K Q_{ij}^{hk*}) \times (\lambda_{ij}^h - \lambda_{ij}^{h*}) \ge 0, \quad \forall \lambda_i^h \ge 0.$$

$$(19)$$

Summation of (19) over all h and over all i, j, gives us:

$$\sum_{h=1}^{H} \sum_{i=1}^{m} \sum_{j=1}^{n} (\bar{Q}_{ij}^{h} - \sum_{k=1}^{K} Q_{ij}^{hk*}) \times (\lambda_{ij}^{h} - \lambda_{ij}^{h*}) \ge 0, \quad \forall \lambda \in R_{+}^{Hmn}.$$
 (20)

Combining (16), (18), and (20), yields variational inequality (14). Necessity has been established.

We now proceed to demonstrate sufficiency. Set $q_i^h = q_i^{h*}$ for all h, i; set $\lambda_{ij}^h = \lambda_{ij}^{h*}$ for all h, i, and set $Q_{ij}^{hk} = Q_{ij}^{hk*}$ for all h, k and i, j except for $h = \tilde{h}$; $k = \tilde{k}$; $i = \tilde{i}$, and $j = \tilde{j}$, and substitute the resultants into variational inequality (14). Such substitutions reduce (14) to:

$$\left(\left[\left(\tilde{\pi}_{\tilde{i}}^{\tilde{h}}(Q^*,q^i)+c_{\tilde{i}\tilde{j}}^{\tilde{h}\tilde{k}}(Q^*)\right)(1+AVE_{\tilde{i}\tilde{j}}^{\tilde{h}})+t_{\tilde{i}\tilde{j}}^{\tilde{h}}-sub_{\tilde{i}\tilde{j}}^{\tilde{h}}\right]e_{\tilde{i}j}+\lambda_{\tilde{i}\tilde{j}}^{\tilde{h}*}-\tilde{\rho}_{\tilde{i}\tilde{j}}^{\tilde{h}}(Q^*,q^*)\right)\times\left(Q_{\tilde{i}\tilde{j}}^{\tilde{h}\tilde{k}}-Q_{\tilde{i}\tilde{j}}^{\tilde{h}\tilde{k}*}\right)\geq0,$$

$$\forall Q_{\tilde{i}\tilde{j}}^{\tilde{h}\tilde{k}}\geq0.$$

$$(21)$$

Clearly, (21) implies that the multicommodity spatial price equilibrium conditions (11) hold.

Letting now $Q_{ij}^{hk} = Q_{ij}^{hk*}$ for all h, k, i, j; similarly, letting $\lambda_{ij}^h = \lambda_{ij}^{h*}$, for all h, i, j, and $q_i^h = q_i^{h*}$, for all h, i such that $h \neq \tilde{h}$ and $i \neq \tilde{i}$, and substituting these values into variational inequality (14), yields:

$$(OC_{\tilde{i}}^{\tilde{h}}(q^*) - \tilde{\pi}_{\tilde{i}}^{\tilde{h}}(Q^*, q^*)) \times (q_{\tilde{i}}^{\tilde{h}} - q_{\tilde{i}}^{\tilde{h}*}) \ge 0, \quad \forall q_{\tilde{i}}^{\tilde{h}} \ge \underline{q}_{\tilde{i}}^{\tilde{h}},$$
(22)

from which it follows that the multicommodity spatial equilibrium conditions (12) also hold.

Finally, letting now $Q_{ij}^{hk} = Q_{ij}^{hk*}$ for all h, k, i, j plus setting $q_i^h = q_i^{h*}$ for all h, i, and then $\lambda_{ij}^h = \lambda_{ij}^{h*}$, for all h except for $h = \tilde{h}$ and for all i except for $i = \tilde{i}$ and for $j = \tilde{j}$, substitution of these values into (14) reduces (14) to:

$$(\bar{Q}_{\tilde{i}\tilde{j}}^{\tilde{h}} - \sum_{k=1}^{K} Q_{\tilde{i}\tilde{j}}^{\tilde{h}\tilde{k}*}) \times (\lambda_{\tilde{i}\tilde{j}}^{\tilde{h}} - \lambda_{\tilde{i}\tilde{j}}^{\tilde{h}*}) \ge 0, \quad \forall \lambda_{\tilde{i}\tilde{j}}^{\tilde{h}} \ge 0,$$
(23)

and, hence, spatial equilibrium conditions (13) must hold. Sufficiency has also been established. \Box

Variational inequality (14) is now put into standard form (cf. Nagurney (1999)), $VI(F, \mathcal{K})$, where one seeks to determine a vector $X^* \in \mathcal{K} \subset \mathbb{R}^N$, such that

$$\langle F(X^*), X - X^* \rangle \ge 0, \quad \forall X \in \mathcal{K},$$
(24)

with F being a given continuous function from \mathcal{K} to $\mathbb{R}^{\mathcal{N}}$, \mathcal{K} being a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denoting the inner product in \mathcal{N} -dimensional Euclidean space.

We define $X \equiv (Q, q, \lambda), \mathcal{K} \equiv \mathcal{K}^1$, and $\mathcal{N} = HKmn + 2Hm$. $F(X) \equiv (F_1(X), F_2(X), F_3(X))$ where $F_1(X)$ consists of the elements: $([(\tilde{\pi}_i^h(Q, q) + c_{ij}^{hk}(Q))(1 + AVE_{ij}^h) + t_{ij}^h - sub_{ij}^h] e_{ij} + \lambda_{ij}^h - \tilde{\rho}_{ij}^h(Q, q)), \forall h, k, \forall i, j$. The components of $F_2(X)$, in turn, are: $(OC_i^h(q) - \tilde{\pi}_i^h(Q, q)), \forall h, i$, whereas the components of $F_3(X)$ are: $(\bar{Q}_{ij}^h - \sum_{k=1}^K Q_{ij}^{hk}), \forall h, \forall i, j$.

VI (14) can, hence, be put into standard form (24).

4. The Algorithm

The modified projection method of Korpelevich (1977) is implemented and applied to solve a series of numerical examples in the next section. This algorithm is guaranteed to converge if the function F(X) that enters the variational inequality problem (14) is monotone and Lipschitz continuous.

The function F(X) is said to be monotone if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \ge 0, \quad \forall X^1, X^2 \in \mathcal{K}.$$
(25)

Also, F(X) is Lipschitz continuous, if there exists an $\eta > 0$, known as the Lipschitz constant, such that

$$||F(X^{1}) - F(X^{2})|| \le \eta ||X^{1} - X^{2}||, \quad \forall X^{1}, X^{2} \in \mathcal{K}.$$
(26)

We now recall the steps of the modified projection method, with τ denoting an iteration counter. Subsequently, we highlight the resolution of these steps in the context of the new model, which consists of closed form expressions for the commodity shipments, the commodity quality levels, as well as the Lagrange multiplers at each iteration. Such closed form expressions demonstrate that the algorithm is easy to implement.

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau = 1$ and let ζ be a scalar such that $0 < \zeta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation

Compute \bar{X}^{τ} by solving the variational inequality subproblem:

$$\langle \bar{X}^{\tau} + \zeta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^{\tau} \rangle \ge 0, \quad \forall X \in \mathcal{K}.$$
(27)

Step 2: Adaptation

Compute X^{τ} by solving the variational inequality subproblem:

$$\langle X^{\tau} + \zeta F(\bar{X}^{\tau}) - X^{\tau-1}, X - X^{\tau} \rangle \ge 0, \quad \forall X \in \mathcal{K}.$$
(28)

Step 3: Convergence Verification

If $|X^{\tau} - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

Because of the structure of the feasible set \mathcal{K}^1 for the international trade spatial price equilibrium model for agri-food products and NTMs, the solution of each of the subproblems in (27) and (28) can be obtained via closed form expressions for the multicommodity shipments, the commodity quality levels, and the Lagrange multipliers as detailed below.

Explicit Formulae at Iteration τ for the Multicommodity Shipments in Step 1

The modified projection method results in the following closed form expressions for (27) for the multicommodity shipments in Step 1 for the solution of variational inequality (14):

$$\bar{Q}_{ij}^{hk\tau} = \max\{0, Q_{ij}^{hk\tau-1} + \zeta(\tilde{\rho}_{ij}^{h}(Q^{\tau-1}, q^{\tau-1}) - \left[(\tilde{\pi}_{i}^{h}(Q^{\tau-1}, q^{\tau-1}) + c_{ij}^{hk}(Q^{\tau-1}))(1 + AVE_{ij}^{h}) + t_{ij}^{h} - sub_{ij}^{h}\right] e_{ij} - \lambda_{ij}^{h\tau-1})\}, \quad \forall h, k, i, j.$$
(29)

Explicit Formulae at Iteration τ for the Multicommodity Quality Levels in Step 1

The closed form expressions for the multicommodity quality levels for (27) for the variational inequality (14) are:

$$\bar{q}_i^{h\tau} = \max\{\underline{q}_i^h, q_i^{h\tau-1} + \zeta(\tilde{\pi}_i^h(Q^{\tau-1}, q^{\tau-1}) - OC_i^h(q^{\tau-1}))\}, \quad \forall h, i.$$
(30)

Explicit Formulae at Iteration τ for the Lagrange Multipliers in Step 1 The closed form expressions for the Lagrange multipliers for (27) for our variational inequality are:

$$\bar{\lambda}_{ij}^{h\tau} = \max\{0, \lambda_{ij}^{h\tau-1} + \zeta(\sum_{k=1}^{K} Q_{ij}^{hk\tau-1} - \bar{Q}_{ij}^{h})\}, \quad \forall h, i, j.$$
(31)

The explicit formulae for the various variables in (28) in Step 2 easily follow.

5. Numerical Examples

In this Section, we present numerical examples with input and output data. The examples are solved using the modified projection method discussed in Section 4. The algorithm is implemented in FORTAN on a Linux system at the University of Massachusetts Amherst. The modified projection method for all the examples is initialized as follows: The commodity flows are initialized to 1.00 whereas the quality levels are initialized to the minimum quality standard (which is 0.00 if no such standard is imposed). The algorithm is considered to have converged if the absolute values of the computed successive iterate variable values are less than or equal to 10^{-8} . The contraction parameter ζ in the algorithm is set to .1.

The numerical examples are all focused on wheat exports from Ukraine. Ukraine, known as the world's breadbasket, has been subject to a major invasion by Russia since February 24, 2022 (see Al Jazeera (2022)). For months transport of agricultural products could not take place on the Black Sea, which was, typically, the most effective, efficient transportation



Lebanon

Figure 2: The Network for International Trade for Set 1 Examples

route for wheat, corn, etc. Ukraine's wheat, in particular, has provided an important input into bread, a food staple in Middle Eastern and North African countries such as Lebanon and Egypt. In July 22, 2022, the brokered agreement, with assistance of the United Nations and Turkey, opened up the Black Sea route with additional checks (cf. UN News (2022)). In our numerical examples, we focus on this period. For the challenges associated with agricultural exports from Ukraine during the war, see Nivievskyi (2022). Whitworth (2022) addresses some of the sanitary and phytosanitary issues impacted by Russia's war on Ukraine.

The exchange rates used in the examples are obtained from the WISE website: https://wise.com/us/; see also, e.g.: https://wise.com/us/currency-converter/usd-to-uah-rate?amount=1000.

Set 1: Quality and Quality Standards

In the first set of examples, we explore quality and quality standards. Ukraine is the wheat exporting country and Lebanon is the importing country. We assume that there are two transportation routes from Ukraine to Lebanon with the first link including the Black Sea route and the second one not. The network topology is depicted in Figure 2. According to the USDA (2022), 97% of the wheat exported from Ukraine is winter wheat. Quality associated with agricultural products can have many dimensions. Here, we assume that the quality of wheat can be captured through the variable of quality. Specific characteristics that enter into quantifying wheat quality and associated numerical values can be found in Ma et al. (2021) and include protein content, gluten content, moisture, etc.

We denote Ukraine by node 1 and Lebanon by node 2. The units are tons.

We consider a single commodity - wheat - and, hence, we remove the superscript h from the notation.

Example 1: Baseline Example for Set 1

The data for Example 1 are as follows.

The supply price function in the Ukrainian currency of hryvnia (UAH) is: $\pi_1(s,q) = .000167s_1 + .001q_1 + 3,364.6000.$

In this example, $\beta_{12}^1 = \beta_{12}^2 = 1.00$ with link 1 being the left link in the network in Figure 2 and link 2 being the link on the right. We, hence, assume in Example 1 that there is no loss in quality as the wheat is transported to the destination.

The demand price function in the Lebanese currency of pounds (LBP) is: $\rho_{12}(d, \hat{q}) = -.082d_{12} + .01\frac{q_1Q_{12}^1 + q_1Q_{12}^2}{Q_{12}^1 + Q_{12}^2} + 796, 162.5000.$

The unit transportation cost functions in hryvnia are:

$$c_{12}^{1}(Q) = .000463Q_{12}^{1} + 14,567.9000, \quad c_{12}^{2}(Q) = .015368Q_{12}^{2} + 13,106.8000$$

The opportunity cost function is: $OC_1(q) = 5q_1$ and the exchange rate $e_{12} = 41.3469$.

In Example 1, there are no additional costs and there is no tariff or subsidy. There is also no quota imposed. We use this example as a baseline and set the minimum quality standard $q^1 = 0.0000$.

The modified projection method computes the following equilibrium solution:

$$Q_{12}^{1*} = 417,556.3125, \quad Q_{12}^{2*} = 107,654.4531,$$

 $q_1^* = 690.6000,$

with corresponding supply and demand of:

$$s_1^* = d_{12}^* = 525,210.7656$$

The supply price in hryvnia is: 3, 453.0010 and the demand price in Lebanese pounds is: 753, 102.1250. Note that, for comparison purposes, the supply price in dollars is: \$94.42 and the demand price in dollars is \$498.08: The first transportation route carries about 4 times the number of tons of wheat as does the second transportation route. The average quality $\hat{q}_1 = 690.6000$. These results are very reasonable as are the volumes of the wheat commodity flows. See also Nagurney et al. (2023).

Example 2: Minimum Quality Standard Imposed

Example 2 has the identical data to the data in Example 1 except that now the minimum quality standard is no longer 0.0000 (which essentially implies that there is no standard) but is now $q^1 = 1,000.0000$.

The computed equilibrium solution is now:

$$Q_{12}^{1*} = 417,468.0938, \quad Q_{12}^{2*} = 107,651.9141,$$

 $q_1^* = 1,000.0000,$

with corresponding supply and demand of:

$$s_1^* = d_{12}^* = 525, 120.0079$$

The supply price in hryvnia is: 3,453.3952 and the demand price in Lebanese pounds is: 753,112.6875.

The quality level in this example is at the minimum imposed standard as is the average quality. The supply and demand of wheat now decrease as do the volumes of wheat shipments on the transportation routes.

Example 3: Consumers are More Responsive to Commodity Quality

Example 3 has the same data as that in Example 2 except that now the demand price function is modified to reflect that consumers in Lebanon are more responsive to the quality of the wheat. The demand price function is now:

$$\rho_{12}(d,\hat{q}) = -.082d_{12} + .1\frac{q_1Q_{12}^1 + q_1Q_{12}^2}{Q_{12}^1 + Q_{12}^2} + 796,162.5000.$$

Note that a buyer of wheat from Ukraine in Lebanon actually rejected a shipment during wartime because of concerns about the delay in transport and the possible impact on quality (see Mathews (2022)).

The computed equilibrium solution is now:

$$Q_{12}^{1*} = 418, 281.3750, \quad Q_{12}^{2*} = 107, 676.3203,$$

 $q_1^* = 1,000.0000,$

with corresponding supply and demand of:

$$s_1^* = d_{12}^* = 525,957.69.$$

The supply price in hryvnia is: 3,453.4351 and the demand price in Lebanese pounds is: 753,134.0000.

The supply and the demand for wheat now increase and consumers in Lebanon pay a higher price for the wheat. The wheat, however, remains at the minimum quality standard of 1,000.0000 as does the average quality.

Example 4: Quality Deterioration in Transport

Example 4 has the same data as that in Example 3 but now we explore the impact of quality deterioration in the wheat, which can occur for many reasons, including delays in transport, time-consuming border checks and processing of certificates, etc.

In Example 4, we have:

$$\beta_{12}^1 = \beta_{12}^2 = .9.$$

The equilibrium solution for Example 4 is:

$$Q_{12}^{1*} = 418, 191.1563, \quad Q_{12}^{2*} = 107, 673.5938,$$

 $q_1^* = 1,000.0000,$

with corresponding supply and demand of:

$$s_1^* = d_{12}^* = 525,864.7501.$$

In Examples 1 through 3 the average quality \hat{q}_{12} was always equal to the equilibrium quality q_1^* since there was no quality deterioration of the wheat. In Example 4, in contrast, $\hat{q}_{12} = 900.0000$.

The supply price of a ton of wheat in hryvnia is now: 3,453.4194 and the demand price in Lebanese pounds is: 753,131.5625. The supply and demand decrease, as compared to the values in Example 3, and the demand price decreases.

Set 2: A New Demand Market and Ad Valorem Equivalents

In Set 2 of examples, we build on Example 5 and include another country on the demand side - that of Egypt.

The network topology is as depicted in Figure 3. Egypt is denoted by node 3.



Figure 3: The Network for International Trade for Set 2 Examples

Example 5: Baseline Example for Set 2

Example 5 has the same data as that in Example 4 except for the additional data associated with Egypt, which is now given. Note that there are two transportation routes from Ukraine to Egypt in Figure 3.

The unit transportation costs for Ukraine to Egypt are:

$$c_{13}^{1}(Q) = .000246Q_{13}^{1} + 14,446.7000, \quad c_{13}^{2}(Q) = .000428Q_{13}^{2} + 13,000.6000.$$

In this example, $\beta_{13}^1 = \beta_{13}^2 = .9$.

The demand price function for Egypt is:

$$\rho_{13}(Q,\hat{q}) = -.000216d_{13} + .2\frac{.9q_1Q_{13}^1 + .9q_1Q_{13}^2}{Q_{13}^1 + Q_{13}^2} + 10,000.6000.$$

The exchange rate $e_{13} = .5236$.

The modified projection method computes the following equilibrium solution:

$$Q_{12}^{1*} = 231, 322.7188, \quad Q_{12}^{2*} = 102, 043.2969,$$

 $Q_{13}^{1*} = 0.0000, \quad Q_{13}^{2*} = 2,996, 647.7500.$
 $q_1^* = 1,000.0000,$

with corresponding supply and demands of:

$$s_1^* = 3,330,013.7500, \quad d_{12}^* = 333,366.0000, \quad d_{13}^* = 2,996,647.7500.$$

The supply price of a ton of wheat in hryvnia is now: 3,921.7124. The demand price in Lebanon in Lebanese pounds is: 768,916.5000 whereas the demand price in Egypt in Egyptian pounds is: 9,533.3232. The supply price has increased as compared to the value in Example 4. The supply of wheat, with an additional demand market, has increased about sixfold. The average quality of the wheat in both Lebanon and Egypt is 900.0000.

Examples 6, 7, and 8: Sensitivity Analysis for Ad Valorem Equivalents

Example 6 has the same data as that in Example 5 except that now we have an ad valorem equivalent of .05. Please see equilibrium conditions (11). We set $AVE_{12} = AVE_{13} = .05$.

Example 7 also has the same data as Example 5 but now we increase the ad valorem equivalents so that: $AVE_{12} = AVE_{13} = .1$.

Example 8 has the same data as Example 5 but now we increase the ad valorem equivalents even further so that: $AVE_{12} = AVE_{13} = .15$.

In Figure 4, the supplies, demands, as well as the supply prices and the demand prices for Examples 5 through 8 are displayed graphically. In these examples, the quality remains at 1,000.0000 in Ukraine, with the average quality in both destination countries being: 900.0000.

As can be seen from the results in Figure 4, the supply of wheat from Ukraine decreases as the values of the AVEs increase. This has implications for global food security since wheat, as an input to bread, is essential for nutrition in Lebanon as well as in Egypt. Furthermore, one can also see from the results in Figure 4 that the demand prices for wheat in both Lebanon and Egypt increase as the AVEs increase, putting further stressors on food security.

Example 9

Example 9, in this set of examples, has the identical data to that of Example 8 except that now the minimum quality standard is no longer 1,000.0000 but is now set to 0.0000 (so, in effect, there is no minimum quality standard).

The modified projection method now yields the following equilibrium solution:

$$Q_{12}^{1*} = 0.00000, \quad Q_{12}^{2*} = 11,305.0000,$$

 $Q_{13}^{1*} = 0.0000, \quad Q_{13}^{2*} = 468,041.2188;$



Figure 4: Sensitivity Analysis for Ad Valorem Equivalents

$$q_1^* = 689.0679,$$

with corresponding supply and demands of:

$$s_1^* = 479,346.2188, \quad d_{12}^* = 11,305.0000, \quad d_{13}^* = 468,041.2188.$$

The supply price of a ton of wheat in hryvnia is: 3,445.3401. The demand price in Lebanon in Lebanese pounds is: 795,297.5000 whereas the demand price in Egypt in Egyptian pounds is: 10,023.5352. The average quality of the wheat in both Lebanon and Egypt is now: 620.1611 and with the equilibrium quality of the wheat in Ukraine being: 689.0679. The supply of wheat in Ukraine in Example 8 was: 575,444.2500, whereas in Example 9 it is: 479,346.2188. The demands in Lebanon and Egypt, in turn, were, respectively, in Example 8: 10,382.2617 and 565,062.0000. The removal of the minimum quality standard results in a decrease in the supply of wheat in Ukraine.

Examples 10, 11, 12, and 13: Sensitivity Analysis for Quality Coefficient in Supply Price Function

In Examples 10 through 13, we investigate the impacts of increasing the quality coefficient in the supply price function of wheat in Ukraine. In the previous examples, the coefficient was .001. We now conduct sensitivity analysis by changing the coefficient to: .01, .1, .2, and .3 in Example 9. The computed supplies and demands in tons of wheat as well as the supply and the demand prices are displayed in Figure 5 with the results for the coefficient equal to .001 also included. In Ukraine, because of the war, it has been more challenging to acquire fertilizer and many of the agricultural fields have been mined by the Russians plus challenges with farm labor remain. These all can factor in to a higher cost associated with quality in terms of the production of wheat, a critical agricultural food staple for many in developing (and also other) countries.



Figure 5: Sensitivity Analysis for Supply Price Quality Coefficient

As can be seen from the results in Figure 5, with the costs associated with wheat quality increasing, as reflected in the supply price for wheat in Ukraine, the supply of wheat decreases while the supply price in Ukraine increases as do the demand prices in both Lebanon and Egypt. These results emphasize that increasing costs associated with producing commodities of quality, such as wheat, as can occur in wartime and also with challenges associated with climate change, food insecurity can be expected to rise, due to a decrease in supply of important agricultural commodities plus a increase in their prices. The computed equilibrium quality increases as does the average quality of the wheat in both Lebanon and Egypt. In Example 10, the equilibrium quality is $q_1^* = 690.0952$ with the equilibrium quality being: 700.5393, 712.5219, and 724.9244 in Examples: 11, 12, and 13, respectively.

Set 3: Subsidies

In Set 3 of numerical examples, we explore the impacts of NTMs in the form of subsidies.

Examples 14, 15, 16, and 17

In this set of examples, we are interested in evaluating (and quantifying) what might be the effects of subsidies that the government of Ukraine or an international organization could provide farmers in Ukraine growing wheat. As we have seen in the Set 2 examples, AVEs can impose additional cost pressures with the consequences being higher prices as well as lower commodity shipment volumes, both of which play a role in exacerbating food insecurity.

Examples 14 through 17 have the same data as that in Example 13, except that now subsidies of the form $sub_{12} = sub_{13} = 100, 200, 300, 400$ are added to construct Examples 14 through 17, respectively.

Here we find something quite interesting - providing such subsidies increases the equilibrium quality of the wheat with: $q_1^* = 728.7834$ in the case of a subsidy of 100; $q_1^* = 732.6473$ in the case of a subsidy of 200; $q_1^* = 736.5016$ in the case of a subsidy of 300, and $q_1^* = 740.3654$ in the case of a subsidy of 400. The average quality of the wheat received in Lebanon and in Egypt from Ukraine, of course, also increases and this is good for the consumers. Of course, the increase in the quality of wheat may be affected not only by policy interventions but also by the natural conditions of the soil (including fertilizers). Farmers can use the available support from the government to improve soil quality and make use of soil amelioration, thereby affecting commodity quality indirectly. In addition, the result may be even more pronounced for some agricultural products and foodstuffs that depend on the quality of inputs, for example.

In Figure 6, for completeness, we display the supply of wheat, the demands in Lebanon and Egypt, and the supply and demand prices for Examples 13 through 17.

As can be seen from the results displayed in Figure 6, subsidizing the wheat benefits both producers and consumers. In addition to the already mentioned increases in the quality of wheat and the average quality of the wheat in both Lebanon and Egypt from Ukraine, we see that the supply of wheat increases, with the supply price increasing but the demand prices decreasing as the subsidies are increased. Hence, farmers in Ukraine benefit from the increase in supply price and they also have an increase in supply whereas consumers benefit in Lebanon and Egypt from reduced prices and increased tons of wheat shipped. These results are quite powerful and speak to the benefits of subsidizing agriculture and



Figure 6: Sensitivity Analysis for Subsidy on Wheat in Ukraine

food production in Ukraine during wartime with results expecting to hold also in times of peace. The subsidies may be a short-term solution to the existing hike in transportation costs in the war. Once the war is over and the most cost-efficient routes can again be used for export shipments, there may be no need for further such government support.

Set 4: Quotas

We now turn to quantifying impacts associated with the imposition of quotas.

Example 18: Quotas

Example 18 has the identical data to that of Example 17 except that now we consider the scenario that the Ukrainian government has imposed quotas on shipments of wheat to both Lebanon and Egypt. Such quotas during wartime can occur because of concern regarding insufficient food for one's own citizens because of disruptions, perishability, etc. For example, there have been multiple cases of Russia destroying grain silos in Ukraine.

The quotas are as follows: $\bar{Q}_{12} = 15,000.0000$ and $\bar{Q}_{13} = 500,000.0000$.

Note that Example 18, since it is constructed from Example 17, includes quality, AVEs,

subsidies, and also quotas (all associated with NTMs).

The modified projection method converges to the following equilibrium solution:

$$Q_{12}^{1*} = 0.00000, \quad Q_{12}^{2*} = 15,000.0000,$$

 $Q_{13}^{1*} = 0.0000, \quad Q_{13}^{2*} = 500,000.0000;$
 $q_1^* = 689.0679,$

with corresponding supply and demands of:

$$s_1^* = 515,000.0000, \quad d_{12}^* = 15,000.0000, \quad d_{13}^* = 500,000.0000.$$

The supply price of a ton of wheat in Ukraine in hryvnia is: 3,670.8564. The demand price in Lebanon in Lebanese pounds is: 794,998.5625 whereas the demand price in Egypt in Egyptian pounds is: 10,024.7500. The average quality of the wheat in both Lebanon and Egypt is now: 660.7542 with the equilibrium quality of the wheat in Ukraine being: 734.1713.

The quota constraints for both Lebanon and Egypt are tight and, hence, the associated Lagrange multipliers are both positive with $\lambda_{12}^* = 5,297.4893$ and $\lambda_{13}^* = 98.0810$. The values of the Lagrange multipliers suggest that, if, given an option, the quota on Lebanon should be relaxed before the quota on Egypt due to the higher value of the Lagrange multiplier associated with the quota for Lebanon.

In Example 17, the equilibrium quality of wheat in Ukraine was: 740.3654, whereas under the quotas in Example 18, the equilibrium quality is now: 689.0679, so the quality decreases as does, of course, the average quality of the wheat in both Lebanon and Egypt. Also, under the quotas, the volumes of wheat shipments decrease: from 19,706.8809 tons of wheat to Lebanon to 15,000.000, and from 669,620.3125 tons in Example 17 to Egypt to only 500,000.0000 tons in Example 18. The demand prices of the wheat in Example 17 in Lebanon and in Egypt were, respectively: 794,613.1875 and 9,989.2275. Hence, the demand prices are higher under the quotas. With fewer tons of the wheat delivered to these MENA countries under the quotas, and at higher prices and of lower quality, such policies can lead to rising food insecurity.

6. Summary, Conclusions, and Suggestions for Future Research

In this paper, we proposed a holistic, general spatial price equilibrium model for international trade with a focus on agricultural products that are subject to non-tariff measures, which are widely applied by governments in this important sector. The modeling and algorithmic framework is based on the theory of variational inequalities. The model extends many earlier supply price equilibrium models to include: exchange rates, ad valorem equivalents (AVEs), which can quantify sanitary and phytosanitary measures, subsidies, and quotas, along with minimum quota standards, while also handling commodity quality and possible deterioration of quality along transportation routes. At the same time, the underlying economic functions are very general, which expands the possible scope of applications.

The international trade spatial price equilibrium model is then illustrated through a series of numerical examples, which also include sensitivity analysis results. Having a rigorous computational mathematical model allows for the determination of both quantitative and qualitative impacts of NTMs, which are policies, on the supplies of commodities, the volumes transported, and the demands, along with the supply prices and the demand prices and the quality of the commodities. Such information is also valuable for assessing the effects of various such policies on food security. Our numerical examples are inspired by the ongoing major invasion of Russia on Ukraine, which is often called the world's breadbasket, and whose wheat is heavily relied upon by, among others, the MENA countries. The results in this paper are also relevant to governmental decision makers and policy makers. We find that as the values of AVEs increase, the supply of wheat from Ukraine decreases, and the demand prices in the MENA countries increase. On the other hand, we find that providing subsidies to producers of wheat in Ukraine increases the quality of the wheat as well as the volume of wheat from Ukraine shipped to Lebanon and Egypt, while the demand prices decrease, and this is good for food security. Quotas, on the other hand, can result not only in lower shipments to the demand markets, but also in these wheat shipments being of lower quality. Since multiple NTMs are often imposed simultaneously in practice, our multicommodity spatial price equilibrium model is relevant to practice and can reveal unexpected results.

There are many possibilities for future research. It would be interesting to construct imperfectly competitive models of oligopolistic competition and NTMs. It would also be worthwhile to develop models, both perfectly competitive and imperfectly competitive ones, with routes consisting of multiple links through different countries and possible distinct associated NTMs that may be applicable. In addition, constructing models with more refined demand price functions for commodities of different quality levels would also be valuable. Finally, investigating the inappropriate use of NTMs by governments, for political purposes, for example, may yield important insights.

Acknowledgments

The authors are grateful to the partnership established between the University of Massachusetts Amherst and the Kyiv School of Economics and the Virtual Scholar Program.

Author Declarations

Funding

The second author was funded, in part, through the University of Massachusetts Amherst - Kyiv School of Economics partnership with funding provided by the former university.

Conflicts of interest/ Competing interests

None.

Ethics approvals

Not applicable.

Availability of data

All data are provided in the article.

References

Al Jazeera, 2022. Russian forces launch full scale invasion of Ukraine. February 24. Available at:

https://www.aljazeera.com/news/2022/2/24/putin-orders-military-operations-in-eastern-uk

Arita, S., Beckmann, J., Mitchell, L., 2016. Sanitary and phytosanitary measures and technical barriers to trade: How much to they impact U.S.-EU agricultural trade? US Department of Agriculture, Economic Research Service, August 1, Washington DC. Available at: https://www.ers.usda.gov/amber-waves/2016/august/sanitary-and-phytosanitary-measures-and

Armington, P.S., 1969. A theory of demand for products distinguished by place of production. IMF Staff Papers 16, 159-178.

Assoua, J.E., Molua, E.L., Nkendah, R., Fani, D.C.R., Tabetando, R., 2022. The effect of sanitary and phytosanitary measures on Cameroon's cocoa exports: An application of the gravity model. Heliyon Jan 13; 8(1): e08754.

Besedina, E., 2015. Non-tariff measures in the context of export promotion policies. Free Network, May 11. Available at:

https://freepolicybriefs.org/2015/05/11/non-tariff-measures-in-the-context-of-export-pr

Besik, D., Nagurney, A., 2017. Quality in competitive fresh produce supply chains with application to farmers' markets. Socio-Economic Planning Sciences 60, 62-76.

Besik, D., Nagurney, A., Dutta, P., 2023. An integrated, multitiered supply chain network model of competing agricultural firms and processing firms: The case of fresh produce and quality. European Journal of Operational Research 307(1), 364-381.

Cadot, O., Gourdon, J., van Tongeren, F., 2018. Estimating ad valorem equivalents of nontariff measures: Combining price-based and quantity-based approaches. OECD Trade Policy Papers, No. 215, OECD Publishing, Paris.

Colajanni, G., Daniele, P., Sciacca, D., 2022. On the provision of services with UAVs in disaster scenarios: A two-stage stochastic approach. Operations Research Forum Article Number 18.

Dafermos, S., Nagurney, A., 1984. Sensitivity analysis for the general spatial economic equilibrium problem. Operations Research 32, 1069-1086.

Daniele, P., 2004. Time-dependent spatial price equilibrium problem: Existence and stability results for the quantity formulation model. Journal of Global Optimization 28(3-4), 283-295.

Devadoss, S., Sabala, E., 2020. Effects of yuan-dollar exchange rate changes on world cotton markets. Journal of Agriculture and Applied Economics 52(3), 420-439.

Dong, F., Jensen, H.H., 2007. Challenges for China's agricultural exports: Compliance with sanitary and phytosanitary measures. Choices, 1st Quarter 22(1), 19-24.

FAO, 2022. The state of agricultural commodity markets 2022: The geography of food and agricultural trade: Policy approaches for sustainable development. Rome, Italy.

Florian, M., Los, M., 1982. A new look at static spatial price equilibrium models. Regional Science and Urban Economics 12, 579-597.

Ghodsi, M., 2023. Exploring the 'non-tariff measures black box': Whose regulatory NTMs on which products improve the imported quality? International Economics 173, 45-67.

Harker, P.T., Editor, 1985. Spatial Price Equilibrium: Advances in Theory, Computation and Application. Springer, Heidelberg, Germany.

Kravchenko, A., Strutt, A., Utoktham, C., Duval, Y., 2022. New price-based ad-valorem equivalent estimates of non-tariff measures. Journal of Global Economic Analysis 7(2), 38-65.

Li, D., Nagurney, A., Yu, M. 2018. Consumer learning of product quality with time delay: Insights from spatial price equilibrium models with differentiated products. Omega 81, 150-168.

Lopez, S.A.I., Rau, M.L., Woltjer, G., 2019. Exploring alternative approaches to estimate the impact of non-tariff measures and further implementation in simulation models. Joint Research Centre (JRC) technical report EUR 29578 EN Publications Office of the European Union, Luxembourg.

Ma, M., Li, Y., Xue, C., Xiong, W., Peng, Z., Han, X., Ju, H., He, Y., 2021. Current situation and key parameters for improving wheat quality in China. Frontiers in Plant Science 12: 638525.

Mathews, S., 2022. Lebanese buyer rejects first batch of Ukrainian grain. Middle East Eye, August 8. Available at:

https://www.middleeasteye.net/news/lebanese-buyer-rejects-first-batch-ukrainian-grain

Nagurney, A., 1999. Network Economics: A Variational Inequality Approach, second and revised edition. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nagurney, A. 2008. Spatial price equilibrium. In: C. Floudas and P. Pardalos, Editors, Encyclopedia of Optimization. Springer, Boston, MA, pp 3646-3652.

Nagurney, A., 2021. A multiperiod supply chain network pptimization model with investments in labor productivity enhancements in an 3ra of COVID-19 and climate change. Operations Research Forum Article Number 68.

Nagurney, A., 2022. Spatial price equilibrium, perishable products, and trade policies in the Covid-19 pandemic. Montes Taurus J. Pure Appl. Math. 4(3), 9-24.

Nagurney, A., Aronson, J., 1988. A general dynamic spatial price equilibrium model: formulation, solution, and computational results. Journal of Computational and Applied Mathematics 22, 339-357.

Nagurney, A., Besik, D., 2022. Spatial price equilibrium networks with flow-dependent arc multipliers. Optimization Letters 16, 2483-2500.

Nagurney, A., Besik, D., Dong, J., 2019. Tariffs and quotas in world trade: A unified variational inequality framework. European Journal of Operational Research 275(1), 347-360.

Nagurney, A., Besik, D., Yu, M., 2018. Dynamics of quality as a strategic variable in complex

food supply chain network competition: The case of fresh produce. Chaos 28, 043124.

Nagurney, A., Hassani, D., Nivievskyi, O., Martyshev, P., 2023. Exchange rates and multicommodity international trade: Insights from spatial price equilibrium modeling with policy instruments via variational inequalities. Journal of Global Optimization DOI https://doi.org/10.1007/s1089 023-01292-x.

Nagurney, A., Li, D., 2016. Competing on Supply Chain Quality: A Network Economics Perspective. Springer International Publishing Switzerland.

Nagurney, A., Li, D., Nagurney, L.S., 2014. Spatial price equilibrium with information asymmetry in quality and minimum quality standards. International Journal of Production Economics 158, 300-313.

Nagurney, A., Nicholson, B.F., Bishop, P.M., 1996. Spatial price equilibrium models with discriminatory ad valorem tariffs: Formulation and comparative computation via variational inequalities. In: Recent Advances in Spatial Equilibrium Modelling, J.C.J.M. Bergh, P. Nijkamp, and P. Rietveld, Editors, Springer, Berlin, Germany, pp 179-200.

Nagurney, A., Salarpour, M., Dong, J., 2022. Modeling of Covid-19 trade measures on essential products: A multiproduct, multicountry spatial price equilibrium framework. International Transactions in Operational Research 29(1), 226-258.

Nagurney, A., Salarpour, M., Dong, J., Nagurney, L.S., 2020. A stochastic disaster relief game theory network model. Operations Research Forum Article Number 10.

Nagurney, A., Takayama, T., Zhang, D., 1995. Massively parallel computation of spatial price equilibrium problems as dynamical systems. Journal of Economic Dynamics and Control 18, 3-37.

Nagurney, A., Thore, S., Pan, J., 1996. Spatial market models with goal targets. Operations Research 44, 393-406.

Nagurney, A., Yu, M., Masoumi, A.H., Nagurney, L.S., 2013. Networks Against Time: Supply Chain Analytics for Perishable Products. Springer New York.

Nivievskyi, O., 2022. Russian War in Ukraine and Global Food Crisis [Webinar]. Public Policy Programs. The Behrend College, Pennsylvania State University Erie. Available at: https://behrend.psu.edu/school-of-humanities-social-sciences/research-outreach/public-policyfund/programs

OECD, 2019. Non-tariff measures. Available at:https://www.oecd.org/trade/topics/

non-tariff-measures/

Samuelson, P.A., 1952. Spatial price equilibrium and linear programming. American Economic Review 42, 283-303.

Sarker, R., Surry, Y., 2005. Product differentiation and trade in agri-food products: Taking stock and looking forward. An invited paper presented on the Theme Day, "Modeling Food and Agricultural Markets" at the 2005 Annual Meeting of the International Agricultural Trade Research Consortium in San Diego California, December 4-6.

Shepotylo, O., 2022. Non-tariff measures and productivity of Ukrainian food processing firms. Vox Ukraine, March 3. Available at:

https://voxukraine.org/en/non-tariff-measures-and-productivity-of-ukrainian-food-proces

Shepotylo, O., Vakhitov, V., Movchan, V., Panga, P., 2022. Non-tariff measures and productivity of Ukrainian food-processing firms. Journal of Agricultural Economics 73(1), 234-256.

Takayama, T., Judge, G.G., 1964. Spatial equilibrium and quadratic programming. Journal of Farm Economics 46(1), 67-93.

Takayama, T., Judge, G.G., 1971. Spatial and Temporal Price and Allocation Models. North-Holland Publishing Company, Amsterdam, The Netherlands.

UN News, 2022. Black Sea grain exports deal 'a beacon of hope' amid Ukraine war - Guterres, July 22. Available at:

https://news.un.org/en/story/2022/07/1123062

UNCTAD, 2010. Non-tariff measures: Evidence from selected developing countries and future research agenda. United Nations publication, New York and Geneva, p. 99.

UNCTAD, 2022. Non-tariff measures from A to Z. United Nations.

UNCTAD and The World Bank, 2017. The unseen impact of non-tariff measures: Insights from a new database. United Nations. Available at:

https://unctad.org/meetings/en/SessionalDocuments/ditc-tab-MC11-UNCTAD-NTMs.pdf

USDA, 2022. Ukraine agricultural production and trade fact sheet. Foreign Agricultural Service, April, Washington DC.

Whitworth, J., 2022. Ukraine tells of war impact on food safety at meeting of World Trade Organization. Food Safety News, July 8. Available at:

https://www.foodsafetynews.com/2022/07/ukraine-tells-of-war-impact-on-food-safety-at-me

World Trade Organization, 1998. Understanding the WTO Agreement on Sanitary and Phytosanitary Measures. May. Available at:

https://www.wto.org/english/tratop_e/sps_e/spsund_e.htm