Exchange Rates and Multicommodity International Trade: Insights from Spatial Price Equilibrium Modeling with Policy Instruments via Variational Inequalities

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Abstract: In this paper, we construct a multicommodity international trade spatial price equilibrium model of special relevance to agriculture in which exchange rates are included along with policy instruments in the form of tariffs, subsidies as well as quotas. The model allows for multiple trade routes between country origin nodes and country destination nodes and these trade routes can include different modes of transportation and transport through distinct countries. We capture the impacts of exchange rates through the definition of effective path costs and identify the governing multicommodity international trade spatial price equilibrium conditions, which are then formulated as a variational inequality problem in product path flows. Existence results are established and a computational procedure presented. The illustrative numerical examples and a case study are inspired by the impacts of the war against Ukraine on agricultural trade flows and product prices. The modeling and algorithmic framework allows for the quantification of the impacts of exchange rates and various trade policies, as well as the addition or deletion of supply markets, demand markets and/or routes, on supply and demand market prices in local currencies, and on the volume of product trade flows with implications for food security.

Key words: exchange rates, spatial price equilibrium, international trade, networks, variational inequalities, agriculture

1. Introduction

Exchange rates represent the value (price) of one currency relative to another currency. They are important economic parameters in international trade, with changes in the exchange rate affecting the decision-making of individuals, businesses, and governments. A separate exchange rate exists for each pair of independent currencies, such as the US dollar and the Ukrainian hryvnia, the hryvnia and the euro, the euro and the Japanese yen, and so on. Changes in the exchange rate have a direct effect on the prices of goods and services produced in a given country relative to those produced in another country. If the US dollar appreciates (the exchange rate increases), the relative price of domestic goods and services increases while the relative price of foreign goods and services falls. The change in relative prices will decrease US exports and will increase its imports.

As noted by Klein (2022), exchange rates reflect the relative demand for different countries' assets and are responsive to current economic conditions and expectations of future conditions, with the US dollar getting stronger over the past year, with the greatest rate of the increase occurring since the Russian invasion of Ukraine on February 24, 2022, with the war still ongoing (see Bilefsky, Perez-Pena, and Nagourney (2022)). In fact, over the period June 1, 2021, through May 31, 2022, the US dollar increased by 12% against the euro; it increased 9% against the British pound and 16% against the yen. In contrast, during the first year of the COVID-19 pandemic, the dollar weakened with respect to the euro, the British pound, and the yen.

Identifying quantitatively the impacts on international trade of exchange rates can provide trade and regulatory bodies with additional information on product trade volumes, consumer prices, plus what can be expected if different trade policies are instituted, as in the form, for example, of subsidies and tariffs, which have become highly relevant as the world continues to battle the COVID-19 pandemic and millions on the planet suffer from hunger and growing food insecurity. Interestingly, the inclusion of exchange rates explicitly into spatial price equilibrium models has been lacking although spatial price equilibrium models have found rich applications in agriculture, mineral, and energy markets (see, e.g., Labys and Yang (1991, 1997), Nagurney (1999), Nagurney and Besik (2022), and the references therein). Devadoss and Sabala (2020), in their recent paper, emphasize that, to the best of their knowledge, no study until theirs had previously used the spatial price equilibrium model to analyze the effects of exchange rate changes, which they consider to be a major contribution. Their study focused on the yuan-dollar exchange rate and cotton markets and proposed a single commodity spatial price equilibrium model. The model in this paper, in contrast, is a multicommodity one with additional significant extensions explicated below.

Garcia-Salazar, Skaggs, and Crawford (2012), using a spatial and intertemporal equilibrium model, analyzed the case of the Mexican corn sector, and, as part of their case study, they assessed two scenarios of Mexican peso/USD exchange rate increases; however, the effect of exchange rate volatility was limited to the relationship between the international price and the local wholesale price of the commodity and did not include transportation costs. Moreover, the effects of policy instruments were not studied.

In this paper, we acknowledge the work of Devadoss and Sabala (2020) and turn to the construction of a general multicommodity international trade spatial price equilibrium model, in which the supply price and demand price functions need not be separable; the unit transportation costs between country are flow-dependent and not fixed, plus there need not be just a single trade route from the product origin country to the destination country. Furthermore, a route can consist of a single link or multiple links. We allow for both subsidies and tariffs and demonstrate how exchange rates can be captured as the products get transported on routes, which can entail transport through different countries with different currencies, an advance not previously captured. As an illustration, consider the blockade for many months of the Black Sea by the Russians in 2022, following the invasion of February 24, 2022, which prevented the export of harvested and stored grain and other agricultural products from Ukraine via the more efficient maritime route and resulted in the use of new routes, which were more time-consuming, inefficient, and costly through the Baltics, Poland, or Romania, onwards (Khurshudyan and Morgunov (2022)). In addition, we include capacities on trade routes. The capacities can also serve as a type of quota.

In this paper, the multicommodity international trade spatial price equilibrium model with exchange rates and policy instruments is constructed, and the governing equilibrium conditions are stated, followed by the derivation of the resulting variational inequality formulation in path flows, which is novel. Existence of an equilibrium solution is established and a computational scheme proposed. This work is inspired by Russia's war on Ukraine and the need to assess its impacts on agricultural trade as well as on food insecurity. Both the illustrative examples and the case study are drawn from the war, which has caused great disruptions globally.

2. Literature Review and Organization of the Paper

The methodology utilized in the construction of the modeling and algorithmic framework for the international trade spatial price equilibrium model with exchange rates and trade policy instruments is the theory of variational inequalities (cf. Nagurney (1999, 2006) and the references therein). In this section, we highlight the most closely related and relevant literature to the research in this paper.

2.1 Variational Inequality Formulations of Spatial Price Equilibrium Models with Multiple Routes

It is important to recognize that spatial price equilibrium models, originating in the seminal contributions of Samuelson (1952) and Takayama and Judge (1964, 1971), are partial equilibrium models and assume perfect competition. Early variational inequality formulations of such problems, which relaxed previously imposed assumptions on the underlying functions that enabled optimization reformulations of the equilibrium conditions, and that also recognized the importance of having alternative routes between supply and demand markets, have included the models of Florian and Los (1982), Dafermos and Nagurney (1984), Friesz, Harker, and Tobin (1984), Harker (1985), Nagurney, Thore, and Pan (1996), and Daniele (2004)). Variational inequality formulations of spatial price equilibrium problems with trade instruments have been constructed by Nagurney, Nicholson, and Bishop (1996), Nagurney, Besik, and Dong (2019), and Nagurney, Salarpour, and Dong (2022). Other variational inequality models of spatial price equilibrium problems have included product quality (Li, Nagurney, and Yu (2018)) and product perishability (see Nagurney and Aronson (1989)), the latter even with policy instruments in Nagurney (2022). However, none of these models incorporated exchange rates, although the underlying applications, as noted earlier, are important to international trade. Recent research in another area of application - that of refugee migration networks - has also utilized variational inequality theory for the formulation, analysis, and solution of such problems in the case of multiple routes from origin nodes to destination nodes in the presence of regulations (see Nagurney, Daniele, and Nagurney (2020)).

2.2 Variational Inequality Formulations of Spatial Oligopolistic Models with Exchange Rates

In terms of imperfectly competitive models under, for example, oligopolistic competition in a supply chain network context, there have been several variational inequality models developed with exchange rates. For example, Liu and Nagurney (2011) investigated the impact of foreign exchange rate uncertainty and competition intensity on supply chain firms engaged in offshore outsourcing activities, with consideration of firms' decision-making as to material procurement, pricing, offshore-outsourcing, transportation, and in-house production under competition and uncertainty in the foreign exchange rate. The transportation costs were considered fixed, however, and were not flow-dependent. Furthermore, there was a single route implicitly assumed between stakeholders. Cruz (2013) constructed a multitiered global supply chain network equilibrium model with exchange rates using variational inequality theory with the inclusion of heterogeneous Corporate Social Responsibility (CSR) activities and noted that CSR activities could be used to potentially mitigate global supply chain risk.

Nagurney and Matsypura (2005), earlier, developed a dynamic, multitiered global supply chain model consisting of manufacturers, distributors, and retailers that included the option of electronic commerce. The variational inequality model had exchange rates and considered decision-making under risk and uncertainty. Cruz, Nagurney, and Wakolbinger (2006), subsequently, proposed a network framework that integrated global supply chain networks with social networks, and incorporated exchange rates. The models developed therein were a static one using variational inequality theory and a dynamic one, where the theory of projected dynamical systems (cf. Nagurney and Zhang (1996)) was utilized. These models, unlike the model in this paper, were imperfectly competitive models. Furthermore, the paths consisted of single links and, hence, would not allow for transportation through different countries, which can, in our framework, take place via different modes of transportation.

2.3 Variational Inequality Formulations of Supply Chain Network Models with Policies and Applications to Agriculture

The research in this paper has been inspired, in part, by the impacts of Russia's war on Ukraine on agricultural trade with ancillary effects on food insecurity. It is worthwhile, hence, to acknowledge the literature on supply chain network models with applications to agriculture that also incorporates policies such as tariffs and quotas using also variational inequality theory. Nagurney, Besik, and Nagurney (2019) constructed a global supply chain network model with profit-maximizing, competing firms in the presence of quantitative trade policy instruments in the form of tariff rate quotas and presented a case study on avocados. Nagurney, Besik, and Li (2019), in turn, also considered competing firms with consumers being responsive to product quality and with a tariff or quota imposed. The authors presented the computed solutions to numerical examples drawn from a soybean application. Yu and Nagurney (2013), Besik and Nagurney (2017), and Besik, Nagurney, and Dutta (2023) formulated agricultural-based competitive supply chain network models under oligopolistic competition in quality of food, such as fresh produce, is subject to decay and perishability. However, trade policies were not considered and exchange rates were not incorporated.

2.4 Organization of the Paper

This paper is organized as follows. In Section 3, the multicommodity international trade spatial price equilibrium model is constructed. It includes subsidies and tariffs, and explicit exchange rates. It also allows for multiple paths (trade routes) between origin country nodes and destination country nodes. Several effective exchange rate concepts are defined to allow for the calculation of the costs and prices in the currency at the destination node relative to the origin node of the product. In addition, illustrative examples are presented to formalize the concepts and the understanding of the framework. The examples are based on the agricultural trade of wheat from Ukraine to Lebanon before the war and during the war, under different scenarios. Section 3 also presents an existence result. Section 4 delineates an algorithm that is then applied in Section 5 to compute solutions to numerical examples comprising a case study focusing on agricultural exports from Ukraine in wartime. Section 6 summarizes our results and presents our conclusions.

3. The Multicommodity International Trade Spatial Price Equilibrium Model

Consider a network consisting of m origin nodes representing different countries in which multiple homogeneous products (commodities) are produced and with n destination nodes denoting possible demand points at which the products can be consumed, with each also denoting a distinct country. Each origin node i is connected to a destination node j via one or more paths, with a typical path denoted by p. Each path represents a trade route and consists of one or more directed links that join nodes in the network. Intermediate nodes in the network, which are transit points, also correspond to countries. Let P_{ij} denote the set of paths connecting the pair of origin/destination country nodes (i, j), with the set of all paths denoted by P. The paths are acyclic. The set P^i denotes all the paths from country i to the destination countries, and P_j denotes the set of paths from all origin countries to destination country j. The network is represented by the graph $G = [N, L]$, where N is the set of nodes in the network and L is the set of links. A typical link is denoted by a and represents transport from a country node at which the link originates to the node denoting the country at which the link terminates. A trade route can entail transportation through multiple countries, depending on the application, and via different modes, such as rail, truck, air, or water (sea, river, etc.). A node can be a country origin node, a country destination node, or a country transit node. There are H commodities, with a typical commodity denoted by h. A hypothetical spatial price network topology is depicted in Figure 1.

Associated with each link $a \in L$ is an exchange rate e_a, reflecting the exchange rate from the country (node) that the link emanates from to the country (node) that it terminates in. Later in this section, we discuss how the unit transportation costs on a link are adapted to capture the exchange rate(s) so that the true cost that will be paid by consumers at the demand markets is quantified. Also, associated with each pair of origin/destination countries

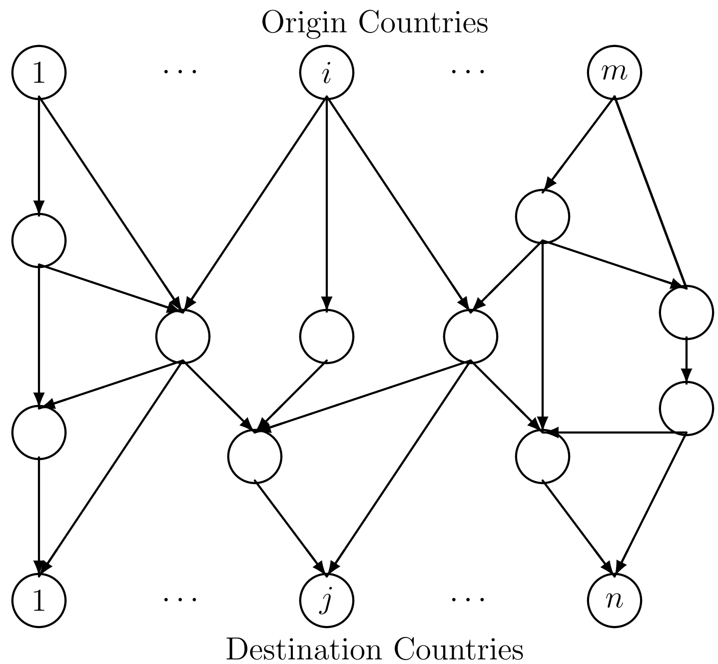

Figure 1: Sample Spatial Price Network Topology with Origin Countries and Destination Countries

(i, j) is the exchange rate e_{ij} for $i = 1, \ldots, m$; $j = 1, \ldots, n$.

The model is quantity-based in that the variables are product flows from origin countries to destination countries. A price and quantity spatial price equilibrium model, which also handles routes between countries, was proposed by Nagurney, Salarpour, and Dong (2022). However, no exchange rates were incorporated therein. Furthermore, the model did not contain subsidies, as the new model here does.

The relevant notation is now presented. Let Q_p^h denote the flow of commodity h on path p and group all such flows into the vector $Q \in R_+^{Hn_P}$, where n_P denotes the number of paths in the network. The flow on a link a of commodity h is denoted by f_a^h and all the link flows are grouped into the vector $f \in R_+^{Hn_L}$, where n_L is the number of links in the network. Let $\bar{Q}_p^h$ denote the upper bound on the flow of commodity h on a path p for all $p \in P$. This upper bound can reflect a capacity and/or a quota. We group the path flow capacities into the vector $\bar{Q} \in R_+^{Hn_P}$.

All vectors are column vectors.

Conservation of Flow Equations

The conservation of flow equations are as follows.

All commodity path flows must be nonnegative:

$$Q_p^h \geq 0, \quad h = 1, \ldots, H; \forall p \in P. \tag{1}$$

The flow on a link a of commodity h, in turn, is equal to the sum of the path flows of the commodity h that use the link, and is given by the expression:

$$f_a^h = \sum_{p \in P} Q_p^h \delta_{ap}, \quad h = 1, \ldots, H; \forall a \in L. \tag{2}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and is 0, otherwise.

The supply of commodity h produced in country i, s_i^h, is equal to the shipments of the commodity from the country to all destination countries:

$$s_i^h = \sum_{p \in P^i} Q_p^h, \quad h = 1, \ldots, H; i = 1, \ldots, m, \tag{3}$$

whereas the demand for commodity h in country j, d_j^h, is equal to the shipments of the commodity from all origin countries to that country:

$$d_j^h = \sum_{p \in P_j} Q_p^h, \quad h = 1, \ldots, H; j = 1, \ldots, n. \tag{4}$$

The supplies of the commodities are grouped into the vector $s \in R_+^{Hm}$ and the demands into the vector $d \in R_+^{Hn}$.

Trade Policy Instruments

Since spatial price equilibrium models have had multiple applications to agricultural markets, and it is well-known that many governments around the world subsidize agriculture, we introduce a subsidy associated with commodity h and imposed by the government in country i, which is denoted by sub_i^h for $h = 1, \ldots, H$; $i = 1, \ldots, m$. The subsidies are assumed to be nonnegative. Also, since tariffs are highly relevant to international trade of agricultural, as well as other, products, we denote the unit tariff levied by country j on commodity h from country i by τ_{ij}^h for $h = 1, \ldots, H$; $i = 1, \ldots, m$; $j = 1, \ldots, n$. Clearly, some countries may decide not to impose a tariff; the same as for a subsidy. Tariffs within a country are not imposed; hence, $\tau_{ii}^h = 0$, $h = 1, \ldots, H$; $i = 1, \ldots, m$.

The Model Supply Price, Demand Price, and Unit Transportation Cost Functions

The supply price function for commodity h of country i is denoted by π_i^h and, in general, the supply price of a commodity h in a country i can depend not only on the supply of the commodity in the country, but also on the supplies of the other commodities in the country as well as on the supplies of all the commodities in all the other countries. Hence, we have that:

$$\pi_i^h = \pi_i^h(s), \quad h = 1, \ldots, H; i = 1, \ldots, m. \tag{5a}$$

With notice of the conservation of flow equations (3), we may define new supply price functions $\tilde{\pi}_i^h$; $h = 1, \ldots, H$; $i = 1, \ldots, m$, such that

$$\tilde{\pi}_i^h(Q) \equiv \pi_i^h(s). \tag{5b}$$

The demand price functions, in turn, can, in general, depend not only on the consumption of the commodity in the particular country (the demand), but also on the demands for the other commodities in the country as well as the demands for the commodities in other countries; that is:

$$\rho_j^h = \rho_j^h(d), \quad h = 1, \ldots, H; j = 1, \ldots, n, \tag{6a}$$

where ρ_j^h denotes the demand price for commodity h in country j.

Making use now of conservation of flow equations (4), we construct equivalent demand price functions $\tilde{\rho}_j^h$; $h = 1, \ldots, H$; $j = 1, \ldots, n$, as follows:

$$\tilde{\rho}_j^h(Q) \equiv \rho_j^h(d). \tag{6b}$$

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With each link $a \in L$, and commodity h, we associate a unit transportation cost c_a^h such that

$$c_a^h = c_a^h(f), \quad h = 1, \ldots, H; \forall a \in L. \tag{7a}$$

Here, we allow the transportation costs to include transaction costs, for the sake of generality and practical relevance. The unit transportation cost on a link a, $\forall a \in L$, for each commodity h; $h = 1, \ldots, H$, is in the currency of the country (node) from which the link emanates. For example, in the case of a ground transportation link in Ukraine, the cost would be in hryvnia. We consider the transportation cost to be a generalized cost that can also include, for example, time, risk, etc. These transportation costs also capture congestion and bottlenecks as well as competition among commodities for transportation on links. Such issues have become quite relevant in wartime for Ukraine in the export of its agricultural products using different modes of transportation and routes.

According to (7a), the unit transportation cost of a commodity between a pair of countries on a link can depend, in general, not only on the flow of commodity on the particular link, but also on the flow of other commodities on the link and on other links.

Because of the conservation of flow equations (2), we can define link unit transportation cost functions $\tilde{c}_a^h(Q)$, $\forall a \in L$, $\forall h$, as:

$$\tilde{c}_a^h(Q) \equiv c_a^h(f). \tag{7b}$$

The supply price, unit transportation cost and the demand price functions are assumed to be continuous.

In the majority of previous studies, as noted in the Introduction, exchange rates were not included in spatial equilibrium models, and it was, hence, assumed that the relevant prices and costs were all in a base currency. Since we believe that evaluating the impacts of changes in exchange rates is important, we include exchange rates in our model explicitly. Furthermore, because the model allows for multiple trade routes between a pair of origin and destination countries, and these can reflect transport through one or more countries, the incorporation of exchange rates needs to be solidified.

Effective Exchange Rates and Transportation Costs

Observe that, in order to appropriately quantify the effective transportation cost on a link a for a commodity h, if a commodity makes use of the link on a path from an origin country node to a destination country node, one needs to calculate the effective exchange rate associated with the commodity on link a being transported onward on path p, which is denoted by e_a^p. Note that e_a^p is the product of the exchange rates on the links on path p that include and follow link a on that path, and is given by:

$$e_a^p \equiv \begin{cases} \displaystyle\prod_{b \in \{a' \geq a\}_p} e_b, & \text{if } \{a' \geq a\}_p \neq \emptyset, \\ \\ 0, & \text{if } \{a' \geq a\}_p = \emptyset, \end{cases} \tag{8}$$

where $\{a' \geq a\}_p$ denotes the set of the links including and following link a in path p, and $\emptyset$ denotes the null set.

If a path consists of a single link a, then, according to (8), $e_a^p = e_a$. The model of Devadoss and Salaba (2020), in effect, had such exchange rates, since each pair of country supply and demand markets was assumed to be joined by a trade route consisting of a single transportation option or link.

The true transportation cost then on link a, $a \in L$, for commodity $h; h = 1, \ldots, H$, when it is used in a path p, is given by the expression:

$$\tilde{c}_a^{hp} = \tilde{c}_a^h(Q)e_a^p. \tag{9}$$

Note that (9) encumbers the exchange rates that are encountered when the commodity takes a particular path p, which can entail transportation links through different countries.

The effective transportation cost on a path, $\tilde{C}_p^h$, $\forall p \in P$, for commodity h; $h = 1, \ldots, H$, is then calculated as:

$$\tilde{C}_p^h = \sum_{a \in L} \tilde{c}_a^{hp} \delta_{ap}; \tag{10}$$

that is, the effective transportation cost on a path, which represents a trade route, is equal to the sum of the effective transportation costs for the commodity on links that make up the path.

In addition, the supply price, as well as the subsidy and tariff, will also need to be converted to the currency associated with the destination node relative to the origin node.

The international trade spatial price network equilibrium conditions are now stated.

Definition 1: The Multicommodity International Trade Spatial Price Network Equilibrium Conditions with Exchange Rates and Under Subsidies, Tariffs and Capacities

A multicommodity path trade flow pattern $Q^* \in R_+^{n_P}$ is an international trade spatial price network equilibrium pattern under subsidies and tariffs with explicit exchange rates and capacities if the following conditions hold: For all pairs of country origin and destination nodes: (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$, and all paths $p \in P_{ij}$ as well as all commodities h; $h = 1, \ldots, H$:

$$(\tilde{\pi}_i^h(Q^*) - sub_i^h + \tau_{ij}^h)e_{ij} + \tilde{C}_p^h(Q^*) \begin{cases} \leq \tilde{\rho}_j^h(Q^*), & \text{if} \quad Q_p^{h*} = \bar{Q}_p^h, \\ = \tilde{\rho}_j^h(Q^*), & \text{if} \quad 0 < Q_p^{h*} < \bar{Q}_p^h, \\ \geq \tilde{\rho}_j^h(Q^*), & \text{if} \quad Q_p^{h*} = 0. \end{cases} \tag{11}$$

According to the multicommodity international trade spatial price equilibrium conditions (11), if there is a positive flow of a commodity between a pair of origin and destination countries on a path, and the product flow is not at the capacity of the path, then the supply price at the origin country of the product minus the subsidy there plus the tariff between the two countries, multiplied by the exchange rate between the two countries plus the effective transportation cost on the path is equal to the demand price at the destination country. If the product flow is at the capacity for the path, then the demand price is greater than or equal to the effective transportation cost on the path and the supply price minus the subsidy plus the tariff (with the latter three terms multiplied by the exchange rate between the origin and destination countries). If the demand price is less than or equal to the summand of the above-noted terms, then it does not make economic sense to trade, and the product flow on that path will be zero.

Theorem 1: Variational Inequality Formulation of the Multicommodity International Trade Spatial Price Network Equilibrium Conditions with Exchange Rates and Under Subsidies, Tariffs and Capacities

A multicommodity path trade flow pattern $Q^* \in K$, where $K \equiv \{Q|0 \leq Q \leq \bar{Q}\}$ is a multicommodity international trade spatial price network equilibrium pattern with exchange rates and under subsidies, tariffs and capacities, according to Definition 1, with equilibrium conditions (11), if and only if it satisfies the variational inequality:

$$\sum_{h=1}^{H}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p\in P_{ij}}\left[(\tilde{\pi}_i^h(Q^*) - sub_i^h + \tau_{ij}^h)e_{ij} + \tilde{C}_p^h(Q^*) - \tilde{\rho}_j^h(Q^*)\right] \times \left[Q_p^h - Q_p^{h*}\right] \geq 0, \quad \forall Q \in K. \tag{12}$$

Proof:

First, we establish necessity; that is, we show that if $Q^* \in K$ satisfies equilibrium conditions (11) then it also satisfies variational equality (12).

Note that, according to equilibrium conditions (11), for a fixed commodity h, a fixed country pair (i, j), and path $p \in P_{ij}$, one must have that:

$$\left[(\tilde{\pi}_i^h(Q^*) - sub_i^h + \tau_{ij}^h)e_{ij} + \tilde{C}_p^h(Q^*) - \tilde{\rho}_j^h(Q^*)\right] \times \left[Q_p^h - Q_p^{h*}\right] \geq 0, \tag{13}$$

for any Q_p^h such that $0 \leq Q_p^h \leq \bar{Q}_p^h$. Indeed, if $Q_p^{h*} = \bar{Q}_p^h$, then, according to (11),

$$\left[(\tilde{\pi}_i^h(Q^*) - sub_i^h + \tau_{ij}^h)e_{ij} + \tilde{C}_p^h(Q^*) - \tilde{\rho}_j^h(Q^*)\right] \leq 0. \tag{14}$$

Considering that $Q_p^h \leq \bar{Q}_p^h$, then $(Q_p^h - Q_p^{h*}) \leq 0$ and, therefore, with the use of (14), we know that (13) holds.

On the other hand, if $0 < Q_p^{h*} < \bar{Q}_p^h$, then, according to (11):

$$\left[(\tilde{\pi}_i^h(Q^*) - sub_i^h + \tau_{ij}^h)e_{ij} + \tilde{C}_p^h(Q^*) - \tilde{\rho}_j^h(Q^*)\right] = 0, \tag{15}$$

and, consequently, (13) also holds.

Finally, if $Q_p^{p*} = 0$, then, according to equilibrium conditions (11):

$$\left[(\tilde{\pi}_i^h(Q^*) - sub_i^h + \tau_{ij}^h)e_{ij} + \tilde{C}_p^h(Q^*) - \tilde{\rho}_j^h(Q^*)\right] \geq 0, \tag{16}$$

and (13) also holds, since $Q_p^H \geq Q_p^{h*}$.

Since (13) holds for any commodity h, and any path p, we can sum (13) over all commodities h, all paths $p \in P_{ij}$, and over all country pairs (i, j), which yields:

$$\sum_{h=1}^{H}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p\in P_{ij}} \left[(\tilde{\pi}_i^h(Q^*) - sub_i^h + \tau_{ij}^h)e_{ij} + \tilde{C}_p^h(Q^*) - \tilde{\rho}_j^h(Q^*)\right] \times \left[Q_p^h - Q_p^{h*}\right] \geq 0, \quad \forall Q \in K, \tag{17}$$

which is precisely variational inequality (12). Necessity has been established.

We now turn to proving sufficiency. In particular, we show that, if $Q^* \in K$ satisfies variational inequality (12), then it also satisfies equilibrium conditions (11).

Let $Q_q^g = Q_q^{g*}$, for all commodities g, $g \neq h$, and for all paths $q \in P_{kl}$, for all kl except for path $p \in P_{ij}$ and substitute the resultants into (12). Then, (12) simplifies to:

$$\left[(\tilde{\pi}_i^h(Q^*) - sub_i^h + \tau_{ij}^h)e_{ij} + \tilde{C}_p^h(Q^*) - \tilde{\rho}_j^h(Q^*)\right] \times \left[Q_p^h - Q_p^{h*}\right] \geq 0, \tag{18}$$

for all $0 \leq Q_p^h \leq \bar{Q}_p^h$. The equilibrium conditions (11) then follow for commodity h, this path $p \in P_{ij}$ and, hence, for all commodities h, and for all paths in P_{ij} plus for all paths in all other country origin/destination pairs. The proof is complete. □

2.1 Illustrative Examples

Ukraine is a major exporter of grain, commonly referred to as the breadbasket of Europe, if not the world. The major invasion of Ukraine by Russia on February 24, 2022, has posed many challenges for Ukrainian farmers and the economy of Ukraine and has exacerbated food insecurity globally. For months, due to the mining of the Black Sea and blockades, which for years had been used in the transport of grain from Ukraine (about 95%) (Mykhaylov

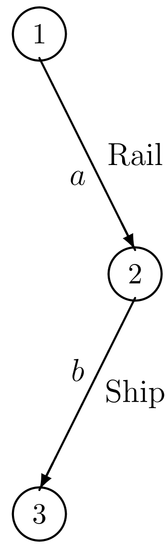

Figure 2: Spatial Price Network Topology for Example 1

(2022)), maritime routes were, in effect, cut off. Several of the Ukrainian ports were also targets and subject to attacks.

We construct four illustrative examples. The examples focus on wheat commodity flows from Ukraine to Lebanon before and after the invasion of Ukraine by Russia on February 24, 2022. Exchange rates throughout the examples are obtained from the website https://wise.com/us/; as an example, see https://wise.com/us/currency-converter/usd-to-uah-rate?amount=1000. Since the focus here is on wheat, the examples are for a single commodity and, hence, the superscript h for a typical commodity h is suppressed.

The local currency codes are: UAH for Ukrainian hryvnia, MDL for the Moldovan leu, RON for the Romanian leu, LBP for the Lebanese pound, and USD for the United States dollar. In these examples, there are no commodity path flow capacities.

Example 1: Pre-Invasion Scenario

Example 1 is a pre-invasion example, based on the scenario of early 2022, as if no major invasion had occurred. Please refer to the spatial price network topology in Figure 2. In the figure, node 1 represents Ukraine. Node 2 is also Ukraine and node 3 represents Lebanon. Link a corresponds to the shipment of wheat within Ukraine via rail to a port on the Black Sea. Link b corresponds to the transport of the wheat from the Ukrainian port via ship to Lebanon.

The unit of commodity flow is a ton. There is a single path $p_1 = (a, b)$. The exchange rate data for Example 1 is drawn from early January 2022. We have that: $e_{13} = 55.0581$,

$e_a = 1.0000$, and $e_b = 55.0581$. The USD to UAH and USD to LBP exchange rates are 27.4619 and 1,512.0000, respectively.

In this simple example, it follows that: $s_1 = Q_{p_1} = d_1$ and $f_a = f_b = Q_{p_1}$.

The supply price function in Ukraine in hryvnia is:

$$\pi_1 = \pi_1(s_1) = .000136s_1 + 7,001.60$$

and, hence,

$$\tilde{\pi}_1(Q_{p_1}) = .000136Q_{p_1} + 7,001.60.$$

The unit transportation cost function on link a in hryvnia is:

$$c_a = c_a(f_a) = .000278f_a + 954.80$$

so that:

$$\tilde{c}_a(Q_{p_1}) = .000278Q_{p_1} + 954.80.$$

The unit transportation cost function on link b in hryvnia is:

$$c_b = c_b(f_b) = .000278f_b + 1,091.20.$$

It follows then that:

$$\tilde{c}_b(Q_{p_1}) = .000278Q_{p_1} + 1,091.20.$$

The demand price function in Lebanon in Lebanese pounds is:

$$\rho_3 = \rho_3(d_3) = -.15d_3 + 602,344.00$$

and, hence:

$$\tilde{\rho}_3(Q_{p_1}) = -.15Q_{p_1} + 602,344.00.$$

The effective exchange rates are:

$$e_a^{p_1} = e_a e_b = 55.0581, \quad e_b^{p_1} = 55.0581.$$

Therefore, the effective link costs are:

$$\tilde{c}_a^{p_1} = e_a^{p_1}\tilde{c}_a = 55.0581\tilde{c}_a, \quad \tilde{c}_b^{p_1} = e_b^{p_1}\tilde{c}_b = 55.0581\tilde{c}_b.$$

The effective path cost on path p_1 is:

$$\tilde{C}_{p_1} = \tilde{c}_a^{p_1} + \tilde{c}_b^{p_1}.$$

Applying the international trade spatial price equilibrium conditions (11), under the assumption of no tariff and no subsidy and no quota, with the above functions, and assuming that $Q^*_{p_1} > 0$, becomes:

$$\tilde{\pi}_1(Q^*_{p_1})e_{13} + \tilde{C}_{p_1}(Q^*_{p_1}) = \tilde{\rho}_3(Q^*_{p_1}),$$

which, in turn, reduces to:

$$.1881Q^*_{p_1} = 104,200.3344,$$

with solution: $Q^*_{p_1} = 553,962.4370$ in tons.

This wheat commodity flow pattern results in a supply and a demand of:

$$s^*_1 = d^*_3 = Q^*_{p_2} = 553,962.4370.$$

Accordingly, the supply and demand prices are:

$$\pi_1(s^*_1) = \tilde{\pi}_1(Q^*_{p_1}) = 7,076.9388 \text{ UAH } = \$257.7002,$$

$$\rho_3(d^*_3) = \tilde{\rho}_3(Q^*_{p_1}) = 519,249.6344 \text{ LBP } = \$343.4190.$$

Additionally, the link costs turn out to be, at the equilibrium:

$$\tilde{c}_a = 1,108.8015 \text{ UAH } = \$40.3759, \quad \tilde{c}_b = 1,245.2015 \text{ UAH } = \$45.3428.$$

The $553,962.4370$ tons of wheat flow is quite reasonable, since, in 2021, Lebanon imported $520,000$ tons of wheat from Ukraine and an even greater harvest was expected in 2022 (Hamdan (2022)). Note that, under this wheat trade flow volume, the supply price in US dollars would be: 257.7002 per ton of wheat in Ukraine (at the farmer level) and the demand price in Lebanon in US dollars would be: 343.4190, which is close to the reported price in 2021 (see Breisinger et al. (2022) and Hamdan (2022)). According to the Associated Press (2022), farmers in Ukraine could get about \$270 per ton of wheat before the invasion. Furthermore, the transportation cost pre-invasion for a ton of wheat in Ukraine to a port was about \$40, as the result in this example (cf. Pratt (2022)).

Example 2: Invasion Scenario

Example 2 considers the invasion (war) scenario after February 24, 2022 (cf. Al Jazeera (2022)) but before the grain shipment agreement, brokered by the United Nations and Turkey, which took effect in late July (UN News (2022)).

During this period, essentially no grain was shipped from Ukraine using a Black Sea route as in Example 1. Instead, one of the alternative routes (cf. Figure 3) consisted of rail

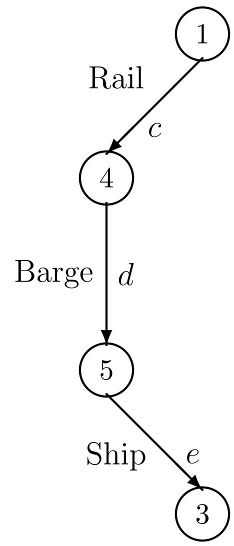

Figure 3: Spatial Price Network Topology for Example 2

transport in Ukraine to Moldova as on link c, followed by transport by barge to Romania as on link d, and then shipment to Lebanon as on link e.

The unit of wheat commodity flow is, again, a ton. There is a single path $p_2 = (c, d, e)$. The exchange rates for Example 2 were obtained from early July, that is, after the invasion but before the shipment agreement. The exchange rates for Example 2 are: $e_{13} = 51.6836$, $e_c = .6528$, $e_d = .2523$, and $e_e = 313.6980$. The USD to UAH, MDL, RON, and LBP exchange rates are 29.2549, 19.1005, 4.8199, and 1,512.0000, respectively. It is worth noting that the exchange rates were essentially the same on July 20, 2022.

In Example 2: $s_1 = Q_{p_2} = d_3$ and $f_c = f_d = f_e = Q_{p_2}$.

The supply price function in Ukrainian hryvnia is:

$$\pi_1(s) = \pi_1(s_1) = .002673 s_1 + 2,806.30$$

and it follows that:

$$\tilde{\pi}_1(Q_{p_2}) = .002673 Q_{p_2} + 2,806.30.$$

The unit transportation cost on link c in hryvnia is:

$$c_c = .002768 f_c + 6,546.50$$

and, hence:

$$\tilde{c}_c = .002768 Q_{p_2} + 6,546.50.$$

The unit transportation cost on link d in Moldovan leus is:

$$c_d = .002172 f_d + 2,324.60$$

and, therefore:

$$\tilde{c}_d = .002172Q_{p_2} + 2,324.60$$

The unit transportation cost on link e in Romanian leus is:

$$c_e = .000257f_e + 345.40.$$

It follows then that:

$$\tilde{c}_e = .000257Q_{p_2} + 345.40.$$

The difference in the cost function on link c in this example and the cost function on link a in Example 1, with both entailing rail transportation in Ukraine, is due to the different rail gauges used in Ukraine and Moldova, which necessitates including loading and unloading costs. Loading and unloading costs are also accounted for in the cost function on link d.

The demand price function in Lebanese pounds is:

$$\rho_3(d) = \rho_3(d_3) = -.17d_3 + 793,747.50$$

so that:

$$\tilde{\rho}_3(Q_{p_2}) = -.17Q_{p_2} + 793,747.50.$$

The effective exchange rates are:

$$e_c^{p_2} = e_c e_d e_e = 51.6665, \quad e_d^{p_2} = e_d e_e = 79.1460, \quad e_e^{p_2} = e_e = 313.6980.$$

The effective link costs are:

$$\tilde{c}_c^{p_2} = e_c^{p_2}\tilde{c}_c = 51.6665\tilde{c}_c, \quad \tilde{c}_d^{p_2} = e_d^{p_2}\tilde{c}_d = 79.1460\tilde{c}_d, \quad \tilde{c}_e^{p_2} = e_e^{p_2}\tilde{c}_e = 313.6980\tilde{c}_e.$$

The effective path cost is:

$$\tilde{C}_{p_2} = \tilde{c}_c^{p_2} + \tilde{c}_d^{p_2} + \tilde{c}_e^{p_2}.$$

As in Example 1, we assume that there is no tariff imposed and no subsidy granted as well as no quota on the commodity path flow. Also, we assume that $Q_{p_2}^* > 0$. Then, according to the international trade spatial price equilibrium conditions (11), and, with notice that here: $Q^* = Q_{p_2}^*$, we have that:

$$\tilde{\pi}_1(Q_{p_2}^*)e_{13} + \tilde{C}_{p_2}(Q_{p_2}^*) = \tilde{\rho}_3(Q_{p_2}^*),$$

and

$$s_1^* = Q_{p_2}^*, \quad d_3^* = Q_{p_2}^*.$$

The above equation, upon substitution of the functions and exchange rates, reduces to:

$$.7036Q^*_{p_2} = 18,138.9902,$$

the solution of which yields: $Q^*_{p_2} = 25,780.2589$ in tons.

This wheat commodity flow pattern results in a supply and a demand of:

$$s^*_1 = d^*_3 = Q^*_{p_2} = 25,780.2589.$$

Accordingly, the supply and demand prices are:

$$\pi_1(s^*_1) = \tilde{\pi}_1(Q^*) = 2,875.2106 \text{ UAH } = \$98.2813,$$

$$\rho_3(d^*_3) = \tilde{\rho}_3(Q^*) = 789,364.8559 \text{ LBP } = \$522.0667.$$

Additionally, the link costs turn out to be, at the equilibrium:

$$\tilde{c}_c = 6,617.8597 \text{ UAH } = \$226.2137, \quad \tilde{c}_d = 2,380.5947 \text{ MDL } = \$124.6358,$$

$$\tilde{c}_e = 352.0255 \text{ RON } = \$73.0358.$$

The wheat flow of 25,780.2589 tons is reasonable since, without access to deep-sea ports on the Black Sea, Ukraine can, at most, export around 10% of what it used to (BBC (2022)). As reported by Nivievskyi (2022), the transportation cost of grain inside Ukraine has jumped to about \$200 which is evident in the result for this example. Furthermore, because of the ongoing war, Ukrainian farmers are earning approximately \$100 per ton of wheat (Arhirova (2022), Brower (2022), Balmforth and Polityuk (2022)), which is similar to the supply price of \$98.2813 in this example. Moreover, with the continuing food crisis in Lebanon, and, as a result of the war, the price of wheat in Lebanon has gone up to more than \$500 per ton (Hernandez (2022), Rose (2022)), which is in accordance with the results for Example 2.

Example 3: Black Sea Grain Initiative in Place

For Example 3, we consider the post-July 22 agreement scenario with maritime transportation from several of the Ukrainian Black Sea ports being, again, possible. The brokered agreement with assistance from the United Nations and Turkey is known as the Black Sea Grain Initiative. Please refer to Figure 4, where the nodes and the links correspond to the same countries and modes of transportation as in Examples 1 and 2.

There are two paths: $p_1 = (a, b)$ and $p_2 = (c, d, e)$. The exchange rates on links are from late August, that is, after the placement of the shipment agreement. The exchange rates

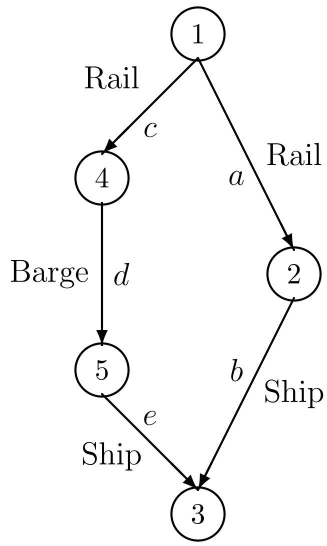

Figure 4: Spatial Price Network Topology for Example 3

are: $e_a = 1.0000$, $e_b = 41.3469$, $e_c = .5291$, $e_d = .2521$, $e_e = 309.8670$, and $e_{13} = 41.3469$. The USD to UAH, MDL, RON, and LBP exchange rates are 36.5686, 19.3500, 4.8794, and 1,512.0000, respectively. It is worth noting that the exchange rates were essentially the same on July 22, 2022. Furthermore, we observe that the USD to UAH exchange rate had a sharp increase from July 20 to 21, 2022.

In Example 3, we have that: $s_1 = Q_{p_1} + Q_{p_2} = d_3$, $f_a = f_b = Q_{p_1}$, and $f_c = f_d = f_e = Q_{p_2}$.

The supply price function in Ukrainian hryvnia is now:

$$\pi_1(s) = \pi_1(s_1) = .000167s_1 + 3,364.60$$

so that:

$$\tilde{\pi}_1(Q_p) = .000167(Q_{p_1} + Q_{p_2}) + 3,364.60.$$

The unit transportation cost on link a in hryvnia is:

$$c_a = .000217f_a + 7,144.80$$

and, hence:

$$\tilde{c}_a = .000217Q_{p_1} + 7,144.80.$$

The unit transportation cost on link b in hryvnia is:

$$c_b = .000246f_b + 7,423.10$$

and, hence:

$$\tilde{c}_b = .000246Q_{p_1} + 7,423.10.$$

The unit transportation cost on link c in hryvnia is:

$$c_c = .003284 f_c + 8,304.80$$

and, hence:

$$\tilde{c}_c = .003284 Q_{p_2} + 8,304.80.$$

The unit transportation cost on link d in Moldovan leus is:

$$c_d = .003097 f_d + 2,397.50$$

and, therefore:

$$\tilde{c}_d = .003097 Q_{p_2} + 2,397.50.$$

The unit transportation cost on link e in Romanian leus is:

$$c_e = .000428 f_e + 361.20.$$

It follows then that:

$$\tilde{c}_e = .000428 Q_{p_2} + 361.20$$

The demand price function in Lebanese pounds is:

$$\rho_3(d) = \rho_3(d_3) = -.082 d_3 + 796,162.50$$

so that:

$$\tilde{\rho}_3(Q_p) = -.082(Q_{p_1} + Q_{p_2}) + 796,162.50.$$

The effective exchange rates for the links are:

$$e_a^{p_1} = e_a e_b = 41.3469, \quad e_b^{p_1} = e_b = 41.3469, \quad e_c^{p_2} = e_c e_d e_e = 41.3319,$$

$$e_d^{p_2} = e_d e_e = 78.1174, \quad e_e^{p_2} = e_e = 309.8670.$$

Therefore, the effective link costs are:

$$\tilde{c}_a^{p_1} = e_a^{p_1} \tilde{c}_a = 41.3469 \tilde{c}_a, \quad \tilde{c}_b^{p_1} = e_b^{p_1} \tilde{c}_b = 41.3469 \tilde{c}_b,$$

$$\tilde{c}_c^{p_2} = e_c^{p_2} \tilde{c}_c = 41.3319 \tilde{c}_c, \quad \tilde{c}_d^{p_2} = e_d^{p_2} \tilde{c}_d = 78.1174 \tilde{c}_d, \quad \tilde{c}_e^{p_2} = e_e^{p_2} \tilde{c}_e = 309.8670 \tilde{c}_e.$$

The effective path costs are:

$$\tilde{C}_{p_1} = \tilde{c}_a^{p_1} + \tilde{c}_b^{p_1}, \quad \tilde{C}_{p_2} = \tilde{c}_c^{p_2} + \tilde{c}_d^{p_2} + \tilde{c}_e^{p_2}.$$

Assuming no tariff and no subsidy and no quotas on the commodity path flows and that both paths are used, that is, the flows on both paths are positive in equilibrium, the equilibrium conditions (11) are, for this example:

$$\tilde{\pi}_1(Q^*)e_{13} + \tilde{C}_{p_1}(Q^*_{p_1}) = \tilde{\rho}_3(Q^*),$$

$$\tilde{\pi}_1(Q^*)e_{13} + \tilde{C}_{p_2}(Q^*_{p_2}) = \tilde{\rho}_3(Q^*),$$

where Q^* is the vector with elements: $Q^*_{p_1}, Q^*_{p_2}$.

After substituting the supply and demand price functions, the effective path costs, and the exchange rate between the Ukrainian (node 1) and Lebanese (node 3) currency, e_{13}, and simplifying, the system of equations to be solved is:

$$.1080Q^*_{p_1} + .0889Q^*_{p_2} = 54,709.2157,$$

$$.0889Q^*_{p_1} + .5991Q^*_{p_2} = 14,583.1302.$$

The solution of the above system of equations yields a negative path flow on path p_2, which is infeasible. Therefore, path p_2 is not used. Then, one has that: $Q^*_{p_1} = 506,566.8120$ and $Q^*_{p_2} = 0.0000$, with the commodity flows, again, in tons. This wheat commodity flow pattern results in the following supply and demand:

$$s^*_1 = d^*_3 = Q^*_{p_1} + Q^*_{p_2} = 506,566.8120,$$

with the supply and demand prices per ton now being:

$$\pi_1(s^*_1) = \tilde{\pi}_1(Q^*) = 3,449.1966 \text{ UAH } = \$94.3212,$$

$$\rho_3(d^*_3) = \tilde{\rho}_3(Q^*) = 754,624.0214 \text{ LBP } = \$499.0899.$$

Additionally, the link costs are:

$$\tilde{c}_a = 7,254.7249 \text{ UAH } = \$198.3867, \quad \tilde{c}_b = 7,547.7154 \text{ UAH } = \$206.3988,$$

$$\tilde{c}_c = 8,304.8000 \text{ UAH } = \$227.1019, \quad \tilde{c}_d = 2,397.5000 \text{ MDL } = \$123.9018,$$

$$\tilde{c}_e = 361.2000 \text{ RON } = \$74.0245.$$

Note that, under this wheat flow pattern, the supply is similar to what Ukraine used to export to Lebanon pre-war. Furthermore, notice that, in this example, with the availability of maritime transportation from Ukraine on the Black Sea, the wheat flow on path p_2 is at 0.0000, which is due to the inefficiency of transporting the grain to a Middle Eastern country by such a route and composition of modes. These results are in accordance with the importance of the grain corridor provided by the brokered agreement for facilitating the export of Ukrainian wheat, as mentioned by Nivievskyi (2022). It also shows that the sustainable operation of the corridor can result in a good part of the wheat demand in crisis-stricken Lebanon being met through the wheat supply in Ukraine, even with high transportation costs.

We note that the prices of wheat products, such as wheat flour and bread, before and early after the agreement, have been relatively stable in Lebanon (Andrée (2022)). Furthermore, Nivievskyi (2022) states that, although the brokered agreement has facilitated the transportation of wheat, the impact on prices is yet to be observed as the war and nearly full storage has kept the prices high. The supply price at \$94.3212 and the demand price at \$499.0899, along with the \$206.3988 cost on link b, reflect these issues and show how the attacks on ports and the meddling of the Russians are preventing the prices from falling, with the associated risk and uncertainty resulting in high transportation costs even through the corridor (Worledge and Belikova (2022)).

Example 4: Example 3 Data with Subsidy

In Example 4, we, again, consider the post-July 22 agreement scenario with maritime transportation via the Black Sea from Ukraine possible; however, a subsidy is introduced in this example and the impact quantified. Good (2022) reports a donation from the US government to the World Food Program for Ukrainian wheat in response to the food insecurity caused by the Russian invasion. We consider the effect of the following subsidy in hryvnia on Ukrainian wheat shipped to Lebanon:

$$sub_1 = 1,000.00.$$

The supply and demand price functions are the same as in Example 3. The link cost functions on paths p_1 and p_2 are also the same as in Example 3. Accordingly, the effective exchange rates for the links, the effective link costs, and the effective path costs remain as in Example 3.

Assuming that both paths are used, that is, the wheat commodity flows on both paths are positive in equilibrium, the equilibrium conditions (11) are, for this example:

$$(\tilde{\pi}_1(Q^*) - sub_1)e_{13} + \tilde{C}_{p_1}(Q^*_{p_1}) = \tilde{\rho}_3(Q^*),$$

$$(\tilde{\pi}_1(Q^*) - sub_1)e_{13} + \tilde{C}_{p_2}(Q^*_{p_2}) = \tilde{\rho}_3(Q^*),$$

where:

$$s_1^* = Q_{p_1}^* + Q_{p_2}^*, \quad d_3^* = Q_{p_1}^* + Q_{p_2}^*,$$

and Q^* is the vector with elements: $Q_{p_1}^*, Q_{p_2}^*$.

After substituting the supply and demand price functions, the subsidy amount, the exchange rate between the Ukrainian (node 1) and Lebanese (node 3) currency, e_{13}, and the effective path costs into the equations, and simplifying, the system of equations to be solved is:

$$.1080Q_{p_1}^* + .0889Q_{p_2}^* = 96,056.1157,$$

$$.0889Q_{p_1}^* + .5991Q_{p_2}^* = 55,930.0302.$$

The solution of the above system of equations yields a negative path flow on path p_2. Accordingly, again, only path p_1 is used, and one has that: $Q_{p_1}^* = 889,408.4787$ and $Q_{p_2}^* = 0.0000$, with all the commodity flows in tons. This wheat commodity flow pattern results in the following supply and demand:

$$s_1^* = d_3^* = Q_{p_1}^* + Q_{p_2}^* = 889,408.4787,$$

with the following supply and demand prices:

$$\pi_1(s_1^*) = \tilde{\pi}_1(Q^*) = 3,513.1312 \text{ UAH } = \$96.0696,$$

$$\rho_3(d_3^*) = \tilde{\rho}_3(Q^*) = 723,231.0047 \text{ LBP } = \$478.3273.$$

Accordingly, the link costs turn out to be:

$$\tilde{c}_a = 7,337.8016 \text{ UAH } = \$200.6585, \quad \tilde{c}_b = 7,641.8944 \text{ UAH } = \$208.9742,$$

$$\tilde{c}_c = 8,304.8000 \text{ UAH } = \$227.1019, \quad \tilde{c}_d = 2,397.5000 \text{ MDL } = \$123.9018,$$

$$\tilde{c}_e = 361.2000 \text{ RON } = \$74.0245.$$

Again, due to the availability of maritime transportation via the Black Sea for grain exports from Ukraine, and the high cost of path p_2, all the wheat flow is on path p_1. However, in this example, the effect of the subsidy enhances the wheat flow, and increases the price that farmers can expect to get for a ton of wheat to \$96.0696, which is of value as the current low supply prices threaten the farmers' ability to buy seed and equipment for the next harvest season (Brower (2022), Arhirova (2022), Balmforth and Polityuk (2022)). The subsidy also helps to reduce the demand price to \$478.3273 which can be of significant importance in countering the food crisis and associated food insecurity in Lebanon (Hernandez (2022), Khoury (2021)). These examples are stylized but illustrative.

Variational inequality (12) is now put into standard form (cf. Nagurney (1999)), VI$(F, \mathcal{K})$, where one seeks to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{19}$$

where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

Specifically, we define $X \equiv Q$, $\mathcal{K} \equiv K$, and $\mathcal{N} = Hn_P$. Plus, $F(X)$ consists of the elements $F^h_p(X) \equiv \left[(\tilde{\pi}^h_i(Q) - sub^h_i + \tau^h_{ij})e_{ij} + \tilde{C}'^h_p(Q) - \tilde{\rho}^h_j(Q)\right]$, $\forall h$, $\forall i, j$, $\forall p \in P_{ij}$. Clearly, VI (11) can be put into standard form (19).

We now establish the existence of a solution to variational inequality (12).

Theorem 2: Existence

There exists a solution to variational inequality (12), with the solution corresponding to an equilibrium according to Definition 1 and equilibrium conditions (11).

Given that the feasible set is closed and bounded, i.e., compact, the existence of a solution to (12) is guaranteed by the standard theory of variational inequalities (see, for instance, Kinderlehrer and Stampacchia (1980) and Nagurney (1999)).

4. The Algorithm

The modified projection method of Korpelevich (1977) is implemented and applied to compute solutions to larger numerical examples in Section 5.

The modified projection method is guaranteed to converge if the function $F(X)$ that enters the variational inequality problem (12) is monotone and Lipschitz continuous.

Recall that F is said to be monotone if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{20}$$

Furthermore, F is Lipschitz continuous, if there exists an $\eta > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \leq \eta \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{21}$$

For completeness, and easy reference, the steps of the modified projection method are delineated below, with t denoting an iteration counter:

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $t = 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^t$ by solving the variational inequality subproblem:

$$\langle \bar{X}^t + \beta F(X^{t-1}) - X^{t-1}, X - \bar{X}^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{22}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^t + \beta F(\bar{X}^t) - X^{t-1}, X - X^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{23}$$

Step 3: Convergence Verification

If $|X^t - X^{t-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $t := t + 1$ and go to Step 1.

Because of the structure of the feasible set $\mathcal{K}$ underlying the multicommodity international trade spatial price equilibrium model with exchange rates, which consists of box type constraints, the solution of each of the subproblems in (22) and (23) can be obtained via closed form expressions for the multicommodity path flows. We make this statement explicit below.

Explicit Formulae at Iteration τ for the Multicommodity Path Flows in Step 1

The algorithm results in the following closed form expressions for (22) for the multicommodity flows in Step 1 for the solution of variational inequality (12):

$$\bar{Q}_p^{ht} = \max\{0, \min\{\bar{Q}_p^h, Q_p^{ht-1} + \beta(\tilde{\rho}_j^h(Q^{t-1}) - (\tilde{\pi}_i^h(Q^{t-1}) + sub_i^h - \tau_{ij}^h)e_{ij} - \tilde{C}_p^h(Q^{t-1}))\}\},$$
$$\forall h, \forall p. \tag{24}$$

The explicit formulae for (23) in Step 2 readily follow.

As noted in Solodov and Tseng (1996), the modified projection method, also referred to as the extragradient method, is easy to implement, requires little computer storage, and also can take advantage of any sparsity or separable structure in F or in $\mathcal{K}$. Furthermore, and, as is the case for the variational inequality (12), if $\mathcal{K}$ is the nonnegative orthant, other solution methods may exist, but they may not be well-suited for large sparse problems. In addition, unlike the projection method (cf. Dafermos (1980)), the modified projection method does not require strong monotonicity of F for convergence. The modified projection method, as stated in Solodov and Tseng (1996), is a very practical method. Its only drawback is its, "at best," linear convergence. The above closed form expressions substantiate the ease of implementation of this algorithm for our spatial price equilibrium model with exchange rates. We emphasize that another positive feature of our model and the proposed algorithm is that the variables include path flow variables and not link flow variables (see also Bertsekas and Gafni (1982)). Of course, once the specific numerical problem is solved, equilibrium link flows can be easily recovered from the equilibrium path flows through the use of the conservation of flow equations (2).

5. Numerical Examples

We now present a series of numerical examples, which are computed using the modified projection method described in the preceding section. The algorithm was coded in FORTRAN and a Linux system at the University of Massachusetts Amherst used for the computations. The algorithm was considered to have converged when the absolute value of each of the computed path flows in two successive iterations differed no more than ϵ with ϵ set to 10^{-7}. The contraction parameter in the modified projection method was set to $\beta = .1$. These numerical examples are extensions of the preceding ones.

Example 5: Brokered Agreement in Place - Two Demand Markets

The spatial price network topology for Example 5 is given in Figure 5. The commodity, as in Examples 1 through 4, is wheat, but now we add a new demand point - Egypt corresponding to node 6. The most vulnerable countries in the world to the impacts of Russia's invasion of Ukraine on Ukrainian wheat exports are those that heavily rely on wheat as a major source of nutrition and are significant net importers of wheat from Ukraine. A host of nations in the Middle East and North Africa are in accord with this description, with Lebanon and Egypt being two representatives from this region. Egypt, on average, imports around 25% of its wheat solely from Ukraine, and the percentage for Lebanon is even higher at more than 60%. In Illustrative Examples 1 through 4, only Lebanon was considered. Adding Egypt, as we do in Example 5, provides additional insights on the assessment of the impacts of the

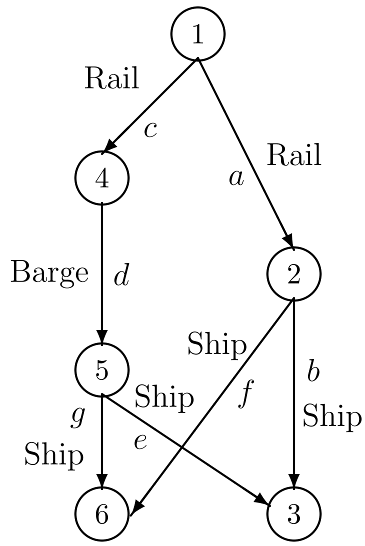

Figure 5: Spatial Price Network Topology for Example 5

war on equilibrium agricultural product trade flows and prices.

In Figure 5, nodes 1 and 2 correspond to Ukraine; node 3 represents Lebanon; node 4 represents Moldova, and node 5 is Romania. Furthermore, the local currency code for the Egyptian pound is EGP.

There are four paths: $p_1 = (a, b)$, $p_2 = (c, d, e)$, $p_3 = (a, f)$, and $p_4 = (c, d, g)$. The exchange rates are derived from late August, the same period as for Examples 3 and 4. The exchange rates are: $e_a = 1.0000$, $e_b = 41.3469$, $e_c = .5291$, $e_d = .2521$, $e_e = 309.8670$, $e_f = .5236$, $e_g = 3.9415$, $e_{13} = 41.3469$, and $e_{16} = .5236$. The USD to UAH, LBP, and EGP exchange rates are 36.5686, 1,512.0000, and 19.1500, respectively

Here, we have that; $s_1 = Q_{p_1} + Q_{p_2} + Q_{p_3} + Q_{p_4}$, $d_3 = Q_{p_1} + Q_{p_2}$, $d_6 = Q_{p_3} + Q_{p_4}$, $f_a = Q_{p_1} + Q_{p_3}$, $f_b = Q_{p_1}$, $f_c = f_d = Q_{p_2} + Q_{p_4}$, $f_e = Q_{p_2}$, $f_f = Q_{p_3}$, and $f_g = Q_{p_4}$.

The supply price function in Ukrainian hryvnia is:

$$\pi_1(s) = \pi_1(s_1) = .000167s_1 + 3,364.60.$$

The unit transportation cost on link a in hryvnia is:

$$c_a(f) = .000217f_a + 7,144.80.$$

The unit transportation cost on link b in hryvnia is:

$$c_b(f) = .000246f_b + 7,423.10.$$

The unit transportation cost on link c in hryvnia is:

$$c_c(f) = .003284 f_c + 8,304.80.$$

The unit transportation cost on link d in Moldovan leu is:

$$c_d(f) = .003097 f_d + 2,397.50.$$

The unit transportation cost on link e in Romanian leus is:

$$c_e(f) = .000428 f_e + 361.20.$$

The unit transportation cost on link f in hryvnia is:

$$c_f(f) = .000246 f_f + 7,023.60.$$

The unit transportation cost on link g in Romanian leus is:

$$c_g(f) = .000428 f_g + 335.20.$$

The demand price function for wheat in Lebanon in Lebanese pounds is:

$$\rho_3(d) = \rho_3(d_3) = -.082 d_3 + 796,162.50.$$

The demand price function for wheat in Egypt in Egyptian pounds is:

$$\rho_6(d) = \rho_6(d_6) = -.000216 d_6 + 10,000.60.$$

The effective exchange rates for the link and path combinations are:

$$e_a^{p_1} = e_a e_b = 41.3469, \quad e_b^{p_1} = e_b = 41.3469,$$

$$e_c^{p_2} = e_c e_d e_e = 41.3319, \quad e_d^{p_2} = e_d e_e = 78.1174, \quad e_e^{p_2} = e_e = 309.8670,$$

$$e_a^{p_3} = e_a e_f = .5236, \quad e_f^{p_3} = e_f = .5236,$$

$$e_c^{p_4} = e_c e_d e_g = .5257, \quad e_d^{p_4} = e_d e_g = .9936, \quad e_g^{p_4} = e_g = 3.9415.$$

Therefore, the effective link cost path combinations are:

$$\tilde{c}_a^{p_1} = e_a^{p_1} \tilde{c}_a = 41.3469 \tilde{c}_a, \quad \tilde{c}_b^{p_1} = e_b^{p_1} \tilde{c}_b = 41.3469 \tilde{c}_b,$$

$$\tilde{c}_c^{p_2} = e_c^{p_2}\tilde{c}_c = 41.3319\tilde{c}_c, \quad \tilde{c}_d^{p_2} = e_d^{p_2}\tilde{c}_d = 78.1174\tilde{c}_d, \quad \tilde{c}_e^{p_2} = e_e^{p_2}\tilde{c}_e = 309.8670\tilde{c}_e,$$

$$\tilde{c}_a^{p_3} = e_a^{p_3}\tilde{c}_a = .5236\tilde{c}_a, \quad \tilde{c}_f^{p_3} = e_f^{p_3}\tilde{c}_f = .5236\tilde{c}_f,$$

$$\tilde{c}_c^{p_4} = e_c^{p_4}\tilde{c}_c = .5257\tilde{c}_c, \quad \tilde{c}_d^{p_4} = e_d^{p_4}\tilde{c}_d = .9936\tilde{c}_d, \quad \tilde{c}_g^{p_4} = e_g^{p_4}\tilde{c}_g = 3.9415\tilde{c}_g.$$

The effective path costs, in turn, are:

$$\tilde{C}_{p_1} = \tilde{c}_a^{p_1} + \tilde{c}_b^{p_1}, \quad \tilde{C}_{p_2} = \tilde{c}_c^{p_2} + \tilde{c}_d^{p_2} + \tilde{c}_e^{p_2}, \quad \tilde{C}_{p_3} = \tilde{c}_a^{p_3} + \tilde{c}_f^{p_3}, \quad C_{p_4} = \tilde{c}_c^{p_4} + \tilde{c}_d^{p_4} + \tilde{c}_g^{p_4}.$$

In this example, we assume that there are no tariffs and no subsidies. There are also no quotas (one can also handle this situation by setting the path quotas at high values).

The modified projection method computes the following commodity path flow pattern, in tons of wheat, to excellent accuracy:

$$Q^*_{p_1} = 302,029.3750, \quad Q^*_{p_2} = 0.0000, \quad Q^*_{p_3} = 1,390,388.5000, \quad Q^*_{p_4} = 0.0000.$$

One can see, from this solution, how important having an unblocked maritime route on the Black Sea is. The corridor provided by the brokered agreement significantly facilitates the transportation of wheat.

The demand market prices at equilibrium in the local currencies are:

$$\rho_3(d^*) = 771,396.0912 \text{ LBP } = \$510.1826, \quad \rho_6(d^*) = 9,700.2754 \text{ EGP } = \$506.5417.$$

The price of a ton in Lebanon increases as compared to the results in Example 3. Egypt essentially "competes" with Lebanon for the wheat. This result also supports the negative effects of war-induced higher prices as compared to the pre-war prices (e.g., around \$343 for Lebanon as shown in Example 1). Furthermore, the demand price of wheat in Egypt is in accord with the post-war reported prices of more than \$470 as compared to less than \$300 pre-war (Galal (2022), El Safty (2022)).

Example 6 - Example 5 with a Subsidy

In order to support farmers, we assume that the Ukrainian government is now subsidizing farmers so that $sub_1 = 1000$ in hryvnia.

Example 6, otherwise, has data identical to that of Example 5.

The modified projection method now yields the following equilibrium commodity path flow pattern of the wheat in tons:

$$Q^*_{p_1} = 557,759.6250, \quad Q^*_{p_2} = 0.0000, \quad Q^*_{p_3} = 2,254,257.0000, \quad Q^*_{p_4} = 0.0000.$$

The most efficient paths only are, again, used and we see a big increase in the wheat flow amounts on paths p_1 and p_3, almost similar to the pre-war import levels (TrendEconomy (2022), IndexMundi (2022)) as reported in Example 1 for Lebanon.

The demand market prices at equilibrium in the local currencies are:

$$\rho_3(d^*) = 750,426.1875 \text{ LBP } = \$496.3136, \quad \rho_6(d^*) = 9,513.6797 \text{ EGP } = \$496.7978.$$

The price of wheat decreases in both countries, under the subsidy, which is of benefit to consumers in the countries. The farmers in Ukraine, now sell the wheat at a lower price (i.e., under $105; although slightly improved compared to Example 5 at under $100) relative to the pre-war supply price of $257.7002 in Example 1, but now export a greater volume of wheat as compared to Example 5. The greater volume of wheat acan assist in reducing food insecurity in Lebanon and Egypt that rely heavily on Ukrainian wheat.

Example 7: Two Demand Markets and Two Commodities

The spatial price network topology for Example 7 is the same as in Figure 5, but here we have two commodities: wheat and corn. In addition to Ukrainian wheat, Lebanon and Egypt are both importers of Ukrainian corn, and the limitations and the uncertainties in the Black Sea caused by the war could result in the two commodities becoming competitive in their supply prices, especially due to the high volumes of grains stuck in storage in Ukraine, and over the transportation links depicted in the spatial price network in Figure 5.

Here, the subscript $h = 1$ refers to wheat, and the subscript $h = 2$ corresponds to corn. There are, again, four paths: $p_1 = (a, b)$, $p_2 = (c, d, e)$, $p_3 = (a, f)$, and $p_4 = (c, d, g)$. The exchange rates are the same as in Examples 5 and 6.

Here, for wheat, we have that: $s^1_1 = Q^1_{p_1} + Q^1_{p_2} + Q^1_{p_3} + Q^1_{p_4}$, $d^1_3 = Q^1_{p_1} + Q^1_{p_2}$, $d^1_6 = Q^1_{p_3} + Q^1_{p_4}$, $f^1_a = Q^1_{p_1} + Q^1_{p_3}$, $f^1_b = Q^1_{p_1}$, $f^1_c = f^1_d = Q^1_{p_2} + Q^1_{p_4}$, $f^1_e = Q^1_{p_2}$, $f^1_f = Q^1_{p_3}$, and $f^1_g = Q^1_{p_4}$.

And, for corn, it follows that: $s^2_1 = Q^2_{p_1} + Q^2_{p_2} + Q^2_{p_3} + Q^2_{p_4}$, $d^2_3 = Q^2_{p_1} + Q^2_{p_2}$, $d^2_6 = Q^2_{p_3} + Q^2_{p_4}$, $f^2_a = Q^2_{p_1} + Q^2_{p_3}$, $f^2_b = Q^2_{p_1}$, $f^2_c = f^2_d = Q^2_{p_2} + Q^2_{p_4}$, $f^2_e = Q^2_{p_2}$, $f^2_f = Q^2_{p_3}$, and $f^2_g = Q^2_{p_4}$.

The wheat supply price function in Ukrainian hryvnia is now:

$$\pi^1_1(s) = \pi^1_1(s^1_1, s^2_1) = .000167 s^1_1 + .000083 s^2_1 + 3,364.60.$$

The corn supply price function in Ukrainian hryvnia is:

$$\pi^2_1(s) = \pi^2_1(s^1_1, s^2_1) = .000054 s^1_1 + .000109 s^2_1 + 4022.50.$$

The unit transportation costs on link a in hryvnia are:

$$c_a^1(f) = .000217f_a^1 + 0.000043f_a^2 + 7,144.80,$$

$$c_a^2(f) = .000047f_a^1 + 0.000236f_a^2 + 7,013.60.$$

The unit transportation costs on link b in hryvnia are:

$$c_b^1(f) = .000246f_b^1 + .000049f_b^2 + 7,423.10,$$

$$c_b^2(f) = .000052f_b^1 + 0.000251f_b^2 + 7,248.30.$$

The unit transportation costs on link c in hryvnia are:

$$c_c^1(f) = .003284f_c^1 + 0.000655f_c^2 + 8,304.80,$$

$$c_c^2(f) = .000639f_c^1 + .003196f_c^2 + 8,0096.60.$$

The unit transportation costs on link d in Moldovan leu are:

$$c_d^1(f) = .003097f_d^1 + .000622f_d^2 + 2,397.50,$$

$$c_d^2(f) = .000575f_d^1 + .002878f_d^2 + 2,251.40.$$

The unit transportation costs on link e in Romanian leu are:

$$c_e^1(f) = .000428f_e^1 + .000086f_e^2 + 361.20,$$

$$c_e^2(f) = .000093f_e^1 + .000461f_e^2 + 352.50.$$

The unit transportation costs on link f in hryvnia are:

$$c_f^1(f) = .000246f_f^1 + .000049f_f^2 + 7,023.60,$$

$$c_f^2(f) = .000051f_f^1 + .000254f_f^2 + 6,892.50.$$

The unit transportation costs on link g in Romanian leu are:

$$c_g^1(f) = .000428f_g^1 + .000086f_g^2 + 335.20,$$

$$c_g^2(f) = .000088f_g^1 + .000441f_g^2 + 326.80.$$

The demand price function for wheat in Lebamon in Lebanese pounds is:

$$\rho_3^1(d) = \rho_3^1(d_3^1) = -.082d_3^1 + 796,162.50.$$

The demand price function for corn in Lebanon in Lebanese pounds is:

$$\rho_3^2(d) = \rho_3^2(d_3^2) = -.43d_3^2 + 718,256.40.$$

The demand price function for wheat in Egyptian pounds is:

$$\rho_6^1(d) = \rho_6^1(d_6^1) = -.000216d_6^1 + 10,000.60.$$

The demand price function for corn in Egyptian pounds is:

$$\rho_6^2(d) = \rho_6^2(d_6^2) = -.000308d_6^2 + 9,900.50.$$

The effective exchange rates for the link and path combinations are the same as in the previous numerical examples. Therefore, the effective link cost path combinations are for commodities $h = 1,2$:

$$\tilde{c}_a^{hp_1} = e_a^{p_1}\tilde{c}_a^h = 41.3469\tilde{c}_a^h, \quad \tilde{c}_b^{hp_1} = e_b^{p_1}\tilde{c}_b^h = 41.3469\tilde{c}_b^h,$$

$$\tilde{c}_c^{hp_2} = e_c^{p_2}\tilde{c}_c^h = 41.3319\tilde{c}_c^h, \quad \tilde{c}_d^{hp_2} = e_d^{p_2}\tilde{c}_d^h = 78.1174\tilde{c}_d^h, \quad \tilde{c}_e^{hp_2} = e_e^{p_2}\tilde{c}_e^h = 309.8670\tilde{c}_e^h,$$

$$\tilde{c}_a^{hp_3} = e_a^{p_3}\tilde{c}_a^h = .5236\tilde{c}_a^h, \quad \tilde{c}_f^{hp_3} = e_f^{p_3}\tilde{c}_f^h = .5236\tilde{c}_f^h,$$

$$\tilde{c}_c^{hp_4} = e_c^{p_4}\tilde{c}_c^h = .5257\tilde{c}_c^h, \quad \tilde{c}_d^{hp_4} = e_d^{p_4}\tilde{c}_d^h = .9936\tilde{c}_d^h, \quad \tilde{c}_g^{hp_4} = e_g^{p_4}\tilde{c}_g^h = 3.9415\tilde{c}_g^h.$$

The effective path costs, in turn, for commodities $h = 1,2$ are:

$$\tilde{C}_{p_1}^h = \tilde{c}_a^{hp_1} + \tilde{c}_b^{hp_1}, \quad \tilde{C}_{p_2}^h = \tilde{c}_c^{hp_2} + \tilde{c}_d^{hp_2} + \tilde{c}_e^{hp_2}, \quad \tilde{C}_{p_3}^1 = \tilde{c}_a^{hp_3} + \tilde{c}_f^{hp_3}, \quad C_{p_4}^h = \tilde{c}_c^{hp_4} + \tilde{c}_d^{hp_4} + \tilde{c}_g^{hp_4}.$$

Again, we assume that there are no tariffs and no subsidies. There are also no quotas.

The modified projection method yielded the following multicommodity equilibrium flow pattern in wheat and corn, respectively:

$$Q_{p_1}^{1*} = 285,284.5625, \quad Q_{p_2}^{1*} = 0.0000, \quad Q_{p_3}^{1*} = 1,288,246.2500, \quad Q_{p_4}^{1*} = 0.0000,$$

$$Q_{p_1}^{2*} = 19,948.1738, \quad Q_{p_2}^{2*} = 0.0000, \quad Q_{p_3}^{2*} = 630,883.1250, \quad Q_{p_4}^{2*} = 0.0000.$$

The above results for Example 7 further substantiate the importance of the maritime routes over the Black Sea for the export of agricultural products from Ukraine, which would even be more the case if maritime freight rates are to decrease. The more efficient paths p_1 and p_3 are, again, in use for wheat trades; however, one can see that the flows decrease as compared to in Example 5, which is an impact of having another type of grain in the network competing with the wheat over supply and trade.

The prices at equilibrium in the local currencies for a ton of wheat in Lebanon and Egypt are:

$$\rho_3^1(d^*) = 772,769.1875 \text{ LBP } = \$511.0907, \quad \rho_6^1(d^*) = 9,722.3388 \text{ EGP } = \$507.6939$$

and for corn:

$$\rho_3^2(d^*) = 772,678.6875 \text{ LBP } = \$511.0308, \quad \rho_6^2(d^*) = 9,706.1885 \text{ EGP } = \$506.8505.$$

The demand prices for wheat are nearly similar to those in Example 5, as both countries highly depend on Ukrainian wheat imports. Furthermore, one can observe the similarly war-induced higher demand prices for corn, although less than wheat. It is also worth noting that even at an increased supply price of around \$114 at the farm's gate, Ukraine remains the supplier providing the cheapest corn in the world (UkrAgroConsult (2022)).

Example 8 - Example 7 with Quotas/Capacities of Product Path Flows

Example 8 has the identical data to the data in Example 7 except that now product path flow quotas/capacities are imposed at the following values:

$$\bar{Q}^1_{p_1} = \bar{Q}^1_{p_2} = 200,000.0000, \quad \bar{Q}^1_{p_3} = \bar{Q}^1_{p_4} = 100,000.0000,$$
$$\bar{Q}^2_{p_1} = \bar{Q}^2_{p_2} = 15,000.0000, \quad \bar{Q}^2_{p_3} = \bar{Q}^2_{p_4} = 600,000.0000.$$

This scenario is inspired by slowdowns in the processing of shipments of agricultural products even after the passage of the Black Sea Grain Agreement. Pre-war, there used to be around 40 inspections a day, but now, due to Russia's meddling, the number has decreased to, on the average, five inspections per day, and even with the establishment of the grain agreement, the Ukrainian Black Sea ports are used up to only half of their capacity (Reuters (2022)).

The modified projection method now converges to the following equilibrium multicommodity flow pattern:

$$Q^{1*}_{p_1} = 200,000.0000, \quad Q^{1*}_{p_2} = 0.0000, \quad Q^{1*}_{p_3} = 100,000.0000, \quad Q^{1*}_{p_4} = 14,006.8184,$$

$$Q_{p_1}^{2*} = 15,000.0000, \quad Q_{p_2}^{2*} = 0.0000, \quad Q_{p_3}^{2*} = 600,000.0000, \quad Q_{p_4}^{2*} = 0.0000.$$

From the numerical results, we see that the product flows for wheat are at their capacities on paths: p_1 and p_3, and the same paths are at their capacities for corn: p_1, p_3. In the case of wheat, we now have positive flow on path p_4, which was not the case in Example 7, but can be compared to Example 2. Clearly, the demand is sufficiently high that an alternative route is now being used. It can also be observed that the alternative route is not used for corn, which, again, is in accord with the high dependence of Lebanon and Egypt on Ukrainian wheat as compared to corn.

The demand market prices at equilibrium in the local currencies for a ton of wheat in Lebanon and Egypt are now:

$$\rho_3^1(d^*) = 779,762.5000 \text{ LBP } = \$515.7159, \quad \rho_6^1(d^*) = 9,975.9746 \text{ EGP } = \$520.9386$$

and for corn:

$$\rho_3^2(d^*) = 774,806.3750 \text{ LBP } = \$512.4380, \quad \rho_6^2(d^*) = 9,715.7002 \text{ EGP } = \$507.3472.$$

We see that the prices of wheat and corn rise in both Lebanon and Egypt under the quota/capacity regime relative to the respective prices in Example 7. However, the increase in the price of wheat is greater than that of corn, which can be traced back to the criticality of wheat as the main source of calories in the two countries. Additionally, this observation indicates that, for example, poor African countries, whose grain imports mostly consist of grain for food (e.g., wheat) and not much grain for feed (e.g., corn), are especially vulnerable to higher wheat prices. Furthermore, the supply price of wheat is at around \$95 relative to the respective supply price of about \$100 in Example 7. Similarly, the supply price of corn decreases from around \$114 in Example 7, to about \$112 in this example. Having such imposed quotas, equivalently, a reduction in capacity on routes has a negative impact on consumers in terms of prices and commodity availability.

6. Summary and Conclusions

International trade of commodities is essential to both producers and consumers. Various recent events of historical significance from the COVID-19 pandemic to Russia's war on Ukraine have demonstrated the criticality of trade for the availability of various commodities. Agricultural commodities, in particular, are necessary for the well-being of people and their food security.

In this paper, we take up the challenge of modeling the inclusion of exchange rates in a general spatial price equilibrium model of international trade. The model allows for multiple routes for commodities from origin countries to destination countries, and these routes, in turn, can consist of multiple transportation links through different countries. We show how exchange rates affect costs on links and paths as well as supply prices, and formalize the definition of the multicommodity spatial price equilibrium with exchange rates and under policies such as tariffs, subsidies, and quotas on commodity path flows.

The governing equilibrium conditions are formulated as a variational inequality problem in commodity path flows, and the existence of an equilibrium is established.

The numerical examples, for which complete input and output data are reported, are drawn from Russia's war on Ukraine, both illustrative examples as well as larger scale ones, which are solved using an implemented computational scheme. The flexibility of the modeling and algorithmic framework allows for the investigation quantitatively of the impacts of different scenarios with features of exchange rates plus various policies and the addition/deletion of markets and trade routes.

The results strongly confirm the importance of efficient transportation routes for trade and also the benefits of subsidies for agricultural trade for both farmers and consumers.

Possible research in the future may include extending the model to assess the competition among commodities with different densities over trade routes. An extended model could also include competition between agricultural and non-agricultural commodities over transportation. Additionally, robust models could be constructed to account for exchange rate fluctuations. Further extensions of the model to address additional issues of food security is also of relevance and interest.

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Data Availability

All the data (input and output) are reported in the paper.

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