An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England

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Abstract: In this paper, we develop a novel electric power supply chain network model with fuel supply markets that captures both the economic network transactions in energy supply markets and the physical network transmission constraints in the electric power network. The theoretical derivation and analysis are done using the theory of variational inequalities. We then apply the model to a specific case, the New England electric power supply chain, consisting of 6 states, 5 fuel types, 82 power generators, with a total of 573 generating units, and 10 demand market regions. The empirical case study demonstrates that the regional electric power prices simulated by the proposed model very well match the actual electricity prices in New England. We also compute the electric power prices and the spark spread, an important measure of the power plant profitability, under natural gas and oil price variations. The empirical examples illustrate that in New England, the market/grid-level fuel competition has become the major factor that affects the influence of the oil price on the natural gas price. Finally, we utilize the model to quantitatively investigate how changes in the demand for electricity influence the electric power and the fuel markets from a regional perspective. The theoretical model can be applied to other regions and multiple electricity markets under deregulation to quantify the interactions in electric power/energy supply chains and their effects on flows and prices.

1. Introduction

Electric power systems provide a critical infrastructure for the functioning of our modern economies and societies. Electric power lights (and cools) our homes, our commercial and industrial enterprises, powers our computers, and enables the production and dissemination of goods and services worldwide. It is undeniably an essential form of energy, whose absence and/or unavailability, can have profound and lasting impacts in both the developed and developing corners of the globe.

In order to understand the availability and, ultimately, reliability and vulnerability of electric power and the underlying systems, one must place the systems in the context of the industrial setting. For example, in the case of the United States, the electric power industry possesses more than half a trillion dollars of net assets, generates $220 billion in annual sales, and consumes almost 40% of domestic primary energy [21, 22]. Currently, the electric power industry in the US is undergoing a deregulation process from once highly regulated, vertically integrated monopolistic utilities to emerging competitive markets [32, 37, 84]. This deregulation process has caused major changes to the electric power industry, and requires a deep and thorough identification of the structure of the emerging electricity supply chains, as well as new paradigms for the modeling, analysis, and computations for electric power markets.

Smeers [77] reviewed a wide range of models of energy markets with various market power assumptions (see also, [1, 9, 10, 30, 56, 70, 76, 84, 85]). Hogan [36] proposed a market power model to study strategic interactions in an electricity transmission network. More recently, Chen and Hobbs [12] proposed an oligopolistic electricity market model with a nitrogen oxide permit market, and provided examples based on the PJM market (Pennsylvania, New Jersey, and Maryland). Neuhoff et al. discussed a variety of important issues and assumptions regarding the Cournot equilibria of deregulated power markets [71]. Nagurney and Matsypura [65], in turn, presented an electric power supply chain network model which provided an integrated perspective for electric power generation, supply, transmission, as well as consumption. Wu et al. [83] considered the generators' generating unit portfolios and reformulated the electric power supply chain network model as a user-optimal transportation network model (see also [63]). Nagurney et al. [64] also established the connections between electric power supply chain networks and transportation networks, and developed a dynamic electric power supply chain network with time-varying demands.

This paper focuses on the relationship and interaction between electric power supply chains and other energy markets. In the US, electric power generation accounts for 30% of the natural gas demand (over 50% in the summer), 90% of the coal demand, and over 45% of the residual fuel oil demand [23]. Moreover, in the US natural gas market, for the past four years, the demand from electric power generation has been significantly and steadily increasing while demands from all the other sectors (industrial, commercial, and residential) have been slightly decreasing [24].

For example, in New England (the northeastern region of the US consisting of the states of Connecticut, Massachusetts, Rhode Island, Vermont, New Hampshire, and Maine), since 1999, approximately 97 percent of all the newly installed generating capacity has depended partially or entirely on natural gas [38]. Hence, various energy markets are inevitably and constantly interacting with electric power supply chains. For instance, from December 1, 2005 to April 1, 2006, the wholesale electricity price in New England decreased by 38% mainly because the delivered natural gas price declined by 45% within the same period. For another example, in August, 2006, the natural gas price jumped 14% because hot weather across the US led to elevated demand for electricity. This high electricity demand also caused the crude oil price to rise by 1.6% [33]. Similarly, the natural gas future price for September 2007 increased by 4.7% mainly because of the forecasted high electricity demands in Northeastern and Mid-western cities due to rising temperatures [72]. However, the quantitative connections between electric power supply chains and other fuel markets are not straightforward and depend on many factors, such as: the generating unit portfolios of power generating companies or generators (gencos), the technological characteristics of generating units as well as the the underlying physical transportation/transmission networks (with their associated capacities).

Moreover, the availability and the reliability of diversified fuel supplies affect not only economic efficiency but also national security. For example, in January 2004, over 7000MW (megawatts or one million watts) of electric power generation, which accounts for almost one fourth of the total capacity of New England, was unavailable during the electric system peak due to the limited natural gas supply [42]. For another instance, the American Association of Railroads has requested that the Federal Energy Regulatory Commission (FERC) investigate the reliability of the energy supply chain with a focus on electric power and coal transportation [6].

The relationships between electric power supply chains and other energy markets have drawn considerable attention from researchers in various fields. Emery and Liu [20] empirically estimated the cointegration of electric power futures and natural gas futures. Routledge, Seppi, and Spatt [73] focused on the connections between natural gas and electricity markets, and studied the equilibrium pricing of electricity contracts (see also [5]). Deng, Johnson, and Sogomonian [18] applied real option theory to develop models that utilize the relationship between fuel prices and electricity prices to value electricity generation and transmission assets. Huntington and Schuler [39] pointed out that historically, the oil price (\$/barrel) and the natural gas price (\$/MMBtu) had a 10:1 relationship because of the responsiveness of dual-fuel generating units in electric power networks (see also [2, 8]). However, due to the decline of the number of dual-fuel plants and the deregulation of the electric power industry, the oil and natural gas prices have decoupled and become complex [34, 7]. The interesting interactions among oil, electric power, and natural gas markets will also be quantitatively investigated using our theoretical model and empirical examples. We note that Matsypura, Nagurney, and Liu [55] proposed the first network model that integrated fuel supplier networks and electric power supply chain networks. However, their model focused on the transactions of the electricity ownership and the economic decision-making processes of the market participants, and did not consider the physical constraints in electricity transmission networks and the electricity demand variations. Furthermore, no empirical results were presented.

In this paper, we develop a novel electric power network model with fuel supply markets that considers both the economic network transactions in energy markets and the physical network transmission constraints in the electric power network which are critical to the understanding of the regional differences in electric power prices [10, 11, 35]. We then apply the model to a specific case, the New England electric power supply chain. In particular, in Example 1 of the case study, we demonstrate that the regional electric power prices simulated by our model match the actual electricity prices in New England very well. In Example 2 of the case study, we utilize the model to compute the spark spread, an important measure of power plant profitability, under natural gas and oil price variations. This example shows how our model can be used to evaluate the profitability and risks of various types of power plants. Additionally, in Example 3, we use our model to show that in New England, the grid/market level fuel competition has become the major factor that determines the influence of oil prices on natural gas prices. The results of Example 3 contribute to the literature regarding the relationship between oil and natural prices [2, 7, 8, 34, 39, 81]. Finally, in Example 4, we utilize the model to quantitatively investigate how changes in the demand for electricity influence the electric power and the fuel markets.

This paper is organized as follows. In Section 2, we propose the integrated electric power supply chain and fuel supply network model. In Section 3, we present a case study of the model applied to the New England electric power supply chain network. The empirical application consists of 6 states, 5 fuel types, 82 power generators, with a total of 573 generating unit and generator combinations, and 10 demand market regions. Section 4 summarizes and concludes the paper and presents suggestions for future research. We provide some qualitative properties of the model and discuss the computation of solutions to the model in the Appendix.

2. The Integrated Electric Power Supply Chain and Fuel Supply Market Network Model

In this section, we develop the electric power supply chain network model which includes regional electricity markets and fuel supply markets.

2.1 A Brief Introduction

The electric power supply chain network model proposed in this paper includes four major components: the fuel supply markets, the power generators, the power buyers/consumers at demand markets, and the independent system operator (ISO).

The power generators or gencos purchase fuels from the supply markets and produce electric power at the generating units. Gencos can sell electric power to the power buyers/consumers at the demand markets directly through bilateral contracts or they can sell to the power pool which is managed by the ISO. Additionally, gencos can also sell their capacities in the regional operating reserve markets. Each electric power generator seeks to determine the optimal production and allocation of the electric power in order to maximize its own profit.

The power buyers/consumers at the demand markets search for the lowest electricity price. They can purchase electric power either directly from the gencos or from the power pool. For example, in the New England electric power supply chain, about $75\% - 80\%$ of electricity is traded through bilateral contracts, while about $20\% - 25\%$ of electricity is traded through the power pool [50]. Thus, the power pools function as markets that balance the residuals of supply and demand, and clear the regional wholesale electricity markets [50].

In most deregulated electricity markets, there is a non-profit independent system operator (ISO) whose major role is to ensure system reliability, and to develop and oversee the wholesale electricity market (e.g. New England ISO, www.iso-ne.com; PJM Interconnection, www.pjm.com; and Electric Reliability Council of Texas, www.ercot.com). Additionally, the ISO schedules all transmission requests and monitors the entire transmission network; it also charges the network users congestion fees if certain transmission interface limits are reached [10, 11, 35]. Moreover, the ISO manages the operating reserve markets where the power generators can get paid for holding back their capacities to help to ensure system reliability. These major features of the ISO are fully reflected in our model.

We let $1, \ldots, a, \ldots, A$ denote the types of fuels. We assume that there are M supply markets for each type of fuel. We assume that the electric power supply chain network includes $1, \ldots, r, \ldots, R$ regions which can be defined based on electricity transmission network interfaces. We let $1, \ldots, g, \ldots, G$ denote the gencos who may own and operate multiple generating units which may use different generating technologies and are located in various regions. For example, the genco, Con Edison Energy, owns one generating unit in New Hampshire, one generating unit in southeastern Massachusetts, and six generating units in western and central Massachusetts [44]. In particular, we let N_{gr} denote the number of generating units owned by genco g in region r. If genco g has no generating unit in region r, then $N_{gr} = 0$. We let $N \equiv \sum_{g=1}^{G} \sum_{r=1}^{R} N_{gr}$ denote the total number of generating units in the network. We assume that in each region there exist K demand market sectors which can be distinguished from one another by the types of associated consumers and the electricity consumption patterns.

The top-tiered nodes in the electric power supply chain network in Figure 1 represent the AM fuel supply markets. The nodes at the second tier in Figure 1 represent the generating units associated with the gencos and regions. The three indices of a generating unit indicate the genco that owns the unit, the region of the unit, and the sequential identifier of the unit, respectively. For example, in region 1, the node on the left denotes genco 1's first generating unit in region 1 while the node on the right denotes genco G's ${N_{G1}}^{th}$ (last) generating unit in region 1. The bottom-tiered nodes in Figure 1 represent the RK region and demand market combinations.

Our model focuses on a single period, the length of which can be from several days to a month. We assume that the fuel prices at the fuel supply markets are relatively stable and do not change throughout the period. However, the electric power demands and prices may exhibit a strong periodic pattern within a day with the highest price typically being two to four times higher than the lowest price. Hence, we allow the electric power prices and demands to vary frequently within the study period. Note that there are two typical ways to represent electric power demand variations: load curves and load duration curves. A load curve plots electricity demands in temporal sequence while a load duration curve sorts and plots demand data according to the magnitude of the demands (e.g. [31, 53]). A point on the load duration curve represents the proportion of time that the demand is above certain level.

In this paper, we use the discretized load duration curve to represent the demand variations within a period. In particular, we divide the load duration curve of the study period into $1, \ldots, w, \ldots, W$ blocks with L_w denoting the time length of block w. In our empirical examples, using the New England electric power supply chain network as a case study in Section 3, we assume that there are six demand levels, consisting of two peak demand levels, two intermediate demand levels, and two low demand levels; hence, $W = 6$.

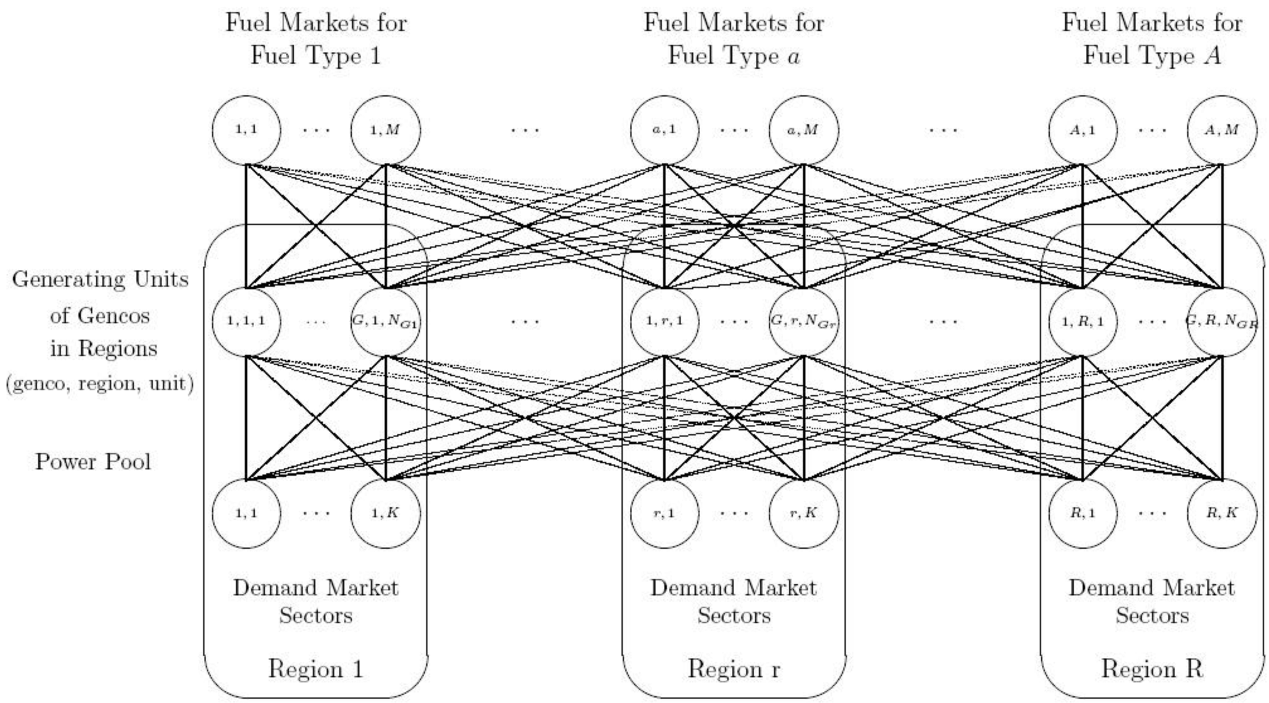

Figure 1: The Electric Power Supply Chain Network with Fuel Supply Markets

We now summarize the critical assumptions of the model:

1. The model focuses on a single period, the length of which can be from several days to a month.

2. The fuel prices are relatively stable and do not fluctuate within the study period while the regional electricity prices fluctuate significantly as the demands vary within the study period. Indeed, in most deregulated electric power markets, the highest electricity price on a day is usually two to four times higher than the lowest electricity price on the same day.

3. The regions can be defined based on the transmission network interfaces. We used the linearized direct current (dc) network to approximate the electricity transmission network. The dc approximation has been widely utilized in congestion management in electricity transmission [10, 11, 12, 13, 35]. For a discussion regarding the dc approximation, see Cheng and Overbye [13].

4. The model assumes perfect competition in the electric power market. The empirical case study in this paper shows that this assumption is appropriate for the New England electric power market since the simulated prices match the actual prices. Additionally, the empirical study by Chen and Hobbs [12] also showed perfect competition to be a reasonable assumption for the Pennsylvania-New Jersey-Maryland electricity market (PJM).

5. Each genco can own and operate multiple generating units which may use different technologies and are located in different regions. A genco is an economic entity and is not restricted to a specific region while a genco's generating units are physically related to regions.

6. Each genco maximizes its own profit. The decisions made by each genco include the quantities of fuels purchased, the production level of each of its generating units at each demand level, the sales of electricity to the power buyers/consumers and to the power pool at each demand level, and the amount of capacity sold at the operating reserve market.

7. Power buyers/consumers search for the lowest electric power price.

8. Power buyers/consumers can purchase electricity directly from the gencos through bilateral contracts or from the power pool. In the electric power supply chain network

equilibrium, the sales of bilateral transactions are mutually determined by gencos and power buyers/consumers.

9. The ISO is a non-profit organization that maintains a competitive wholesale electricity market and ensures system security and reliability.

10. The ISO develops and oversees competitive power pools as well as maintains market clearance at each power pool.

11. The ISO schedules all transmission requests, monitors the transmission network, and charges congestion fees if certain transmission interface limits are reached.

12. The ISO manages operating reserve market to ensure system reliability. The above assumptions regarding the ISO are made based on the descriptions on the ISO New England website (www.iso-ne.org).

In this paper, our model is developed based on the variational inequality theory. The finite-dimensional variational inequality problem, $VI(F,\mathcal{K})$, is to determine a vector $X^* \in \mathcal{K} \subset R^n$, such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{1}$$

where F is a given continuous function from $\mathcal{K}$ to R^n, $\mathcal{K}$ is a given closed convex set, and $< \cdot, \cdot >$ denotes the inner product in Euclidian space.

The variational inequality formulation allows for a unified treatment of equilibrium and optimization problems, and is closely related to many mathematical problems, such as, constrained and unconstrained optimization problems, fixed point problems, and complementarity problems. For a complete introduction of the finite-dimensional variational inequality theory, we refer the readers to the book by Nagurney [57]. For studies regarding variational inequalities and dynamical systems, see [19, 66]. For more applications of variational inequalities in the areas of transportation, supply chains, and electric power networks, see [58, 59, 64, 65].

2.2 The Model of the Integrated Electric Power Supply Chain Network with Fuel Supply Markets

We now present the electric power supply chain network model. We describe the behavior of the fuel suppliers, the gencos, the ISO, and the power buyers/consumers at demand markets. We then state the equilibrium conditions for the electric power supply chain network and provide the variational inequality formulation.

Before we introduce the model, we would like to explain the use of r_1 and r_2 in the notation of the model. In general, a transaction of electricity is related to two regions (nodes): the region of injection where the generating unit is located and the region of withdrawal where the power buyers/consumers are located. In order to avoid confusion of the two regions, we use r_1 and r_2 to represent the region of injection and the region of withdrawal, respectively.

In our notation system, the m^{th} supply market of fuel a is denoted by am; the u^{th} generating unit owned by genco g in region r_1 is denoted by gr_1u; and the k^{th} demand market sector in region r_2 is denoted by r_2k.

For succinctness, we give the notation for the model in Tables 1, 2, 3, and 4. An equilibrium solution is denoted by "*". In Table 2, we also explain who determines the values of which decision variables. In particular, Q^1, Y^1, and Z are the vectors of decision variables of the power generators, whereas Y^2 is the vector of decision variables determined by the power buyers/consumers. The vector Q^2, in turn, is determined by both the power generators and the power buyers/consumers. In equilibrium, the decisions of the power generators and the power buyers/consumers regarding Q^2 have to coincide, which means that the bilateral contract quantities are mutually determined by the power generators and the power buyers/consumers.

Under the perfect competition assumption, both the power generators and the power buyers/consumers are price takers. Hence, the equilibrium values of the prices: ρ_1, ρ_2, ρ_3, and ρ_4, denoted by "*", are not determined by individual generators or power buyers/consumers, but, rather, through the solution of the complete model (see also [16, 17, 54, 61, 64, 83]). After Theorem 1 we discuss how to recover these equilibrium prices from the equilibrium solution of the entire model. All vectors are assumed to be column vectors, except where noted. If a variable or cost function is related to the transaction of fuels, the superscript indicates the fuel market while the subscript indicates the generating unit and the demand level; if a variable or cost function is related to the bilateral transaction of electric power, the superscript indicates the generating unit while the subscript indicates the demand market and the demand level; and if a variable or cost function is related to electricity production or operating reserve, there is no superscript while the subscript indicates the generating unit as well as the demand level. The actual units for the prices, the demands, and the electric power flow transactions used in practice are explicated in Section 3, which contains the empirical case study and the associated examples.

Table 1: Parameters in the Electric Power Supply Chain Network Model

Notation	**Definition**
$c^{am}_{gr_1uw}$	The unit transportation/transaction cost between the m^{th} supply market of fuel a and the u^{th} generating unit of genco g in region r_1 at demand level w.
L_w	The time duration of demand level w.
$TCap_b$	The interface (flowgate) limit of interface b.
Cap_{gr_1u}	The generating capacity of the u^{th} generating unit of genco g in region r_1.
OP_{gr_1u}	The maximum level of operating reserve that can be provided by the u^{th} generating unit of genco g in region r_1.
OPR_{r_1w}	The operating reserve requirement of region r_1 at demand level w.
β_{gr_1ua} $(a \neq 0)$	The nonnegative conversion rate (the inverse of the heat rate) at the u^{th} generating unit of genco g in region r_1 if the generating unit utilizes fuel a; β_{gr_1ua} is equal to zero if the generating unit does not use fuel a. We assume that for units using renewable technologies, all β_{gr_1ua}s are equal to zero.
β_{gr_1u0}	The renewable unit indicator. β_{gr_1u0} is equal to one if the u^{th} generating unit of genco g in region r_1 utilizes renewable technologies and zero otherwise. For generating units using renewable technologies, all β_{gr_1ua}s are equal to zero.
$\alpha_{r_1r_2b}$	The impact of transferring one unit electricity from region r_1 to region r_2 on interface b. $\alpha_{r_1r_2b}$ is equal to $PTDF_{r_2b} - PTDF_{r_1b}$ where $PTDF_{rb}$ denotes the power transmission distribution factor of region (node) r for interface limit b. In particular, $PTDF_{rb}$ is defined as the quantity of power flow (MW) through the critical link of interface b induced by a 1 MW injection at node r [9, 10, 30].
d_{r_2kw}	The fixed demand at demand market sector k in region r_2 at demand level w.
κ_{r_2w}	The transmission loss factor of region r_2 at demand level w.

Table 2: Decision Variables in the Electric Power Supply Chain Network Model

Notation	**Definition**
Q^1	$AMNW$-dimensional vector of fuel flows between fuel supply markets and generating units within the entire period with component $magr_1uw$ denoted by $q^{am}_{gr_1uw}$ and denoting the transactions between the m^{th} supply market of fuel a and the u^{th} generating unit of genco g in region r_1 at demand level w. In equilibrium, Q^{1*} is determined by the power generators.
q_w	N-dimensional vector of the power generators' electric power outputs at demand level w with components q_{gr_1uw} denoting the power generation at the u^{th} generating unit of genco g in region r_1 at demand level w. We group the q_w at all demand levels w into vector q. In equilibrium, q^*_w is determined by the power generators.
Q^2_w	NRK-dimensional vector of electric power flows between generating units and demand markets at demand level w with component gr_1ur_2k denoted by $q^{gr_1u}_{r_2kw}$ and denoting the transactions between the u^{th} generating unit of genco g in region r_1 and demand market sector k in region r_2 at demand level w. We group the Q^2_w at all demand levels w into vector Q^2. In equilibrium, the quantities of bilateral contracts, Q^{2*}_w, are mutually determined by the power generators and the power buyers/consumers at the demand markets.
Y^1_w	NR-dimensional vector of electric power transactions between power generators and regional power pools at demand level w with component gr_1ur_2 denoted by $y^{gr_1u}_{r_2w}$ and denoting the transactions between generating unit u of generator g in region r_1 and the regional power pool in region r_2 at demand level w. We group the Y^1_w at all demand levels w into vector Y^1. In equilibrium, Y^{1*}_w is determined by the power generators.
Y^2_w	R^2K-dimensional vector of electric power transactions between demand markets and regional power pools at demand level w with component r_1r_2k denoted by $y^{r_1}_{r_2kw}$ and denoting the transactions between demand market sector k in region r_2 and the regional power pool in region r_1 at demand level w. We group the Y^2_w at all demand levels w into vector Y^2. In equilibrium, Y^{2*}_w is determined by the power buyers/consumers at the demand markets.
Z_w	N-dimensional vector of regional operating reserves with component gr_1u denoted by z_{gr_1uw} and denoting the operating reserve held by generating unit u of genco g in region r_1 at demand level w. We group the Z_w at all demand levels w into vector Z. In equilibrium, Z^{1*}_w is determined by the power generators.

Table 3: Endogenous Prices and Shadow Prices of the Electric Power Supply Chain Network Model

Notation	Definition
$\rho^{am}_{gr_1uw}$	The price paid by generating unit u of genco g in region r_1 at demand level w in transacting with the m^{th} supply market of fuel a. We group all $\rho^{am}_{gr_1uw}$s into vector ρ_1.
$\rho^{gr_1u}_{r_2kw}$	The unit electricity price received by generating unit u of genco g in region r_1 for the transaction with power buyers/consumers in demand market sector k in region r_2 at demand level w. We group all $\rho^{gr_1u}_{r_2kw}$s into vector ρ_2.
ρ_{r_2w}	The unit electricity price at location r_2 on the electricity pool market at demand level w. We group all ρ_{r_2w}s into vector ρ_3.
ρ_{r_2kw}	The unit electric power price paid by the buyers/consumers at demand market sector k in region r_2 at demand level w. We group all ρ_{r_2kw}s into vector ρ_4.
φ_{r_1w}	The unit price of capacity on the regional operating reserve market in region r_1 at demand level w. We group all φ_{r_1w}s into vector φ.
μ_{bw}	The unit congestion charge of interface (flowgate) b at demand level w. We group all μ_{bw}s into vector μ.
η_{gr_1uw}	The shadow price associated with the capacity constraint of generating unit u of generator g in region r_1 at demand level w. We group all η_{gr_1uw}s into vector η.
λ_{gr_1uw}	The shadow price associated with the maximum operation reserve constraint of generating unit u of generator g in region r_1 at demand level w. We group all λ_{gr_1uw}s into vector λ.
γ_{gr_1uw}	The shadow price associated with the production constraint of generating unit u of generator g in region r_1 at demand level w. We group all γ_{gr_1uw}s into vector γ.
θ_{gr_1uw}	The shadow price associated with the fuel conversion constraint of generating unit u of generator g in region r_1 at demand level w. We group all θ_{gr_1uw}s into vector θ.
$\psi^{gr_1u}_{r_2w}$	The marginal cost of generating unit u of generator g in region r_1 supplying electricity to the power pool in region r_2 at demand level w. We group all $\psi^{gr_1u}_{r_2w}$s into vector ψ.

Table 4: Cost and Price Functions of the Electric Power Supply Chain Network Model

Notation	**Definition**
$\pi_{am}(Q^1)$	The inverse supply function (price function) at the m^{th} supply market of fuel a.
$f_{gr_1uw}(q_{gr_1uw})$	The generating cost of generating unit u of genco g in region r_1 at demand level w.
$c^{gr_1u}_{r_2kw}(q^{gr_1u}_{r_2kw})$	The transaction/transmission cost incurred at generating unit u of genco g in region r_1 in transacting with demand market sector k in region r_2 at demand level w.
$c^{gr_1u}_{r_2w}(y^{gr_1u}_{r_2w})$	The transaction/transmission cost incurred at generating unit u of genco g in region r_1 in selling electricity to region r_2 through the power pool at demand level w.
$c_{gr_1uw}(z_{gr_1uw})$	The operating reserve cost at generating unit u of genco g in region r_1 at demand level w.
$\hat{c}^{gr_1u}_{r_2kw}(Q^2_w)$	The unit transaction/transmission cost incurred by power buyers in demand market sector k in region r_2 in transacting with generating unit u of genco g in region r_1 at demand level w.
$\hat{c}^{r_1}_{r_2kw}(Y^2_w)$	The unit transaction/transmission cost incurred by power buyers in demand market sector k in region r_2 when purchasing electricity from region r_1 through the power pool at demand level w.

The Equilibrium Conditions for the Fuel Supply Markets

We first describe the equilibrium conditions for the fuel supply markets. The typical fuels used for electric power generation include coal, natural gas, residual fuel oil (RFO), distillate fuel oil (DFO), jet fuel, and uranium. The gencos take into account the prices of fuels at energy markets and the transportation/distribution costs in making their economic decisions (see also [28, 29, 74]).

We use the inverse supply function (price function), $\pi_{am}(Q^1)$, to model the fuel price at each energy fuel market am. Since this paper focuses on the electric power supply chain, the fuel demands from other sectors are considered exogenous and are included as parameters of $\pi_{am}(Q^1)$.

$\pi_{am}(Q^1)$ is assumed to be a non-decreasing continuous function [80, 82]. A special case is where $\pi_{am}(Q^1) = \bar{\pi}_{am}$ in which case the fuel price is fixed and equal to $\bar{\pi}_{am}$.

The (spatial price) equilibrium conditions (cf. [57]) for suppliers at fuel supply market

am; $a = 1, ..., A$; $m = 1, ..., M$, take the form: for each generating unit gr_1u; $g = 1, ..., G$; $r_1 = 1, ..., R$; $u = 1, ..., N_{gr_1}$, and at each demand level w:

$$\pi_{am}(Q^{1*}) + c^{am}_{gr_1uw} \begin{cases} = \rho^{am*}_{gr_1uw}, & \text{if} \quad q^{am*}_{gr_1uw} > 0, \\ \geq \rho^{am*}_{gr_1uw}, & \text{if} \quad q^{am*}_{gr_1uw} = 0. \end{cases} \tag{2}$$

In equilibrium, conditions (2) must hold simultaneously for all the fuel supply market and generating unit pairs and at all demand levels. We can express these equilibrium conditions as the following variational inequality (see, e.g., [57]): determine $Q^{1*} \in \mathcal{K}^1$, such that

$$\sum_{w=1}^{W} \sum_{a=1}^{A} \sum_{m=1}^{M} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \left[\pi_{am}(Q^{1*}) + c^{am}_{gr_1uw} - \rho^{am*}_{gr_1uw}\right] \times [q^{am}_{gr_1uw} - q^{am*}_{gr_1uw}] \geq 0, \, \forall Q^1 \in \mathcal{K}^1, \tag{3}$$

where $\mathcal{K}^1 \equiv \{Q^1 | Q^1 \in R^{AMNW}_+\}$.

The Behavior of the Power Generators and Their Optimality Conditions

Recall that the equilibrium prices are indicated by "*". Under the assumption that each individual genco is a profit-maximizer and may own multiple generating units in various regions, the optimization problem of genco g can be expressed as follows:

$$\text{Maximize} \quad \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \rho^{gr_1u*}_{r_2kw} q^{gr_1u}_{r_2kw}$$

$$+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \rho^*_{r_2w} y^{gr_1u}_{r_2w} + \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \varphi^*_{r_1w} z_{gr_1uw} - \sum_{w=1}^{W} \sum_{a=1}^{A} \sum_{m=1}^{M} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \rho^{am*}_{gr_1uw} q^{am}_{gr_1uw}$$

$$- \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} f_{gr_1uw}(q_{gr_1uw}) - \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{k=1}^{K} c^{gr_1u}_{r_2kw}(q^{gr_1u}_{r_2kw})$$

$$- \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} c^{gr_1u}_{r_2w}(y^{gr_1u}_{r_2w}) - \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} c_{gr_1uw}(z_{gr_1uw})$$

$$- \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{b=1}^{B} \mu^*_{bw} \alpha_{r_1r_2b} [\sum_{k=1}^{K} q^{gr_1u}_{r_2kw} + y^{gr_1u}_{r_2w}] \tag{4}$$

subject to:

$$L_w(\sum_{r_2=1}^{R} \sum_{k=1}^{K} q^{gr_1u}_{r_2kw} + \sum_{r_2=1}^{R} y^{gr_1u}_{r_2w}) = L_w q_{gr_1uw}, \quad r_1 = 1, ..., R; \; u = 1, ..., N_{gr_1}; \; w = 1, ..., W, \tag{5}$$

$$L_w q_{gr_1uw} = \sum_{a=1}^{A} \beta_{gr_1ua} \sum_{m=1}^{M} q^{am}_{gr_1uw} + L_w \beta_{gr_1u0} q_{gr_1uw}, \quad r_1 = 1, ..., R;\ u = 1, ..., N_{gr_1};\ w = 1, ..., W, \tag{6}$$

$$q_{gr_1uw} + z_{gr_1uw} \leq Cap_{gr_1u}, \quad r_1 = 1, ..., R;\ u = 1, ..., N_{gr_1};\ w = 1, ..., W, \tag{7}$$

$$z_{gr_1uw} \leq OP_{gr_1u}, \quad r_1 = 1, ..., R;\ u = 1, ..., N_{gr_1};\ w = 1, ..., W, \tag{8}$$

$$q^{gr_1u}_{r_2kw} \geq 0, \quad r_1 = 1, ..., R;\ u = 1, ..., N_{gr_1};\ r_2 = 1, ..., R;\ k = 1, ..., K;\ w = 1, ..., W, \tag{9}$$

$$q^{am}_{gr_1uw} \geq 0, \quad a = 1, ..., A;\ m = 1, ..., M;\ r_1 = 1, ..., R;\ u = 1, ..., N_{gr_1}; w = 1, \ldots, W, \tag{10}$$

$$y^{gr_1u}_{r_2w} \geq 0, \quad r_1 = 1, ..., R;\ u = 1, ..., N_{gr_1};\ r_2 = 1, ..., R;\ w = 1, .., W, \tag{11}$$

$$z_{gr_1uw} \geq 0, \quad r_1 = 1, ..., R;\ u = N_{gr_1};\ w = 1, ..., W. \tag{12}$$

The first three terms in the objective function (4) represent the revenues from bilateral transactions with the demand markets, the energy pool sales, and the regional operating reserve market, respectively. The fourth term is the total payout to the fuel suppliers. The fifth, sixth, and seventh terms represent the generating cost, the transaction costs of bilateral contracts with the demand markets, and the transaction costs of selling electric power to energy pools, respectively. The eighth term represents the costs of providing operating reserves. The last term of the objective function (4) represents the cost of congestion charges where $\sum_{b=1}^{B} \mu^*_{bw} \alpha_{r_1r_2b}$ is equivalent to the congestion charge of transferring one unit electricity from region r_1 to region r_2. Note that $\alpha_{r_1r_2b}$ is equal to $PTDF_{r_2b} - PTDF_{r_1b}$ where $PTDF_{rb}$ denotes the power transmission distribution factor of region (node) r for interface limit b. In particular, $PTDF_{rb}$ is defined as the quantity of power flow (MW) through the critical link of interface b induced by a 1 MW injection at node r [10, 11, 13, 35]. Here, the gencos have to pay the transmission right costs for bilateral transactions.

Next, we explain the constraints that genco g must satisfy when it maximizes its profit. Constraint (5) states that at each generating unit the total amount of electric power sold cannot exceed the total production of electric power.

Constraint (6) models the production of electricity at each generating unit. If a generating unit uses fossil fuel, at each demand level, the quantity of electricity produced is equal to the quantity of electricity converted from the fuels. In constraint (6), β_{gr_1ua} is equal to the nonnegative conversion rate (the inverse of the *heat rate*) at generating unit u of genco g

in region r_1 if the unit utilizes fuel a, and is equal to zero otherwise; β_{gr_1u0} is equal to one if the generating unit utilizes renewable technologies and is zero otherwise. For units using renewable technologies, all β_{gr_1ua}s are equal to zero. Note that for renewable generating units, constraint (6) will automatically hold. In the electric power industry, generating units that burn the same type of fuel may have very different average heat rates depending on the technologies that the generating units use. For example, the heat rates of large natural gas generating units range from 5500 Btu/kWh to 20500 Btu/kWh while the heat rates of large oil generating units vary from 6000 Btu/kWh to 25000 Btu/kWh (e.g. [80]).

Constraint (7) states that the sum of electric power generation and operating reserve cannot exceed the generating unit capacity, Cap_{gr_1u}. Constraint (8), in turn, states that the operating reserve provided by generating unit u of genco g in region r_1 cannot exceed the maximum level of operating reserve of that unit, OP_{gr_1u}.

We assume that the generating cost and the transaction cost functions for the generating units are continuously differentiable and convex (see also [55, 64, 65]), and that the gencos compete non-cooperatively in a Nash manner[67, 68]. The optimality conditions for all power generators simultaneously, under the above assumptions (see also [3, 4, 27, 57]), coincide with the solution of the following variational inequality: determine $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Z^*, \eta^*, \lambda^*) \in \mathcal{K}^2$ satisfying

$$\sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \left[\frac{\partial f_{gr_1uw}(q^*_{gr_1uw})}{\partial q_{gr_1uw}} + \eta^*_{gr_1uw} \right] \times [q_{gr_1uw} - q^*_{gr_1uw}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \left[\frac{\partial c^{gr_1u}_{r_2kw}(q^{gr_1u*}_{r_2kw})}{\partial q^{gr_1u}_{r_2kw}} + \sum_{b=1}^{B} \mu^*_{bw} \alpha_{r_1r_2b} - \rho^{gr_1u*}_{r_2kw} \right] \times [q^{gr_1u}_{r_2kw} - q^{gr_1u*}_{r_2kw}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \left[\frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u*}_{r_2w})}{\partial y^{gr_1u}_{r_2w}} + \sum_{b=1}^{B} \mu^*_{bw} \alpha_{r_1r_2b} - \rho^*_{r_2w} \right] \times [y^{gr_1u}_{r_2w} - y^{gr_1u*}_{r_2w}]$$

$$+ \sum_{w=1}^{W} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{a=1}^{A} \sum_{m=1}^{M} \rho^{am*}_{gr_1uw} \times [q^{am}_{gr_1uw} - q^{am*}_{gr_1uw}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \left[\frac{\partial c_{gr_1uw}(z^*_{gr_1uw})}{\partial z_{gr_1uw}} + \lambda^*_{gr_1uw} + \eta^*_{gr_1uw} - \varphi^*_{r_1w} \right] \times [z_{gr_1uw} - z^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[Cap_{gr_1u} - q^*_{gr_1uw} - z^*_{gr_1uw}\right] \times [\eta_{gr_1uw} - \eta^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[OP_{gr_1u} - z^*_{gr_1uw}\right] \times [\lambda_{gr_1uw} - \lambda^*_{gr_1uw}] \geq 0,\ \forall (Q^1, q, Q^2, Y^1, Z, \eta, \lambda) \in \mathcal{K}^2, \tag{13}$$

where $\mathcal{K}^2 \equiv \{(Q^1, q, Q^2, Y^1, Z, \eta, \lambda) | (Q^1, q, Q^2, Y^1, Z, \eta, \lambda) \in R_+^{AMNW+NRKW+NRW+4NW}$, and (5) and (6) hold$\}$.

Note that under the same assumptions, the optimality conditions can be also written as the following variational inequality: determine $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Z^*, \eta^*, \lambda^*, \gamma^*, \theta^*) \in \mathcal{K}^3$ satisfying

$$\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[\theta^*_{gr_1uw} + \frac{\partial f_{gr_1uw}(q^*_{gr_1uw})}{\partial q_{gr_1uw}} + \eta^*_{gr_1uw} - \gamma^*_{gr_1uw}\right] \times [q_{gr_1uw} - q^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R}\sum_{k=1}^{K} \left[\gamma^*_{gr_1uw} + \frac{\partial c^{gr_1u}_{r_2kw}(q^{gr_1u*}_{r_2kw})}{\partial q^{gr_1u}_{r_2kw}} + \sum_{b=1}^{B} \mu^*_{bw}\alpha_{r_1r_2b} - \rho^{gr_1u*}_{r_2kw}\right] \times [q^{gr_1u}_{r_2kw} - q^{gr_1u*}_{r_2kw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R} \left[\gamma^*_{gr_1uw} + \frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u*}_{r_2w})}{\partial y^{gr_1u}_{r_2w}} + \sum_{b=1}^{B} \mu^*_{bw}\alpha_{r_1r_2b} - \rho^*_{r_2w}\right] \times [y^{gr_1u}_{r_2w} - y^{gr_1u*}_{r_2w}]$$

$$+\sum_{w=1}^{W}\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{a=1}^{A}\sum_{m=1}^{M} \left[\rho^{am*}_{gr_1uw} - \beta_{gr_1ua}\theta^*_{gr_1uw}\right] \times [q^{am}_{gr_1uw} - q^{am*}_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[\frac{\partial c_{gr_1uw}(z^*_{gr_1uw})}{\partial z_{gr_1uw}} + \lambda^*_{gr_1uw} + \eta^*_{gr_1uw} - \varphi^*_{r_1w}\right] \times [z_{gr_1uw} - z^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[q^*_{gr_1uw} - \sum_{r_2=1}^{R}\sum_{k=1}^{K} q^{gr_1u*}_{r_2kw} - \sum_{r_2=1}^{R} y^{gr_1u*}_{r_2w}\right] \times [\gamma_{gr_1uw} - \gamma^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[\sum_{a=1}^{A} \beta_{gr_1ua} \sum_{m=1}^{M} q^{am*}_{gr_1uw} + L_w\beta_{gr_1u0}q^*_{gr_1uw} - L_w q^*_{gr_1uw}\right] \times [\theta_{gr_1uw} - \theta^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[Cap_{gr_1u} - q^*_{gr_1uw} - z^*_{gr_1uw}\right] \times [\eta_{gr_1uw} - \eta^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[OP_{gr_1u} - z^*_{gr_1uw}\right]\times[\lambda_{gr_1uw}-\lambda^*_{gr_1uw}] \geq 0,\ \forall (Q^1, q, Q^2, Y^1, Z, \eta, \lambda, \gamma, \theta) \in \mathcal{K}^3, \tag{13a}$$

where γ and θ denote the vectors of the shadow prices of constraints (5) and (6), respectively; and $\mathcal{K}^3 \equiv \{(Q^1, q, Q^2, Y^1, Z, \eta, \lambda, \gamma, \theta) | (Q^1, q, Q^2, Y^1, Z, \eta, \lambda, \gamma, \theta) \in R_+^{AMNW+NRKW+NRW+4NW} \times R^{2NRW}\}$. Variational inequality (13a) will be used to prove Lemma 1 and Lemma 2 as well as to recover ρ_2^* from the equilibrium solution of the complete model.

We note that the shadow price of constraint (6), θ_{gr_1uw}, can be interpreted as *the marginal fuel cost*, which is the fuel cost of producing one more unit of electric power at the u^{th} generating unit of genco g in region r_1 at demand level w. We now let $\psi^{gr_1u}_{r_2w}$ denote *the marginal supply cost* of the u^{th} generating unit of genco g in region r_1 to market k at region r_2 at demand level w, and let $\psi^{gr_1u}_{r_2w}$ be equal to the sum of the marginal fuel cost, θ_{gr_1uw}, the marginal operational cost, $\frac{\partial f_{gr_1uw}(q_{gr_1uw})}{\partial q_{gr_1uw}}$, the marginal transaction cost, $\frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u}_{r_2w})}{\partial y^{gr_1u}_{r_2w}}$, and the unit congestion charge, $\sum_{b=1}^{B} \mu_{bw}\alpha_{r_1r_2b}$:

$$\psi^{gr_1u}_{r_2w} \equiv \theta_{gr_1uw} + \frac{\partial f_{gr_1uw}(q_{gr_1uw})}{\partial q_{gr_1uw}} + \frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u}_{r_2w})}{\partial y^{gr_1u}_{r_2w}} + \sum_{b=1}^{B} \mu_{bw}\alpha_{r_1r_2b}. \tag{14}$$

The ISO's Role

The major role of the ISO is to ensure that the system reliability conditions are met, and to provide and oversee the competitive wholesale market (www.iso-ne.com).

The ISO achieves economic efficiency by *developing* and *overseeing* competitive power pools as well as maintaining market clearance at each power pool. For example, according to the company profile of ISO New England, one of its primary responsibilities is "*Development, oversight and fair administration of New England's wholesale electricity marketplace, through which bulk electric power has been bought, sold and traded since 1999. These competitive markets provide positive economic and environmental outcomes for consumers and improve the ability of the power system to meet ever-increasing demand efficiently*" (see: www.iso-ne.com).

Because of the development and oversight of the competitive wholesale market by the ISO, we can assume that at power pools the gencos compete with one another in a noncooperative manner in the sense of Nash [67, 68], and have incorporated this competition in (13).

The ISO ensures that the regional electricity markets $r = 1, \dots, R$ clear at each demand level $w = 1, \dots, W$, that is,

$$L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} y_{rw}^{gr_1u*} \begin{cases} = L_w \sum_{r_2=1}^{R} \sum_{k=1}^{K} y_{r_2kw}^{r*}, & \text{if} \quad \rho_{rw}^* > 0, \\ \geq L_w \sum_{r_2=1}^{R} \sum_{k=1}^{K} y_{r_2kw}^{r*}, & \text{if} \quad \rho_{rw}^* = 0. \end{cases} \tag{15}$$

The left-hand side of constraint (15) represents the total quantity of electric power sold by power sellers at region r through the power pool, and the right-hand side represents the total amount of electric power purchased by power buyers/consumers from region r through the power pool. The ISO ensures that the demand must be satisfied (cf. (15)) and oversees the power pool.

We now state and prove:

Lemma 1

In equilibrium, if the u^{th} generating unit of genco g in region r_1 supplies electricity to the power pool in region r_2 at demand level w, the marginal supply cost of the generating unit, $\psi_{r_2w}^{gr_1u*}$, is less than or equal to the clearing price at the power pool, $\rho_{r_2w}^*$.

Proof: See Appendix.

Lemma 2

In equilibrium, if at the power pool in region r_2 at demand level w, the u^{th} generating unit of genco g in region r_1 has available capacity, the marginal supply cost of the generating unit, $\psi_{r_2w}^{gr_1u*}$, is greater than or equal to the market clearing price at the power pool, $\rho_{r_2w}^*$.

Proof: See Appendix.

Proposition 1

In equilibrium, the following conditions must hold at the power pool:

(1) The demand at the power pool is satisfied;

(2) If a generating unit supplies electricity to the power pool in region r_2 at demand level w, the marginal supply cost of the generating unit is less than or equal to the clearing price at the power pool;

(3) If at the power pool in region r_2 at demand level w a generating unit has available capacity, the marginal supply cost of the generating unit is greater than or equal to the market clearing price at the power pool.

Proof: Proposition 1 can be directly obtained from equation (15), Lemma 1, and Lemma 2. Q.E.D.

In equilibrium, the ISO quotes the market clearing price, $\rho^*_{r_2w}$, so that the total demand at the power pool is satisfied and the economic efficiency is achieved. Note that, in equilibrium, the outcome of the competitive power pool (cf. Proposition 1) is equivalent to the result of choosing power plants from the lowest to the highest marginal supply costs till the demand is satisfied. In each regional power pool, all power suppliers will be paid at the *same* unit price which is equal to the market clearance price in that region.

Moreover, the ISO also manages the operating reserve markets where the gencos can get paid for holding back their capacities to help to ensure system reliability. We have assumed that the generators compete non-cooperatively in the operating reserve markets in a Nash manner [67, 68]. The ISO needs to ensure that the regional operating reserve markets $r_1 = 1, \dots, R$ clear at each demand level $w = 1, \dots, W$, that is,

$$L_w \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} z^*_{gr_1uw} \begin{cases} = L_w OPR_{r_1w}, & \text{if} \quad \varphi^*_{r_1w} > 0, \\ \geq L_w OPR_{r_1w}, & \text{if} \quad \varphi^*_{r_1w} = 0. \end{cases} \tag{16}$$

The ISO also manages transmission congestion and imposes congestion fees, which not only ensure system security, but also eliminate inter-region arbitrage opportunities and enhance economic efficiency of the system. We use a linearized direct current network to approximate the transmission network, and assume that the ISO charges network users congestion fees in order to ensure that the interface limits are not violated [10, 11, 35]. In our model, the following conditions must hold for each interface b and at each demand level w, where $b = 1, \ldots, B$; $w = 1, \ldots, W$:

$$L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} [\sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} q_{r_2kw}^{gr_1u*} + \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} y_{r_2w}^{gr_1u*} + \sum_{k=1}^{K} y_{r_2kw}^{r_1*}]\alpha_{r_1r_2b} \begin{cases} = L_w TCap_b, & \text{if} \quad \mu_{bw}^* > 0, \\ \leq L_w TCap_b, & \text{if} \quad \mu_{bw}^* = 0. \end{cases} \tag{17}$$

In equilibrium, conditions (15), (16), and (17) must hold simultaneously. We can express these equilibrium conditions using the following variational inequality: determine $(\mu^*, \rho_3^*, \varphi^*) \in R_+^{WB+2WR}$, such that

$$\sum_{w=1}^{W} L_w \sum_{b=1}^{B} [TCap_b - \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} [\sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} q_{r_2kw}^{gr_1u*} + \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} y_{r_2w}^{gr_1u*} + \sum_{k=1}^{K} y_{r_2kw}^{r_1*}]\alpha_{r_1r_2b}] \times [\mu_{bw} - \mu_{bw}^*]$$

$$+ \sum_{w=1}^{W} L_w \sum_{r=1}^{R} [\sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} y_{rw}^{gr_1u*} - \sum_{r_2=1}^{R} \sum_{k=1}^{K} y_{r_2kw}^{r*}] \times [\rho_{rw} - \rho_{rw}^*]$$

$$+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} [\sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} z_{gr_1uw}^* - OPR_{r_1w}] \times [\varphi_{r_1w} - \varphi_{r_1w}^*] \geq 0, \quad \forall (\mu, \rho_3, \varphi) \in R_+^{BW+2RW}. \tag{18}$$

Equilibrium Conditions for the Demand Markets

Next, we describe the equilibrium conditions at the demand markets. We group the consumers who have similar consumption patterns into the same demand market sector. The consumers search for the lowest electricity cost which is equal to the sum of the electricity price and the transaction cost.

We assume that all demand market sectors in all regions have fixed and known demands, and that the following conservation of flow equations, hence, must hold for all regions $r_2 = 1, \ldots, R$, all demand market sectors $k = 1, \ldots, K$, and at all demand levels $w = 1, \ldots, W$:

$$\sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} q_{r_2kw}^{gr_1u*} + \sum_{r_1=1}^{R} y_{r_2kw}^{r_1*} = (1 + \kappa_{r_2w}) d_{r_2kw}, \tag{19}$$

where d_{r_2kw} denotes the demand at market sector k in region r_2 at demand level w, and κ_{r_2w} denotes the transmission loss factor which is usually between $1\% - 6\%$.

We also assume that all the unit transaction cost functions $\hat{c}^{gr_1u}_{r_2kw}(Q^2_w)$s and $\hat{c}^{r_1}_{r_2kw}(Y^2_w)$s are continuous and nondecreasing.

The equilibrium conditions for consumers at demand market sector k in region r_2 take the form (see [16, 17, 61, 64, 74, 78, 83]): for each generating unit gr_1u; $g = 1, ..., G$; $r_1 = 1, ..., R$; $u = 1, ..., N_{gr_1}$, and each demand level w; $w = 1, \ldots, W$:

$$L_w[\rho^{gr_1u*}_{r_2kw} + \hat{c}^{gr_1u}_{r_2kw}(Q^{2*}_w)] \begin{cases} = L_w\rho^*_{r_2kw}, & \text{if} \quad q^{gr_1u*}_{r_2kw} > 0, \\ \geq L_w\rho^*_{r_2kw}, & \text{if} \quad q^{gr_1u*}_{r_2kw} = 0; \end{cases} \tag{20}$$

and

$$L_w[\rho^*_{r_1w} + \sum_{b=1}^{B} \mu^*_{bw}\alpha_{r_1r_2b} + \hat{c}^{r_1}_{r_2kw}(Y^{2*}_w)] \begin{cases} = L_w\rho^*_{r_2kw}, & \text{if} \quad y^{r_1*}_{r_2kw} > 0, \\ \geq L_w\rho^*_{r_2kw}, & \text{if} \quad y^{r_1*}_{r_2kw} = 0. \end{cases} \tag{21}$$

Conditions (20) state that, in equilibrium, if power buyers/consumers at demand market sector k in region r_2 purchase electricity from generating unit u of genco g in region r_1, then the price the consumers pay is exactly equal to the sum of the electricity price and the unit transaction cost. However, if the electricity price plus the transaction cost is greater than the price the buyers/consumers are willing to pay at the demand market, there will be no transaction between this generating unit/demand market pair. Conditions (21) state that power buyers/consumers in demand markets need to also consider congestion fees when they purchase electric power from other regions through the power pool.

In equilibrium, conditions (20) and (21) must hold simultaneously for all demand markets in all regions. We can express these equilibrium conditions using the following variational inequality: determine $(Q^{2*}, Y^{2*}) \in \mathcal{K}^4$, such that

$$\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R}\sum_{k=1}^{K} \left[\rho^{gr_1u*}_{r_2kw} + \hat{c}^{gr_1u}_{r_2kw}(Q^{2*}_w)\right] \times [q^{gr_1u}_{r_2kw} - q^{gr_1u*}_{r_2kw}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}\sum_{k=1}^{K} \left[\rho^*_{r_2w} + \sum_{b=1}^{B}\mu^*_{bw}\alpha_{r_1r_2b} + \hat{c}^{r_1}_{r_2kw}(Y^{2*}_w)\right] \times [y^{r_1}_{r_2kw} - y^{r_1*}_{r_2kw}] \geq 0,$$

$$\forall (Q^2, Y^2) \in \mathcal{K}^4, \tag{22}$$

where $\mathcal{K}^4 \equiv \{(Q^2, Y^2) | (Q^2, Y^2) \in R^{NRKW+R^2KW}_+$ and (19) holds$\}$. Note that since the conservation of flow equation (19) assumes fixed demands, ρ^*_4 cancels out in (22).

The Equilibrium Conditions for the Electric Power Supply Chain Network

In equilibrium, the optimality conditions for all gencos, the equilibrium conditions for all fuel supply markets, all demand market sectors, and the independent system operator must be simultaneously satisfied so that no decision-maker has any incentive to unilaterally alter its transactions. We now formally state the equilibrium conditions for the electric power supply chain with fuel supply markets as follows.

Definition 1: Electric Power Supply Chain Network Equilibrium

The equilibrium state of the electric power supply chain network with fuel supply markets is one where the fuel and electric power flows and the prices satisfy the sum of conditions (3), (13), (18), and (22).

We now state and prove:

Theorem 1: Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium Model with Fuel Suppliers

The equilibrium conditions governing the electric power supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*, \eta^*, \lambda^*, \mu^*, \rho_3^*, \varphi^*) \in \mathcal{K}^5$ *satisfying*

$$\sum_{w=1}^{W}\sum_{a=1}^{A}\sum_{m=1}^{M}\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[\pi_{am}(Q^{1*}) + c_{gr_1uw}^{am}\right] \times [q_{gr_1uw}^{am} - q_{gr_1uw}^{am*}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[\frac{\partial f_{gr_1uw}(q^*_{gr_1uw})}{\partial q_{gr_1uw}} + \eta^*_{gr_1uw}\right] \times [q_{gr_1uw} - q^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R}\sum_{k=1}^{K} \left[\frac{\partial c_{r_2kw}^{gr_1u}(q_{r_2kw}^{gr_1u*})}{\partial q_{r_2kw}^{gr_1u}} + \sum_{b=1}^{B}\mu^*_{bw}\alpha_{r_1r_2b} + \hat{c}_{r_2kw}^{gr_1u}(Q_w^{2*})\right] \times [q_{r_2kw}^{gr_1u} - q_{r_2kw}^{gr_1u*}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R} \left[\frac{\partial c_{r_2w}^{gr_1u}(y_{r_2w}^{gr_1u*})}{\partial y_{r_2w}^{gr_1u}} + \sum_{b=1}^{B}\mu^*_{bw}\alpha_{r_1r_2b} - \rho^*_{r_2w}\right] \times [y_{r_2w}^{gr_1u} - y_{r_2w}^{gr_1u*}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[\frac{\partial c_{gr_1uw}(z^*_{gr_1uw})}{\partial z_{gr_1uw}} + \lambda^*_{gr_1uw} + \eta^*_{gr_1uw} - \varphi^*_{r_1w}\right] \times [z_{gr_1uw} - z^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}\sum_{k=1}^{K}\left[\rho^*_{r_1w}+\hat{c}^{r_1}_{r_2kw}(Y^{2*}_w)+\sum_{b=1}^{B}\mu^*_{bw}\alpha_{r_1r_2b}\right]\times[y^{r_1}_{r_2kw}-y^{r_1*}_{r_2kw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\left[Cap_{gr_1u}-q^*_{gr_1uw}-z^*_{gr_1uw}\right]\times[\eta_{gr_1uw}-\eta^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\left[OP_{gr_1u}-z^*_{gr_1uw}\right]\times[\lambda_{gr_1uw}-\lambda^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{b=1}^{B}[TCap_b-\sum_{r_1=1}^{R}\sum_{r_2=1}^{R}[\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}}\sum_{k=1}^{K}q^{gr_1u*}_{r_2kw}+\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}}y^{gr_1u*}_{r_2w}+\sum_{k=1}^{K}y^{r_1*}_{r_2kw}]\alpha_{r_1r_2b}]\times[\mu_{bw}-\mu^*_{bw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r=1}^{R}[\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}y^{gr_1u*}_{rw}-\sum_{r_2=1}^{R}\sum_{k=1}^{K}y^{r*}_{r_2kw}]\times[\rho_{rw}-\rho^*_{rw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}[\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}}z^*_{gr_1uw}-OPR_{r_1}]\times[\varphi_{r_1w}-\varphi^*_{r_1w}]\geq 0,$$

$$\forall(Q^1,q,Q^2,Y^1,Y^2,Z,\eta,\lambda,\mu,\rho_3,\varphi)\in\mathcal{K}^5, \tag{23}$$

where $\mathcal{K}^5\equiv\{(Q^1,q,Q^2,Y^1,Y^2,Z,\eta,\lambda,\mu,\rho_3,\varphi)|(Q^1,q,Q^2,Y^1,Y^2,Z,\eta,\lambda,\mu,\rho_3,\varphi)$ $\in R_+^{AMNW+NRKW+NRW+4NW+R^2KW+BW+2RW}$ *and* (5), (6), *and* (19) *hold*$\}$.

Proof: We first establish that Definition 1 implies variational inequality (23). Indeed, summation of (3), (13), (18), and (22), after algebraic simplifications, yields (23).

Now we prove the converse, that is, that a solution to (23) satisfies the sum of (3), (13), (18), and (22), and is, hence, an equilibrium.

To variational inequality (23), add $\rho^{am*}_{gr_1uw}-\rho^{am*}_{gr_1uw}$ to the term in the first brackets preceding the first multiplication sign, and add $\rho^{gr_1u*}_{r_2kw}-\rho^{gr_1u*}_{r_2kw}$ in the brackets preceding the third multiplication sign. The addition of such terms does not change (23) since the value of these terms is zero, and yields the sum of (3), (13), (18), and (22) after simple algebraic simplifications. Q.E.D.

We now discuss how to recover the equilibrium prices: ρ^*_1, ρ^*_2, and ρ^*_4 (see also [16, 17, 61]). In order to recover the vector of $\rho^{am*}_{gr_1uw}$s, ρ^*_1, one can (after solving variational inequality (23) for the particular numerical problem) set $\rho^{am*}_{gr_1uw}=\pi_{am}(Q^{1*})+c^{am}_{gr_1uw}$, for any a, m, g, r_1, u, and w such that $q^{am*}_{gr_1uw}>0$ (cf. (2)).

We now describe how to recover the vector of $\rho^{gr_1u*}_{r_2kw}$s, ρ^*_2. First, one can recover θ^* by setting $\theta^*_{gr_1uw} = \frac{\rho^{am*}_{gr_1uw}}{\beta_{gr_1ua}}$, for any a, m, g, r_1, u, and w such that $q^{am*}_{gr_1uw} > 0$ (cf. 13a). One can then recover γ^* by setting $\gamma^*_{gr_1uw} = \theta^*_{gr_1uw} + \frac{\partial f_{gr_1uw}(q^*_{gr_1uw})}{\partial q_{gr_1uw}} + \eta^*_{gr_1uw}$ for any g, r_1, u, and w such that $q^*_{gr_1uw} > 0$ (cf. 13a). Finally, we can recover ρ^*_2 by setting $\rho^{gr_1u*}_{r_2kw} = \gamma^*_{gr_1uw} + \frac{\partial c^{gr_1u}_{r_2kw}(q^{gr_1u*}_{r_2kw})}{\partial q^{gr_1u}_{r_2kw}} + \sum_{b=1}^{B} \mu^*_{bw}\alpha_{r_1r_2b}$, for any g, r_1, u, r_2, k, and w such that $q^{gr_1u*}_{r_2kw} > 0$ (cf. (13a)).

One can recover the vector of $\rho^*_{r_2kw}$s, ρ^*_4, either by setting $\rho^*_{r_2kw} = \rho^{gr_1u*}_{r_2kw} + \hat{c}^{gr_1u}_{r_2kw}(Q^{2*}_w)$, for any g, r_1, u, r_2, k, and w such that $q^{gr_1u*}_{r_2kw} > 0$ or by setting $\rho^*_{r_2kw} = \rho^*_{r_1w} + \sum_{b=1}^{B} \mu^*_{bw}\alpha_{r_1r_2b} + \hat{c}^{r_1}_{r_2kw}(Y^{2*}_w)$, for any r_1, r_2, k, and w such that $y^{r_1*}_{r_2kw} > 0$ (cf. (20) and (21)).

Under the above pricing mechanisms, the optimality conditions (13), as well as the equilibrium conditions (2), (18), and (22) also hold separately (as well as for each individual decision-maker).

The variational inequality problem (23) can be rewritten in standard variational inequality form (cf. [57]) as follows: determine $X^* \in \mathcal{K}$ satisfying

$$\left\langle F(X^*)^T, X - X^* \right\rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{24}$$

where $X \equiv (Q^1, q, Q^2, Y^1, Z, Y^2, \eta, \lambda, \mu, \rho_3, \varphi)^T$, $\mathcal{K} \equiv \mathcal{K}^5$, and

$$F(X) \equiv (F^{am}_{gr_1uw}, F^{gr_1u}_{w}, F^{gr_1u}_{r_2kw}, F^{gr_1u}_{r_2w}, F^{zgr_1u}_{w}, F_{r_1r_2kw}, F_{\lambda gr_1u}, F_{\eta gr_1u}, F_{bw}, F_{rw}, F_{r_1w})$$

with indices $a = 1, \ldots, A$; $m = 1, \ldots, M$; $w = 1, \ldots, W$; $r_1 = 1, \ldots, R$; $r_2 = 1, \ldots, R$; $r = 1, \ldots, R$; $g = 1, \ldots, G$; $u = 1, \ldots, N_{gr_1}$; $k = 1, \ldots, K$; $b = 1, \ldots, B$, and the functional terms preceding the multiplication signs in (23), respectively. Here $< \cdot, \cdot >$ denotes the inner product in Ω-dimensional Euclidian space where $\Omega = AMNW + NRKW + NRW + 4NW + R^2KW + BW + 2RW$.

We provide some qualitative properties of the model and discuss the computation of solutions to the model in the Appendix. In particular, we provide the conditions of existence of a solution to the variational inequality (23). We also show that in our case study, $F(X)$ that enters the variational inequality is monotone, and the Jacobian of $F(x)$ is positive semidefinite. Additionally, we propose a computational method that converges to a solution of the model provided that $F(X)$ is monotone and Lipschitz continuous, and a solution exists,

3. Empirical Case Study and Examples

In this section, we present the results for four empirical examples based on the New England electric power market and fuel markets data. In Example 1, we show that the regional electric power prices simulated by our theoretical model very well match the actual electricity prices in New England. In Example 2, we conduct sensitivity analysis for electricity prices under natural gas and oil price variations. Based on the sensitivity analysis, we also compute the spark spread, an important measure of the power plant profitability, under gas and oil price variations. In Example 3, we investigate how the natural gas price is influenced by the oil price through electric power markets. In particular, we present examples that show that in New England the fuel competition at electric power generation markets has become the major factor affecting the relationship between oil and gas prices. In Example 4, we apply our model to investigate how the changes in the demands for electricity affect the electric power and fuel supply markets. Throughout this section, we use the demand market prices, $\rho^*_{r_2kw}$; $w = 1, \ldots, W$; $r_2 = 1 \ldots, R$; and $k = 1, \ldots, K$, as the simulated regional electric power prices. Note that the model developed in this paper can be easily expanded to include multiple electricity markets and can be applied to larger areas where deregulation is taking place.

Data

The data that we used for the d_{r_2kw}s (see (19)) were New England day-ahead hourly zonal demands. We downloaded the data from the ISO New England hourly demand and price datasets [46] and the Connecticut Valley Electric Exchange [14]. The regional demands were adjusted based on the imported and the exported electric power between New England and the surrounding regions [47].

In New England, there are 82 gencos who own and operate 573 generating units (G=82, N=573). We obtained the electric generating unit data including the heat rates (the inverse of the β_{gr_1ua}s), the generating costs, the f_{gr_1uw}s, the fuel types, the capacities, the Cap_{gr_1u},

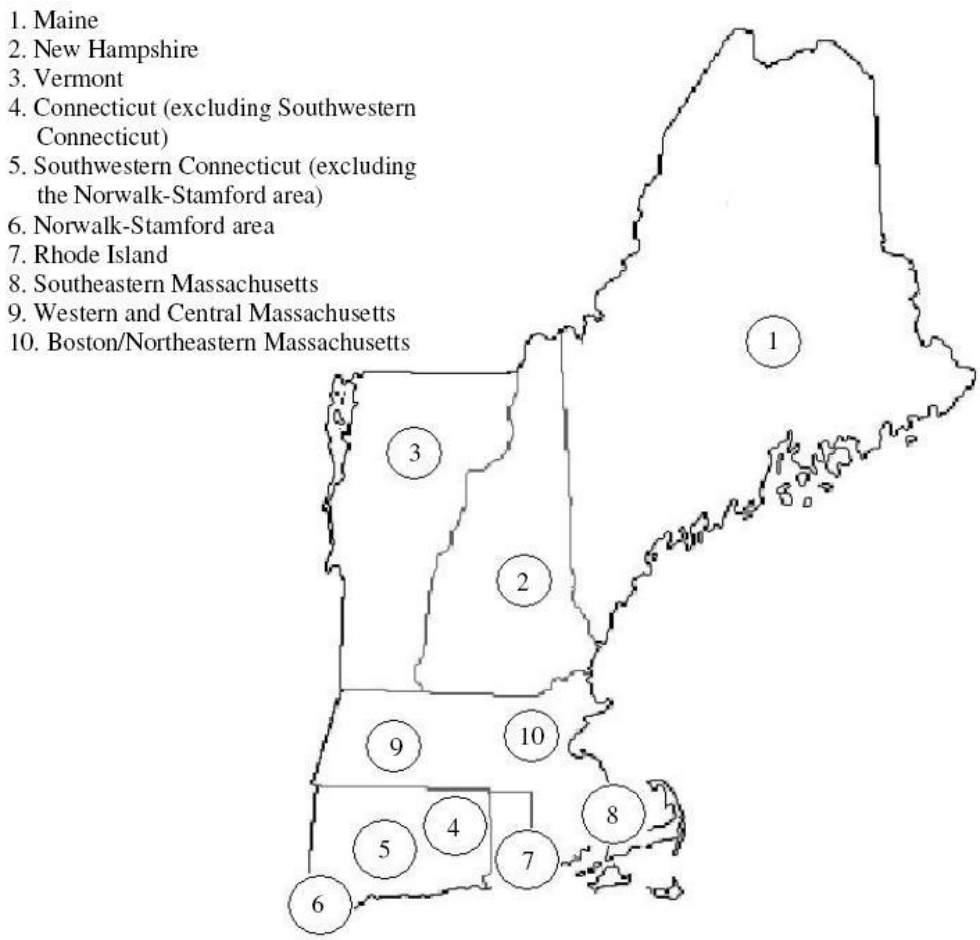

Figure 2: The Ten Regions of the New England Electric Power Supply Chain

and the locations from the following sources: (1) the national electric energy data system [80], (2) the ISO New England seasonal claimed capacity reports [44], and (3) the New England FERC natural gas infrastructure report [26]. We adjusted the generation capacities based on the capacity availability factor data obtained from the ISO New England website [40]. In order to simplify the computations, we did not include the power plants whose capacities were less than 1MW since the total combined capacity of those small power plants only accounted for less than 0.2% of the total generating capacity in New England. The transaction/transmission costs: the $c^{gr_1u}_{r_2kw}$s, the $c^{gr_1u}_{r_2w}$s, the $\hat{c}^{gr_1u}_{r_2kw}$s, and the $\hat{c}^{r_1}_{r_2kw}$s, were estimated based on the average transmission costs obtained from the ISO New England website [51].

We considered 5 types of fuels: natural gas, residual fuel oil (sulfur≤1%), distillate fuel oil, jet fuel, and coal (A=5). We downloaded the monthly delivered fuel price data for each state of New England from the Energy Information Administration website [25]. Hence, M=number of states=6; $a = 1, \ldots, 5$. Note that in the case study we used actual delivered regional fuel price data to set the fuel prices and the constructed price functions, the π_{am}s, for each market (these prices already included the transportation costs, the $c^{am}_{gr_1uw}$s, from other areas to New England). For more fuel transportation and transmission rate data see the website of the Federal Regulation and Oversight of Energy (www.ferc.gov).

We approximated the physical transmission constraints (the $TCap_b$s and the $\alpha_{r_1r_2b}$s) using the interface limits provided in the ISO New England regional system plan [43], the ISO New England research report, "Determination of 2006-2015 transfer limits" [45], and the day-ahead and real-time limit data [48]. Based on the interface constraints and the demand/price data, we divided the whole area into ten regions (R=10): 1. Maine, 2. New Hampshire, 3. Vermont, 4. Connecticut (excluding Southwest Connecticut), 5. Southwestern Connecticut (excluding the Norwalk-Stamford area), 6. Norwalk-Stamford area, 7. Rhode Island, 8. Southeastern Massachusetts, 9. Western and Central Massachusetts, 10. Boston/Northeast Massachusetts, as shown in Figure 2. We used an aggregated demand market to represent the demand for each region (K=1).

Based on [75], we assumed that the transmission loss factor, the κ_{r_2w}, is 4% for the highest demand level and 3% for the other demand levels. We did not include the regional operating reserve markets in the case study version of the model because before 2007 New England did not have such markets. The New England ISO, instead, designated several generating units as second-contingency units to support system reliability. We obtained this information from the ISO New England website [41]. We used the operating reserve requirements for summer 2007 to approximate the operating reserve requirements for summer 2006 [49], and assigned the local operating reserves to the second-contingency units.

We tested the model on the data of July 2006 which included $24 \times 31 = 744$ hourly demand/price scenarios. We sorted the scenarios based on the total hourly demand, and constructed the load duration curve. We divided the duration curve into 6 blocks ($L_1 = 94$ hours, and $L_w = 130$ hours; $w = 2, ..., 6$) and calculated the average regional demands and the average weighted regional prices for each block. In our model, all cost functions and fuel price functions are assumed linear based on the data and the literature [24, 80, 82]. We then implemented the model and the modified projection method in Matlab (see www.mathworks.com). Moreover, in Steps 1 and 2 of the modified projection method (see the Appendix), due to the special structure of the underlying feasible set, the subproblems are completely separable and can be solved as W transportation network problems with the prices in each subproblem solvable in closed form (see, e.g., [58, 59, 63, 64, 65, 83]). In particular, in Steps 1 and 2, we applied the general equilibration algorithm (cf. [57]) to fully exploit the structure of the network subproblems [57]. The demand market prices $\rho^*_{r_2kw}$ for all $r_2,\ k,\ w$ can be recovered from the path costs of the active paths in the reformulated path flow formulation, or from conditions (20) or (21). Each example in this section was solved within 300 minutes on a Lenovo laptop with the 2.1GHz Core(TM)2 Duo CPU.

Example 1: Regional Electric Power Supply Chain Simulation

We first set the fuel prices of each regional market equal to the actual regional delivered fuel prices which were obtained from the Energy Information Administration website [25]. We downloaded the hourly locational marginal price (LMP) data from the ISO New England website [46]. For each block, we used the average regional demand data as model input to compute the regional electricity prices. The average regional demands for each block are shown in Table 5. Tables 6 and 7, and Figure 3 compare the simulated prices and the actual weighted average LMPs in the ISO day-ahead market. In Tables 6 and 7, the simulated prices of Connecticut were the weighted average of the prices of regions 4, 5, and 6.

In Figure 3, each individual point represents a simulated and actual price pair for one region at one demand level. The diagonal line in Figure 3 indicates perfect matches. Tables 6 and 7, and Figure 3 show that the simulated average regional prices match the actual prices very well at all demand levels except that at the lowest demand level the simulated prices are higher than the actual prices. This may be due to the following reason: in this specific example, the prices of electricity at the lowest demand level are determined by those natural gas generating units that have lowest generating costs (lowest heat rates). Note that a generating unit's generating cost is approximately equal to the product of the fuel cost and the average heat rate of that generating unit. Due to the limitation of the data, we used the average natural gas price at the regional markets to estimate the fuel costs for those generating units. However, in reality, some generating units may be able to purchase natural gas at lower costs. Therefore, in our model, the lowest generating costs of those natural gas generating units may be overestimated.

Additionally, both actual and simulated electricity prices have significant geographic differences due to the limited interfaces' capacities in the physical transmission network. We can also see that the simulated prices have smaller regional differences compared to the actual prices. This is because the PTDF data of the transmission network are not available to the public and we used the interface limit data in [43, 45, 48] to approximate the transmission constraints.

Table 5: Average Regional Demands for Each Demand Level (MWh)

Region	**Block 1**	**Block 2**	**Block 3**	**Block 4**	**Block 5**	**Block 6**
1	1512	1425	1384	1292	1051	889
2	1981	1868	1678	1481	1193	1005
3	774	760	717	654	560	500
4	2524	2199	2125	1976	1706	1432
5	2029	1798	1636	1485	1257	1065
6	1067	931	838	740	605	509
7	1473	1305	1223	1112	952	801
8	2787	2478	2315	2090	1736	1397
9	2672	2457	2364	2262	2448	2186
10	4383	4020	3684	3260	2744	2384
Total	21201	19241	17963	16350	14252	12168

Table 6: Actual Regional Prices ($/MWh)

Region	**Block 1**	**Block 2**	**Block 3**	**Block 4**	**Block 5**	**Block 6**
ME	96.83	72.81	59.78	52.54	45.79	36.70
NH	102.16	77.17	63.07	56.31	48.20	38.35
VT	105.84	80.69	65.32	58.39	49.71	39.24
CT	133.17	112.25	86.85	65.97	50.92	39.97
RI	101.32	75.66	61.84	56.06	47.55	37.94
SE MA	101.07	75.78	62.09	56.27	47.54	38.05
WC MA	104.15	79.19	64.49	58.41	49.25	39.53
NE MA	109.29	83.96	63.93	63.02	48.11	38.22

Table 7: Simulated Regional Prices ($/MWh)

Region	**Block 1**	**Block 2**	**Block 3**	**Block 4**	**Block 5**	**Block 6**
ME	97.07	82.66	62.40	53.49	51.00	51.00
NH	97.39	82.66	62.40	53.49	51.00	51.00
VT	97.39	82.66	62.40	53.49	51.00	51.00
CT	127.48	114.96	67.62	62.61	51.00	51.00
RI	97.39	82.66	62.40	56.65	51.00	51.00
SE MA	97.39	82.66	62.40	56.65	51.00	51.00
WC MA	97.39	82.66	62.40	56.65	51.00	51.00
NE MA	99.90	78.43	62.40	56.65	51.00	51.00

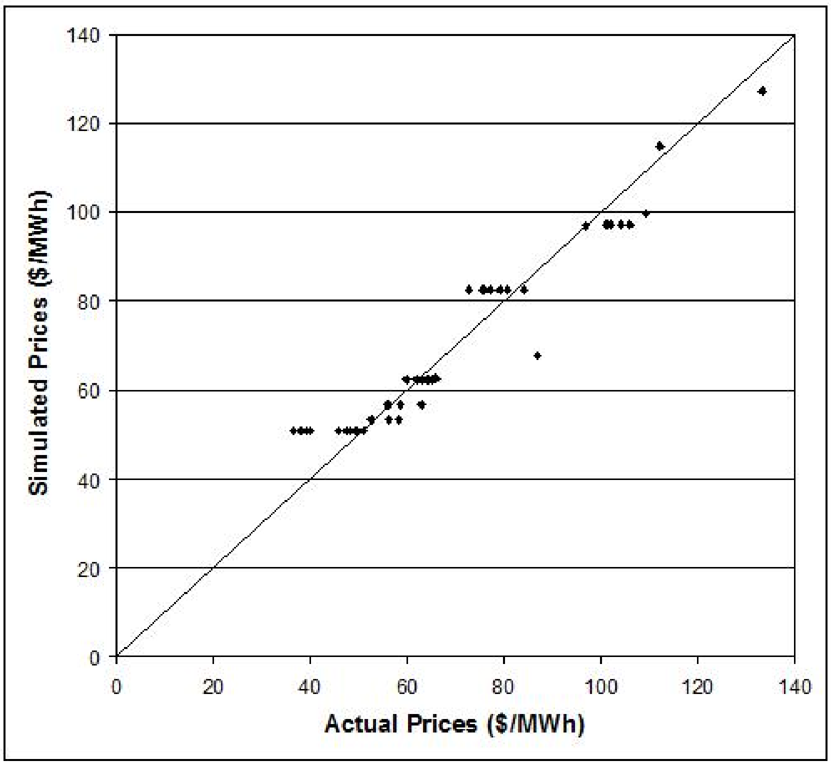

Figure 3: Actual Prices Vs. Simulated Prices ($/MWh)

Example 2: Peak Electric Power Prices under Fuel Price Variations

The second example consists of two parts. In the first part of Example 2, we conducted sensitivity analysis for the electricity prices under fuel price variations. The sensitivity analysis shows how fuel diversification can mitigate fuel price shocks. In the second part of this example, based on the results obtained in the first part, we calculated the spark spread, an important measure of the power plant profitability, under fuel price variations. These results are especially helpful for investors and managers in evaluating the profitability as well as managing the risk.

In New England, natural gas and oil are the most important fuels for electric power generation. Natural gas units and oil units generate 38% and 24% of electric power in New England, respectively [38]. Moreover, generating units that burn gas or oil set electric power market price 85% of the time [44].

We used the same demand data, but varied the prices of natural gas and residual fuel oil. We assumed that the percentage change of distillate fuel oil and jet fuel prices were the same as that of the residual fuel oil price. Tables 8, 9, and 10 present the average electricity prices at peak, intermediate, and low demand levels under oil/gas price variations, respectively. The surfaces in Figures 4, 5, and 6 represent the average electricity prices under different natural gas and oil price combinations at the three demand levels. Note that, if the price of one type of fuel is fixed, the electricity price changes less percentage-wise than the other fuel price does. For example, at the peak demand level, when the residual fuel price is fixed and equal to \$7/MMBtu, if the natural gas price increases 160% from \$5/MMBtu to \$13/MMBtu the electric power price increases \$8.51/MMBtu to \$11.43/MMBtu which is about 34.41%. This is mainly because fuel diversity can mitigate fuel price shocks.

In the second part, we demonstrated how the results obtained in the first part could be used to compute the spark spread under fuel price variations. The spark spread of a power plant represents the gross income of the power plant under certain market conditions, and is defined as follows:

$$Spark\ Spread = Electricity\ Price - Heat\ Rate\ of\ the\ Power\ Plant\ \times\ Fuel\ Price. \quad (25)$$

The spark spread has been widely utilized in evaluating the profitability and value of power plants as well as in managing financial risks [18, 73, 79]. In the second part of Example 2, we studied the spark spread of a generic combined-cycle natural gas power plant at different demand levels under fuel price variations. Based on the data, we assumed that the heat rate of the combined-cycle natural gas power plant is 7.5 MMBtu/MWh. The electricity and natural gas fuel prices used in equation (25) were obtained from the first part of Example 2. Tables 11, 12, and 13 present the spark spread of the combined-cycle natural gas power plant at peak, intermediate, and low demand levels under oil/gas price variations, respectively. Note that a positive spark spread indicates that the power plant is profitable under the given fuel price and demand combination.

The results show that at all demand levels, the profitability of the combined-cycle natural gas power plant increases as the residual fuel oil price increases while the profitability decreases as the natural gas price increases. Moveover, the combined-cycle natural gas power plant is always profitable at the peak demand levels. However, at the intermediate demand level, the profitability diminishes in the scenarios where the residual fuel oil price is low and the natural gas price is high. At the low demand level, the power plant is profitable only in the scenarios with low natural gas prices. Similar analysis can be conducted to study spark spreads for various types of power plants for the purpose of investment evaluation and risk management.

Table 8: Average Electricity Prices at Peak Demand Level under Fuel Price Variations

Electricity Price (cents/kWh)		Residual Fuel Oil Prices ($/MMBtu) 4.00	5.00	6.00	7.00	8.00
	5.00	5.46	6.18	6.42	8.51	9.66
	7.00	6.27	6.70	6.91	8.62	9.66
Natural Gas	9.00	7.72	7.95	8.01	9.01	9.84
($/MMBtu)	11.00	9.04	9.38	9.53	10.24	10.53
	13.00	10.29	10.75	11.02	11.43	11.58

Table 9: Average Electricity Prices at Intermediate Demand Level under Fuel Price Variations

Electricity Price (cents/kWh)		Residual Fuel Oil Prices ($/MMBtu) 4.00	5.00	6.00	7.00	8.00
	5.00	4.59	5.12	5.57	5.62	7.10
	7.00	5.39	5.63	5.73	6.64	7.51
Natural Gas	9.00	6.85	6.85	6.85	7.46	7.46
($/MMBtu)	11.00	8.30	8.30	8.30	8.30	8.30
	13.00	9.76	9.76	9.76	9.76	9.76

Table 10: Average Electricity Prices at Low Demand Level under Fuel Price Variations

Electricity Price (cents/kWh)		Residual Fuel Oil Prices ($/MMBtu) 4.00	5.00	6.00	7.00	8.00
	5.00	4.07	4.15	4.15	4.15	4.42
	7.00	5.29	5.39	5.39	5.39	5.56
Natural Gas	9.00	5.92	6.38	6.55	6.85	6.85
($/MMBtu)	11.00	6.33	7.00	7.00	8.19	8.30
	13.00	7.58	7.62	7.62	9.18	9.63

Table 11: Spark Spread at Peak Demand Level under Fuel Price Variations

Electricity Price (cents/kWh)		RFO Prices ($/MMBtu) 4.00	5.00	6.00	7.00	8.00
	5.00	1.71	2.43	2.67	4.67	5.91
	7.00	1.02	1.45	1.66	3.37	4.41
Natural Gas	9.00	0.97	1.20	1.26	2.26	3.09
($/MMBtu)	11.00	0.79	1.13	1.28	1.99	2.28
	13.00	0.54	1.00	1.24	1.68	1.83

Table 12: Spark Spread at Intermediate Demand Level under Fuel Price Variations

Electricity Price (cents/kWh)		Residual Fuel Oil Prices ($/MMBtu) 4.00	5.00	6.00	7.00	8.00
	5.00	0.84	1.37	1.82	1.87	3.35
	7.00	0.14	0.38	0.48	1.39	2.26
Natural Gas	9.00	0.10	0.10	0.10	0.71	1.21
($/MMBtu)	11.00	0.05	0.05	0.05	0.17	0.63
	13.00	0.01	0.01	0.01	0.01	0.01

Table 13: Spark Spread at Low Demand Level under Fuel Price Variations

Electricity Price (cents/kWh)		Residual Fuel Oil Prices ($/MMBtu) 4.00	5.00	6.00	7.00	8.00
	5.00	0.32	0.40	0.40	0.40	0.67
	7.00	0.04	0.14	0.14	0.14	0.31
Natural Gas	9.00	-0.83	-0.37	-0.20	0.10	0.10
($/MMBtu)	11.00	-1.92	-1.25	-1.25	-0.06	0.05
	13.00	-2.17	-2.17	-2.13	-0.57	-0.12

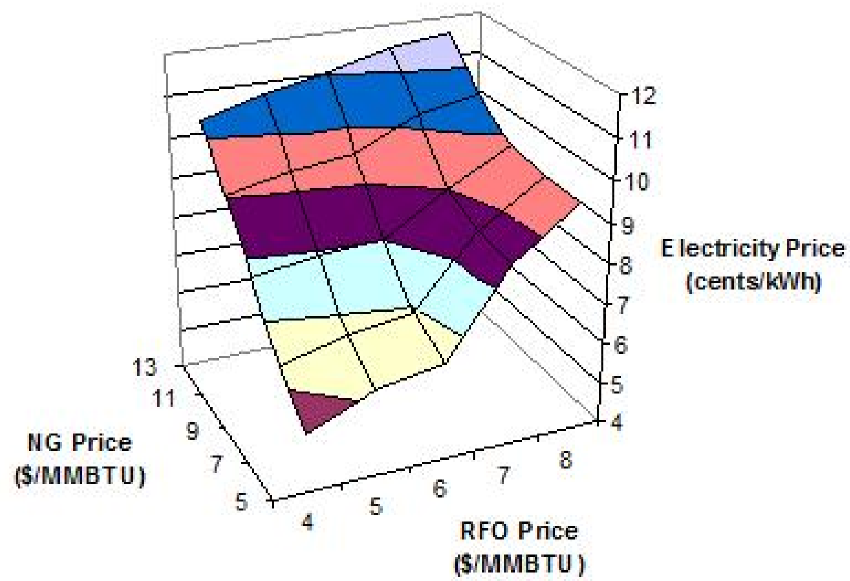

Figure 4: The Average Electric Power Price at Peak Demand Level under Fuel Price Variations

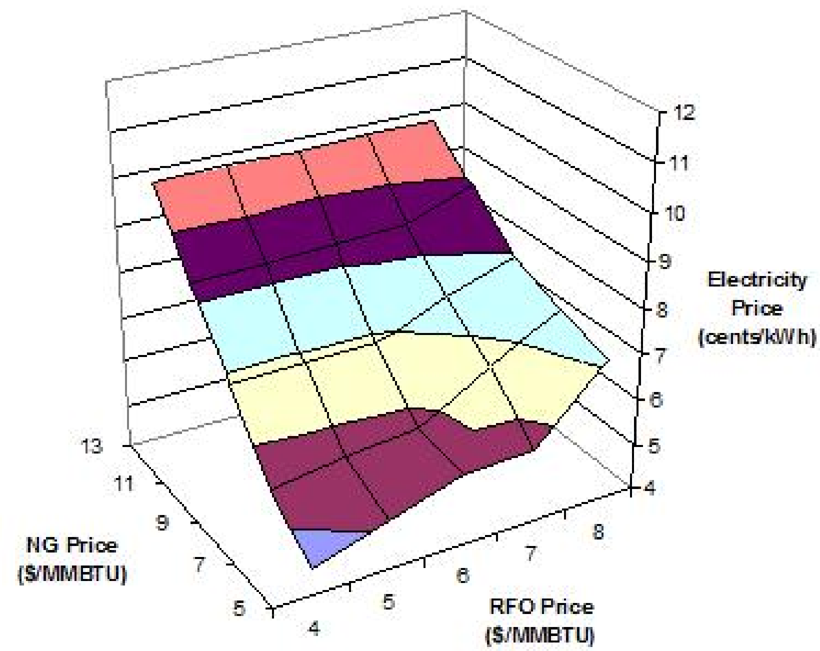

Figure 5: The Average Electric Power Price at Intermediate Demand Level under Fuel Price Variations

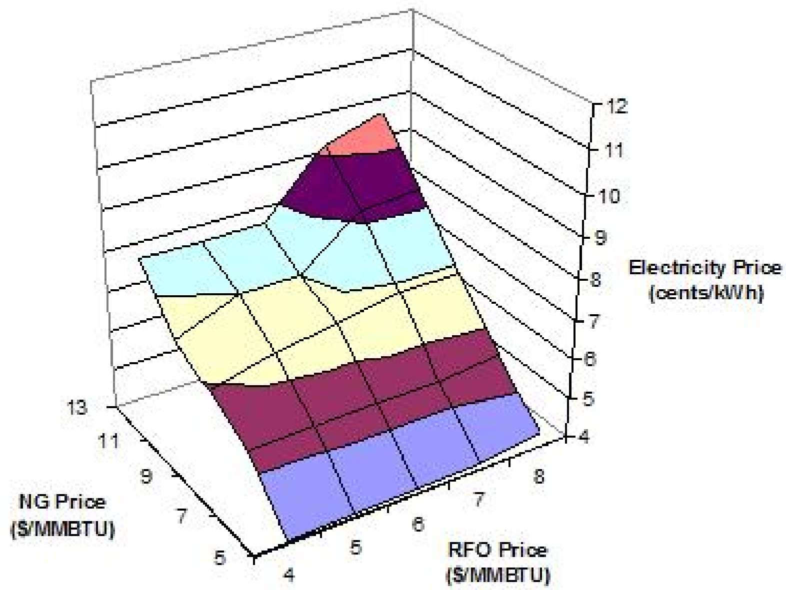

Figure 6: The Average Electric Power Price at Low Demand Level under Fuel Price Variations

Example 3: The Interactions Among Electric Power, Natural Gas, and Oil Markets

Next, we utilized the model as well as the New England electricity and fuel market data to explore the impact of electric power markets on the relationship between the natural gas and oil markets. In particular, we present examples where the natural gas price can be influenced by the residual fuel oil price through electric power markets.

The connection between the natural gas price and the oil price has drawn considerable attention from researchers and practitioners (see, for example, [7, 8, 34, 39, 81]). The understanding of this relationship is important for both energy market participants and policy makers to evaluate the risks in various energy markets. Historically, it was widely thought that the crude oil price ($/barrel) is approximately ten times as high as the natural gas price ($/MMBtu). Huntington and Schuler [39] pointed out this price relationship was due to the responsiveness of dual-fuel (natural gas and residual fuel oil) generating units and industrial users because the less expensive fuel to generate electric power would be chosen. However, recently, many studies found that the two prices had decoupled because of the decline of the number of dual-fuel plants in electric and industrial sectors [7, 8, 34, 81]. These studies also investigated the cointegration relationship between the two prices using various statistical regression methods ([7, 8, 34, 81]). Brown [7] found that the natural gas and oil prices continued to be connected with a complex relationship. Hartley, Medlock, and Rosthal [34] discovered that oil price influenced natural gas price through the competition of natural gas and RFO in electric power generation at both plant and grid/market levels. The authors also pointed out that the evolving relationship between natural gas and oil prices was a result of technological changes in the electric power sector.

In summary, the statistical studies in the literature discovered that after the decoupling of the oil and natural gas prices there was still a statistical connection between the oil and natural gas price series, which was due to the fuel competition between RFO and natural gas at both plant and market/grid levels in the electric power industry.

However, these studies only used regression models to study the statistical relationship which could not reveal how the grid/market-level competition in electric power generation take place, and how much such competition contributes to the influence of oil price on the natural gas price. Given the trend of deregulation of power markets, it is important to understand how the grid/market-level fuel competition takes place in the electric power market and how much it affects the natural gas price.

In competitive electric power markets, the market mechanism automatically selects electricity from generating units that have lower costs and that use less expensive fuels. Therefore, if the residual fuel oil price decreases, the demand for residual fuel oil will increase, which will reduce the natural gas demand and put downward pressure on the natural gas price.

We present two examples to demonstrate how our theoretical model can be used to quantify this impact. In Example 3.1, we allowed the dual-fuel generating units to freely switch between their primary fuels and alternative fuels, while in Example 3.2, we assumed that the dual-fuel generating units had to use their primary fuels and could not switch to alternative fuels. Based on the findings in Hartley, Medlock, and Rosthal [34], we assumed that the natural gas price was elastic and the residual oil price was exogenous.

Base on the literature [82] and the New England natural gas consumption data in 2006, we assumed that the natural gas price function took the form:

$$\pi_{GASm}(h) = 7.29\frac{\sum_{w=1}^{6}\sum_{m=1}^{6}\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} q_{gr_1uw}^{GASm} + 30.81}{61.15}; \quad m = 1, \ldots, 6. \tag{26}$$

Tables 14 and 15 present the results of Examples 3.1 and 3.2. Figure 7 compares the impacts of the RFO price on the natural gas price in the two examples

Based on the results of Examples 3.1 and 3.2, we concluded that in the New England electric power market, the grid/market-level competition had become the major factor affecting the influence of the RFO price on the natural gas price while the plant-level switch only had minor contribution to the relationship. Note that this model can be applied to other deregulated electric power markets to study how much both levels of fuel competitions contribute to the cointegration relationship between natural gas and oil prices.

Table 14: The Price Changes of Natural Gas and Electric Power Under Residual Fuel Oil Price Variation (with Dual-Fuel Plants)

RFO Price ($/MMBtu)	2.00	4.00	6.00	8.00	10.00	12.00	14.00
NG Demand (Billion MMBtu)	21.94	24.40	35.17	39.86	44.51	45.06	45.06
NG Price ($/MMBtu)	6.47	6.77	8.09	8.67	9.24	9.31	9.31
EP (Peak Level) (c/kWh)	7.23	7.50	9.13	10.57	11.46	12.37	12.73
EP (Intermediate Level) (c/kWh)	4.96	5.29	6.76	8.38	9.37	9.51	9.62
EP (Low Level) (c/kWh)	4.81	5.12	6.22	6.87	7.31	7.39	7.39
NG=Natural Gas, RFO=Residual Fuel Oil, EP=Average Electricity Price							

Table 15: The Price Changes of Natural Gas and Electric Power Under Residual Fuel Oil Price Variation (without Dual-Fuel Plants)

RFO Price ($/MMBtu)	2.00	4.00	6.00	8.00	10.00	12.00	14.00
NG Demand (Billion MMBtu)	21.29	25.78	34.88	38.87	40.03	40.03	40.03
NG Price ($/MMBtu)	6.39	6.94	8.06	8.55	8.69	8.69	8.69
EP (Peak Level) (c/kWh)	7.32	8.14	9.78	10.42	11.70	13.28	15.40
EP (Intermediate Level) (c/kWh)	4.91	5.51	6.74	8.27	9.20	9.47	9.60
EP (Low Level) (c/kWh)	4.70	5.14	6.38	6.78	6.89	6.89	6.89
NG=Natural Gas, RFO=Residual Fuel Oil, EP=Average Electricity Price							

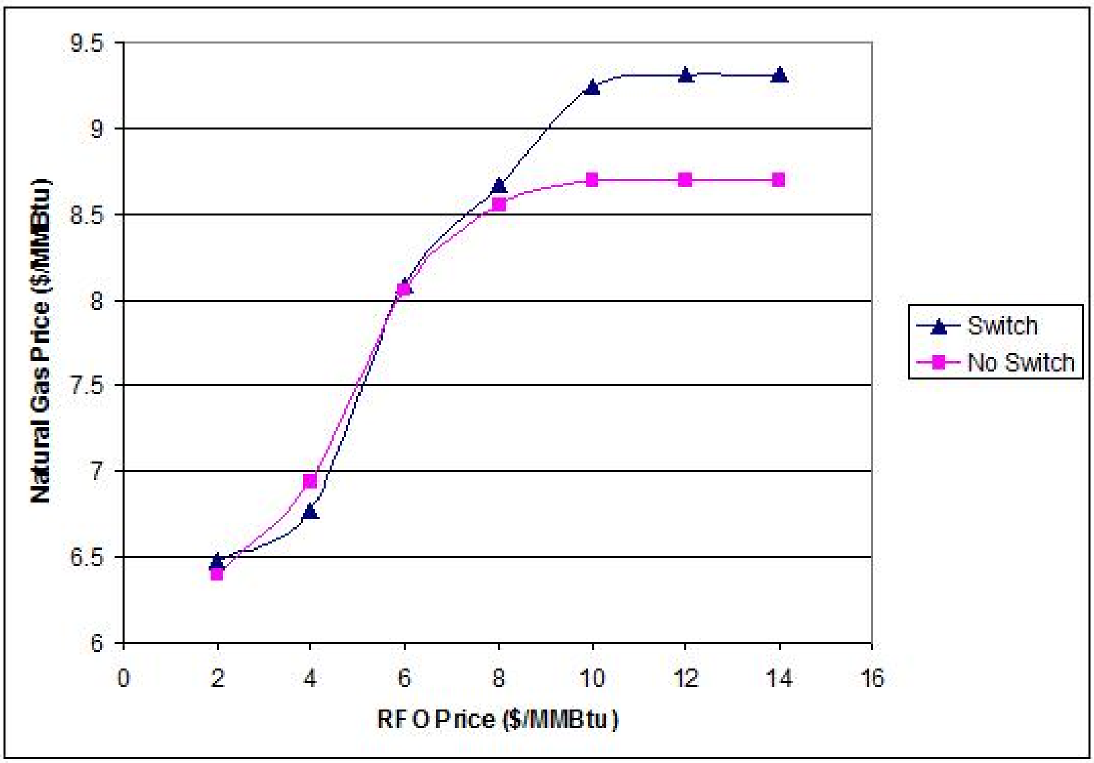

Figure 7: Natural Gas Price under Residual Fuel Oil Price Variations

Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

We now present an example that illustrates the impact of electricity demand changes on the electric power and natural gas markets. When electricity demands increase (or decrease), the electric power prices will increase (or decrease) due to two reasons: 1. generating units with higher generating costs (e.g. heat rates) have to operate more (or less) frequently; 2. the demands for various fuels will also rise which may result in higher (or lower) fuel prices/costs. The magnitude of fuel price changes may depend on the elasticities of fuel prices as well as the fuel competition in power generation which we have demonstrated in Examples 3.1 and 3.2. Next, we apply our theoretical model to show how these interactions between electric power and fuel markets lead to an equilibrium of the entire power supply chain network. The inverse demand function of natural gas takes the same form as that in Example 3. The other fuel prices are exogenous.

We first let the RFO price equal \$10/MMbtu and then increase the electric power demand in each block in each region by 5%. Tables 16 and 17 show the electric power prices, the natural gas prices, and the demands before and after the increases in the electric power demands.

In Tables 16 and 17, higher electricity demands resulted in higher natural gas prices, which was consistent with the interaction between the electric power market and the natural gas market in the reality. For example, in August, 2006, the natural gas price soared by 14% because hot weather across the US led to high electricity demand [33]. Similarly, in July 2007, the natural gas future price for September 2007 increased by 4.7% mainly because of the forecasted high electricity demands in Northeastern and Mid-western cities due to rising temperatures [72].

Table 16: Prices Before the Demand Increase ($/MWh)

Region	**Block 1**	**Block 2**	**Block 3**	**Block 4**	**Block 5**	**Block 6**
ME	105.28	105.28	101.00	81.19	69.90	66.63
NH	109.15	105.39	101.00	81.19	70.83	66.63
VT	109.15	105.39	101.00	81.19	70.83	66.63
CT	137.13	133.34	101.07	84.51	70.83	66.63
RI	109.15	105.39	101.00	81.19	70.83	66.63
SE MA	109.15	105.39	101.00	81.19	70.83	66.63
WC MA	109.15	105.39	101.00	81.19	70.83	66.63
NE MA	122.52	105.39	101.00	81.19	70.83	66.63
NG Demand	40.03 Billion MMBtu					
NG Price	8.69 $/MMBtu					

Table 17: Prices after the Demand Increase ($/MWh)

Region	**Block 1**	**Block 2**	**Block 3**	**Block 4**	**Block 5**	**Block 6**
ME	105.28	105.28	105.28	100.62	74.64	68.34
NH	113.35	105.28	105.28	100.62	74.64	68.34
VT	113.35	105.28	105.28	100.62	74.64	68.34
CT	140.57	134.71	132.61	100.77	74.64	68.34
RI	113.35	105.28	105.28	100.62	74.64	68.34
SE MA	113.35	105.28	105.28	100.62	74.64	68.34
WC MA	113.35	105.28	105.28	100.62	74.64	68.34
NE MA	122.52	112.34	105.28	100.62	74.64	68.34
NG Demand	41.86 Billion MMBtu					
NG Price	8.91 $/MMBtu					

4. Summary, Conclusions, and Future Research

In this paper, we proposed a new model of electric power supply chain networks with fuel markets, which considers both economic transaction networks and physical transmission networks. We derived the optimality conditions of the decision-makers and proved that the governing equilibrium conditions satisfy a variational inequality problem. We also provided some qualitative properties of the model and proposed a computational method. We then conducted a case study where our theoretical model was applied to the New England electric power market and fuel supply markets. The model provides a good simulation of the actual regional electric power prices in New England. We also conducted sensitivity analysis in order to investigate the electric power price and the spark spread under fuel price variations. Additionally, we utilized the model to show how natural gas prices can be influenced by oil prices through electric power networks and markets. In particular, we showed that in New England, the market/grid-level fuel competition has become the major factor affecting the influence of the RFO price on the natural gas price. Finally, we applied our model to quantitatively demonstrate how changes in the demand for electricity influence the electric power and fuel markets.

The model and results presented in this paper are useful in determining and quantifying the interactions between electric power flows and prices and the various fuel supply markets. Such information is important to policy makers who need to ensure system reliability as well as for the energy asset owners and investors who need to manage risk and evaluate their assets.

For future research, several extensions can be developed based on this model. One can add maximum amounts of fuel supplies of the fuel markets to the model to study the system reliability under limited fuel supplies; secondly, one can expand the model to include multiple electric power markets, and to consider broader areas, such as a country or multiple countries. In addition, one can incorporate the price relationship results obtained using this model to other risk management and asset pricing models.

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Appendix

Lemma 1

In equilibrium, if the u^{th} generating unit of genco g in region r_1 supplies electricity to the power pool in region r_2 at demand level w, the marginal supply cost of the generating unit, $\psi_{r_2w}^{gr_1u*}$, is less than or equal to the market clearing price at the power pool, $\rho_{r_2w}^*$.

Proof: We first show the following equilibrium conditions hold for all generating units at all demand levels:

$$\theta_{gr_1uw}^* + \frac{\partial f_{gr_1uw}(q_{gr_1uw}^*)}{\partial q_{gr_1uw}} + \eta_{gr_1uw}^* \begin{cases} = \gamma_{gr_1uw}^*, & \text{if} \quad q_{gr_1uw}^* > 0, \\ \geq \gamma_{gr_1uw}^*, & \text{if} \quad q_{gr_1uw}^* = 0. \end{cases} \qquad (A.1)$$

Since variational inequality (13a) holds for all $(Q^1, q, Q^2, Y^1, Z, \eta, \lambda, \gamma, \theta) \in \mathcal{K}^3$, we let $q_{gr_1uw} = q_{gr_1uw}^*$ if $g \neq \bar{g}$, $r_1 \neq \bar{r}_1$, $u \neq \bar{u}$, and $w \neq \bar{w}$, let $q_{r_2kw}^{gr_1u} = q_{r_2kw}^{gr_1u*}$, let $y_{r_2w}^{gr_1u} = y_{r_2w}^{gr_1u*}$, $q_{gr_1uw}^{am} = q_{gr_1uw}^{am*}$, $z_{gr_1uw} = z_{gr_1uw}^*$, $\gamma_{gr_1uw} = \gamma_{gr_1uw}^*$, $\theta_{gr_1uw} = \theta_{gr_1uw}^*$, $\eta_{gr_1uw} = \eta_{gr_1uw}^*$, and $\lambda_{gr_1uw} = \lambda_{gr_1uw}^*$ for all a, m, g, r_1, u, r_2, k, w. Variational inequality (13a) then reduces to:

$$\left[\theta_{\bar{g}\bar{r}_1\bar{u}\bar{w}}^* + \frac{\partial f_{\bar{g}\bar{r}_1\bar{u}\bar{w}}(q_{\bar{g}\bar{r}_1\bar{u}\bar{w}}^*)}{\partial q_{\bar{g}\bar{r}_1\bar{u}\bar{w}}} + \eta_{\bar{g}\bar{r}_1\bar{u}\bar{w}}^* - \gamma_{\bar{g}\bar{r}_1\bar{u}\bar{w}}^*\right] \times [q_{\bar{g}\bar{r}_1\bar{u}\bar{w}} - q_{\bar{g}\bar{r}_1\bar{u}\bar{w}}^*] \geq 0, \ \forall \ q_{\bar{g}\bar{r}_1\bar{u}\bar{w}} \geq 0. \qquad (A.2)$$

It is easy to verify that the inequality (A.2) is equivalent to (A.1). Moreover, since $\bar{g}$, $\bar{r}_1$, $\bar{u}$, and $\bar{w}$ are chosen arbitrarily, condition (A.1) holds for all generating units at all demand levels. Using the same method, we can also show that

$$\gamma_{gr_1uw}^* + \frac{\partial c_{r_2w}^{gr_1u}(y_{r_2w}^{gr_1u*})}{\partial y_{r_2w}^{gr_1u}} + \sum_{b=1}^{B} \mu_{bw}^* \alpha_{r_1r_2b} \begin{cases} = \rho_{r_2w}^*, & \text{if} \quad y_{r_2w}^{gr_1u*} > 0, \\ \geq \rho_{r_2w}^*, & \text{if} \quad y_{r_2w}^{gr_1u*} = 0. \end{cases} \qquad (A.3)$$

Now, we prove that if the u^{th} generating unit of genco g in region r_1 supplies electricity to the power pool in region r_2 the marginal supply price, $\psi_{r_2w}^{gr_1u*}$, is less than or equal to the market clearing price at the power pool, $\rho_{r_2w}^*$. Note that if the u^{th} generating unit of genco g in region r_1 supplies electric power to the power pool in region r_2, then $q_{gr_1uw}^* > 0$ and

$y_{r_2w}^{gr_1u*} > 0$, which implies that

$$\theta^*_{gr_1uw} + \frac{\partial f_{gr_1uw}(q^*_{gr_1uw})}{\partial q_{gr_1uw}} + \eta^*_{gr_1uw} = \gamma^*_{gr_1uw} \tag{A.4}$$

and

$$\gamma^*_{gr_1uw} + \frac{\partial c_{r_2w}^{gr_1u}(y_{r_2w}^{gr_1u*})}{\partial y_{r_2w}^{gr_1u}} + \sum_{b=1}^{B} \mu^*_{bw}\alpha_{r_1r_2b} = \rho^*_{r_2w}. \tag{A.5}$$

After substituting (A.4) into (A.5), we get

$$\theta^*_{gr_1uw} + \frac{\partial f_{gr_1uw}(q^*_{gr_1uw})}{\partial q_{gr_1uw}} + \eta^*_{gr_1uw} + \frac{\partial c_{r_2w}^{gr_1u}(y_{r_2w}^{gr_1u*})}{\partial y_{r_2w}^{gr_1u}} + \sum_{b=1}^{B} \mu^*_{bw}\alpha_{r_1r_2b} = \rho^*_{r_2w}; \tag{A.6}$$

equivalently,

$$\psi_{r_2w}^{gr_1u*} + \eta^*_{gr_1uw} = \rho^*_{r_2w}. \tag{A.7}$$

Recall that $\eta^*_{gr_1uw}$ is the shadow price of capacity constraint (7), and is greater than or equal to zero. Therefore, we have:

$$\psi_{r_2w}^{gr_1u*} \le \rho^*_{r_2w}. \tag{A.8}$$

Q.E.D.

Lemma 2

In equilibrium, if at the power pool in region r_2 at demand level w, the u^{th} generating unit of genco g in region r_1 has available capacity, the marginal supply cost of the generating unit, $\psi_{r_2w}^{gr_1u*}$, is greater than or equal to the market clearing price at the power pool, $\rho^*_{r_2w}$.

Proof: Using the same method in the proof of Lemma 1, we can show that

$$q^*_{gr_1uw} + z^*_{gr_1uw} \begin{cases} = Cap_{gr_1u}, & \text{if} \quad \eta^*_{gr_1uw} > 0, \\ \le Cap_{gr_1u}, & \text{if} \quad \eta^*_{gr_1uw} = 0. \end{cases} \tag{A.9}$$

In equilibrium, the u^{th} generating unit of genco g in region r_1 having available capacity at demand level w means that $q^*_{gr_1uw} + z^*_{gr_1uw} < Cap_{gr_1u}$, which implies that $\eta^*_{gr_1uw} = 0$. Using (A.1) and (A.3), we obtain

$$\theta^*_{gr_1uw} + \frac{\partial f_{gr_1uw}(q^*_{gr_1uw})}{\partial q_{gr_1uw}} + \eta^*_{gr_1uw} + \frac{\partial c_{r_2w}^{gr_1u}(y_{r_2w}^{gr_1u*})}{\partial y_{r_2w}^{gr_1u}} + \sum_{b=1}^{B} \mu^*_{bw}\alpha_{r_1r_2b} \ge \rho^*_{r_2w}. \tag{A.10}$$

Since $\eta^*_{gr_1uw} = 0$, we get

$$\theta^*_{gr_1uw} + \frac{\partial f_{gr_1uw}(q^*_{gr_1uw})}{\partial q_{gr_1uw}} + \frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u*}_{r_2w})}{\partial y^{gr_1u}_{r_2w}} + \sum_{b=1}^{B} \mu^*_{bw} \alpha_{r_1r_2b} \geq \rho^*_{r_2w}, \qquad (A.11)$$

equivalently,

$$\psi^{gr_1u*}_{r_2w} \geq \rho^*_{r_2w}. \qquad (A.12)$$

Q.E.D.

Qualitative Properties

We now provide some qualitative properties of the solution to variational inequality (24); equivalently, (23). We can derive existence of a solution X^* to (24) simply from the assumption of continuity of functions that enter $F(X)$, which is the case in this model [52, 57]. We now state the following theorems.

Theorem 2: Existence

If $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*,\ \eta^*, \lambda^*, \mu^*, \rho^*_3, \varphi^*)$ satisfies variational inequality (23) then $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*)$ is a solution to the variational inequality problem: determine $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*) \in \mathcal{K}^5$ satisfying

$$\sum_{w=1}^{W}\sum_{a=1}^{A}\sum_{m=1}^{M}\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[\pi_{am}(Q^{1*}) + c^{am}_{gr_1uw}\right] \times [q^{am}_{gr_1uw} - q^{am*}_{gr_1uw}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[\frac{\partial f_{gr_1uw}(q^*_{gr_1uw})}{\partial q_{gr_1uw}}\right] \times [q_{gr_1uw} - q^*_{gr_1uw}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R}\sum_{k=1}^{K} \left[\frac{\partial c^{gr_1u}_{r_2kw}(q^{gr_1u*}_{r_2kw})}{\partial q^{gr_1u}_{r_2kw}} + \hat{c}^{gr_1u}_{r_2kw}(Q^{2*}_w)\right] \times [q^{gr_1u}_{r_2kw} - q^{gr_1u*}_{r_2kw}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R} \frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u*}_{r_2w})}{\partial y^{gr_1u}_{r_2w}} \times [y^{gr_1u}_{r_2w} - y^{gr_1u*}_{r_2w}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \frac{\partial c_{gr_1uw}(z^*_{gr_1uw})}{\partial z_{gr_1uw}} \times [z_{gr_1uw} - z^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}\sum_{k=1}^{K} \hat{c}^{r_1}_{r_2kw}(Y^{2*}_w) \times [y^{r_1}_{r_2kw} - y^{r_1*}_{r_2kw}] \geq 0, \; \forall (Q^1, q, Q^2, Y^1, Y^2, Z) \in \mathcal{K}^5, \quad (A.13)$$

where $\mathcal{K}^5 \equiv \{(Q^1, q, Q^2, Y^1, Y^2, Z) | (Q^1, q, Q^2, Y^1, Y^2, Z) \in R_+^{AMNW+NRKW+NRW+2NW+R^2KW}$ *and* (5), (6), (7), (8), (19), *and*

$$L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} y^{gr_1u}_{rw} \geq L_w \sum_{r_2=1}^{R}\sum_{k=1}^{K} y^{r}_{r_2kw}, \forall r; \forall w, \quad (A.14)$$

$$L_w \sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}} z_{gr_1uw} \geq L_w OPR_{r_1w}, \forall r_1; \forall w, \quad (A.15)$$

$$and \;\; L_w \sum_{r_1=1}^{R}\sum_{r_2=1}^{R} [\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}}\sum_{k=1}^{K} q^{gr_1u}_{r_2kw} + \sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}} y^{gr_1u}_{r_2w} + \sum_{k=1}^{K} y^{r_1}_{r_2kw}] \alpha_{r_1r_2b} \leq L_w TCap_b, \; \forall b; \forall w \quad (A.16)$$

are satisfied$\}$.

A solution to (A.13) is guaranteed to exist provided that $\mathcal{K}^5$ *is nonempty. Moreover, if* $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*)$ *is a solution to (A.13), there exist* $(\eta^*, \lambda^*, \mu^*, \rho_3^*, \varphi^*) \in R_+^{2NW+BW+2RW}$ *with* $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*, \; \eta^*, \lambda^*, \mu^*, \rho_3^*, \varphi^*)$ *being a solution to variational inequality (23).*

Proof: The proof is an analog of the proof of Theorem 3 in [62]. Q.E.D.

Since all the constraints of $\mathcal{K}^5$ are linear, it is easy to verify the existence of a feasible point in $\mathcal{K}^5$. In our case study, because $\mathcal{K}^5$ is nonempty for the New England electric power supply chain, the existence of a solution is guaranteed for each empirical example in Section 5.

We now recall the concept of monotonicity and state an additional theorem.

Theorem 3: Monotonicity

Suppose that all cost functions in the model are continuously differentiable and convex; all unit cost functions are monotonically increasing, and the inverse price functions at the fuel supply markets are monotonically increasing. Then the vector F that enters the variational inequality (23) as expressed in (24) is monotone, that is,

$$\left\langle (F(X') - F(X''))^T, X' - X'' \right\rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}, X' \neq X''. \tag{A.17}$$

Proof:

We prove Theorem 3 by expanding

$$\left\langle (F(X') - F(X''))^T, X' - X'' \right\rangle \tag{A.18}$$

$$= \sum_{w=1}^{W}\sum_{a=1}^{A}\sum_{m=1}^{M}\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[\pi_{am}(Q^{1'}) + c^{am}_{gr_1uw} - \pi_{am}(Q^{1''}) - c^{am}_{gr_1uw}\right] \times [q^{am'}_{gr_1uw} - q^{am''}_{gr_1uw}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} [\frac{\partial f_{gr_1uw}(q'_{gr_1uw})}{\partial q_{gr_1uw}} - \frac{\partial f_{gr_1uw}(q''_{gr_1uw})}{\partial q_{gr_1uw}}] \times [q'_{gr_1uw} - q''_{gr_1uw}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R}\sum_{k=1}^{K} [\frac{\partial c^{gr_1u}_{r_2kw}(q^{gr_1u'}_{r_2kw})}{\partial q^{gr_1u}_{r_2kw}} + \sum_{b=1}^{B} \mu^{'}_{bw}\alpha_{r_1r_2b} + \hat{c}^{gr_1u}_{r_2kw}(Q^{2'}_w)$$

$$- \frac{\partial c^{gr_1u}_{r_2kw}(q^{gr_1u''}_{r_2kw})}{\partial q^{gr_1u}_{r_2kw}} - \sum_{b=1}^{B} \mu^{''}_{bw}\alpha_{r_1r_2b} - \hat{c}^{gr_1u}_{r_2kw}(Q^{2''}_w)] \times [q^{gr_1u'}_{r_2kw} - q^{gr_1u''}_{r_2kw}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R} [\frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u'}_{r_2w})}{\partial y^{gr_1u}_{r_2w}} + \sum_{b=1}^{B} \mu^{'}_{bw}\alpha_{r_1r_2b} - \rho^{'}_{r_2w} - \frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u''}_{r_2w})}{\partial y^{gr_1u}_{r_2w}} - \sum_{b=1}^{B} \mu^{''}_{bw}\alpha_{r_1r_2b} + \rho^{''}_{r_2w}]$$

$$\times [y^{gr_1u'}_{r_2w} - y^{gr_1u''}_{r_2w}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[\frac{\partial c_{gr_1uw}(z'_{gr_1uw})}{\partial z_{gr_1uw}} - \varphi'_{r_1w} - \frac{\partial c_{gr_1uw}(z''_{gr_1uw})}{\partial z_{gr_1uw}} + \varphi''_{r_1w}\right] \times [z'_{gr_1uw} - z''_{gr_1uw}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}\sum_{k=1}^{K} [\rho^{'}_{r_2w} + \hat{c}^{r_1}_{r_2kw}(Y^{2'}_w) + \sum_{b=1}^{B} \mu^{'}_{bw}\alpha_{r_1r_2b}$$

$$-\rho^{''}_{r_2w} - \hat{c}^{r_1}_{r_2kw}(Y^{2''}_w) - \sum_{b=1}^{B} \mu^{'}_{bw}\alpha_{r_1r_2b}] \times [y^{r'_1}_{r_2kw} - y^{r''_1}_{r_2kw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{b=1}^{B}[TCap_b - \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}[\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}}\sum_{k=1}^{K} q^{gr_1u'}_{r_2kw} + \sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}} y^{gr_1u'}_{r_2w} + \sum_{k=1}^{K} y^{r'_1}_{r_2kw}]\alpha_{r_1r_2b}$$

$$-TCap_b + \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}[\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}}\sum_{k=1}^{K} q^{gr_1u''}_{r_2kw} + \sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}} y^{gr_1u''}_{r_2w} + \sum_{k=1}^{K} y^{r''_1}_{r_2kw}]\alpha_{r_1r_2b}] \times [\mu^{'}_{bw} - \mu^{''}_{bw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r=1}^{R}[\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} y^{gr_1u'}_{rw} - \sum_{r_2=1}^{R}\sum_{k=1}^{K} y^{r'}_{r_2kw} - \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} y^{gr_1u''}_{rw} + \sum_{r_2=1}^{R}\sum_{k=1}^{K} y^{r''}_{r_2kw}] \times [\rho^{'}_{rw} - \rho^{''}_{rw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r=1}^{R}[\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}} z'_{gr_1uw} - OPR_{r_1} - \sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}} z''_{gr_1uw} + OPR_{r_1}] \times [\varphi^{'}_{r_1w} - \varphi^{''}_{r_1w}], \quad (A.19)$$

which, after algebraic simplification, yields

$$\sum_{w=1}^{W}\sum_{a=1}^{A}\sum_{m=1}^{M}\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[\pi_{am}(Q^{1'}) + c^{am}_{gr_1uw} - \pi_{am}(Q^{1''}) - c^{am}_{gr_1uw}\right] \times [q^{am'}_{gr_1uw} - q^{am''}_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}[\frac{\partial f_{gr_1uw}(q'_{gr_1uw})}{\partial q_{gr_1uw}} - \frac{\partial f_{gr_1uw}(q''_{gr_1uw})}{\partial q_{gr_1uw}}] \times [q'_{gr_1uw} - q''_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R}\sum_{k=1}^{K}[\frac{\partial c^{gr_1u}_{r_2kw}(q^{gr_1u'}_{r_2kw})}{\partial q^{gr_1u}_{r_2kw}} + \hat{c}^{gr_1u}_{r_2kw}(Q^{2'}_w) - \frac{\partial c^{gr_1u}_{r_2kw}(q^{gr_1u''}_{r_2kw})}{\partial q^{gr_1u}_{r_2kw}} - \hat{c}^{gr_1u}_{r_2kw}(Q^{2''}_w)] \times [q^{gr_1u'}_{r_2kw} - q^{gr_1u''}_{r_2kw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R}[\frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u'}_{r_2w})}{\partial y^{gr_1u}_{r_2w}} - \frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u''}_{r_2w})}{\partial y^{gr_1u}_{r_2w}}] \times [y^{gr_1u'}_{r_2w} - y^{gr_1u''}_{r_2w}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\left[\frac{\partial c_{gr_1uw}(z'_{gr_1uw})}{\partial z_{gr_1uw}} - \frac{\partial c_{gr_1uw}(z''_{gr_1uw})}{\partial z_{gr_1uw}}\right] \times [z'_{gr_1uw} - z''_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}\sum_{k=1}^{K}\left[\hat{c}^{r_1}_{r_2kw}(Y^{2'}_w) - \hat{c}^{r_1}_{r_2kw}(Y^{2''}_w)\right] \times [y^{r'_1}_{r_2kw} - y^{r''_1}_{r_2kw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{b=1}^{B}\sum_{r_1=1}^{R}\sum_{r_2=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}}\sum_{k=1}^{K} \alpha_{r_1r_2b}[q^{gr_1u'}_{r_2kw} - q^{gr_1u''}_{r_2kw}] \times [\mu^{'}_{bw} - \mu^{''}_{bw}]$$

$$-\sum_{w=1}^{W} L_w \sum_{b=1}^{B}\sum_{r_1=1}^{R}\sum_{r_2=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}}\sum_{k=1}^{K} \alpha_{r_1r_2b}[q^{gr_1u'}_{r_2kw} - q^{gr_1u''}_{r_2kw}] \times [\mu^{'}_{bw} - \mu^{''}_{bw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{b=1}^{B}\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}} \alpha_{r_1 r_2 b}[y_{r_2 w}^{gr_1 u'} - y_{r_2 w}^{gr_1 u''}] \times [\mu_{bw}^{'} - \mu_{bw}^{''}]$$

$$-\sum_{w=1}^{W} L_w \sum_{b=1}^{B}\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}} \alpha_{r_1 r_2 b}[y_{r_2 w}^{gr_1 u'} - y_{r_2 w}^{gr_1 u''}] \times [\mu_{bw}^{'} - \mu_{bw}^{''}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}\sum_{k=1}^{K} \alpha_{r_1 r_2 b}[y_{r_2 kw}^{r_1'} - y_{r_2 kw}^{r_1''}] \times [\mu_{bw}^{'} - \mu_{bw}^{''}]$$

$$-\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}\sum_{k=1}^{K} \alpha_{r_1 r_2 b}[y_{r_2 kw}^{r_1'} - y_{r_2 kw}^{r_1''}] \times [\mu_{bw}^{'} - \mu_{bw}^{''}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r=1}^{R}\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} [y_{rw}^{gr_1 u'} - y_{rw}^{gr_1 u''}] \times [\rho_{rw}^{'} - \rho_{rw}^{''}]$$

$$-\sum_{w=1}^{W} L_w \sum_{r=1}^{R}\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} [y_{rw}^{gr_1 u'} - y_{rw}^{gr_1 u''}] \times [\rho_{rw}^{'} - \rho_{rw}^{''}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r=1}^{R}\sum_{r_2=1}^{R}\sum_{k=1}^{K} [y_{r_2 kw}^{r'} - y_{r_2 kw}^{r''}] \times [\rho_{rw}^{'} - \rho_{rw}^{''}]$$

$$-\sum_{w=1}^{W} L_w \sum_{r=1}^{R}\sum_{r_2=1}^{R}\sum_{k=1}^{K} [y_{r_2 kw}^{r'} - y_{r_2 kw}^{r''}] \times [\rho_{rw}^{'} - \rho_{rw}^{''}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}} [z_{gr_1 uw}' - z_{gr_1 uw}''] \times [\varphi_{r_1 w}^{'} - \varphi_{r_1 w}^{''}]$$

$$-\sum_{w=1}^{W} L_w \sum_{r=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}} [z_{gr_1 uw}' - z_{gr_1 uw}''] \times [\varphi_{r_1 w}^{'} - \varphi_{r_1 w}^{''}] \qquad (A.20)$$

which is equal to

$$\sum_{w=1}^{W}\sum_{a=1}^{A}\sum_{m=1}^{M}\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[\pi_{am}(Q^{1'}) - \pi_{am}(Q^{1''})\right] \times [q_{gr_1 uw}^{am'} - q_{gr_1 uw}^{am''}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} [\frac{\partial f_{gr_1 uw}(q_{gr_1 uw}')}{\partial q_{gr_1 uw}} - \frac{\partial f_{gr_1 uw}(q_{gr_1 uw}'')}{\partial q_{gr_1 uw}}] \times [q_{gr_1 uw}' - q_{gr_1 uw}'']$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R}\sum_{k=1}^{K} [\frac{\partial c_{r_2 kw}^{gr_1 u}(q_{r_2 kw}^{gr_1 u'})}{\partial q_{r_2 kw}^{gr_1 u}} + \hat{c}_{r_2 kw}^{gr_1 u}(Q_w^{2'}) - \frac{\partial c_{r_2 kw}^{gr_1 u}(q_{r_2 kw}^{gr_1 u''})}{\partial q_{r_2 kw}^{gr_1 u}} - \hat{c}_{r_2 kw}^{gr_1 u}(Q_w^{2''})]$$

$$
\times [q^{gr_1u'}_{r_2kw} - q^{gr_1u''}_{r_2kw}]
$$

$$
+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} [\frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u'}_{r_2w})}{\partial y^{gr_1u}_{r_2w}} - \frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u''}_{r_2w})}{\partial y^{gr_1u}_{r_2w}}] \times [y^{gr_1u'}_{r_2w} - y^{gr_1u''}_{r_2w}]
$$

$$
+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \left[\frac{\partial c_{gr_1uw}(z'_{gr_1uw})}{\partial z_{gr_1uw}} - \frac{\partial c_{gr_1uw}(z''_{gr_1uw})}{\partial z_{gr_1uw}} \right] \times [z'_{gr_1uw} - z''_{gr_1uw}]
$$

$$
+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \left[\hat{c}^{r_1}_{r_2kw}(Y^{2'}_w) - \hat{c}^{r_1}_{r_2kw}(Y^{2''}_w) \right] \times [y^{r'_1}_{r_2kw} - y^{r''_1}_{r_2kw}]. \qquad (A.21)
$$

Hence, if all cost functions in the model are continuously differentiable and convex; all unit cost functions are nondecreasing, and the inverse supply functions are nondecreasing, then (A.21), equivalently, (A.18) are both greater than or equal to zero, and, therefore, $F(X)$ is monotone. Q.E.D.

In our case study for New England, $F(X)$ is monotone and the Jacobian of $F(X)$ is uniformly positive semidefinite. Moreover, $F(X)$ is linear and, hence, Lipschitz continuous (see [57]).

Computational Method

We now consider the computation of solutions to variational inequality (23). In particular, we recall the modified projection method [57]. The method converges to a solution of the model provided that $F(X)$ is monotone and Lipschitz continuous, and a solution exists, which is the case for our empirical application. For problems with special structure or special cost function specifications, other (decomposition-type) algorithms may be exploited. Next, we present the modified projection method.

The Computational Procedure

Step 0: Initialization

Start with an $X^0 \in \mathcal{K}$ and select ω, such that $0 < \omega \leq \frac{1}{L}$, where L is the Lipschitz constant for function $F(X)$. Let $\mathcal{T} = 1$.

Step 1: Construction and Computation

Compute $\bar{X}^{\mathcal{T}-1}$ by solving the variational inequality subproblem:

$$\left\langle (\bar{X}^{\mathcal{T}-1} + (\omega F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}))^T, X' - \bar{X}^{\mathcal{T}-1} \right\rangle \geq 0, \quad \forall X' \in \mathcal{K}. \tag{A.22}$$

Step 2: Adaptation

Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\left\langle (X^{\mathcal{T}} + (\omega F(\bar{X}^{\mathcal{T}-1}) - X^{\mathcal{T}-1}))^T, X' - X^{\mathcal{T}} \right\rangle \geq 0, \quad \forall X' \in \mathcal{K}. \tag{A.23}$$

Step 3: Convergence Verification

If $||X^{\mathcal{T}} - X^{\mathcal{T}-1}||_\infty \leq \epsilon$ with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$ and go to Step 1. (We set the parameter $\omega = 0.05$ and the tolerance $\epsilon = 0.001$ for all computations of the numerical examples in Section 5.)

Note that the subproblems in Steps 1 and 2 above are separable quadratic programming problems and, hence, there are numerous algorithms that can be used to solve these embedded subproblems.